I. INTRODUCTION

The concept of spectrum spears in many different areas of physics and of engineering sciences. For instance, a cantilever beam behaves like a vibrating chord (piano, guitar,…). Sine Pythagoras, we know that a system cannot oscillate at any random frequency, but is characterized by its eigenmodes. For a given length l, there exists a fundamental period proportional to \( 2\pi \), and harmonics of period proportional to \( 2\pi/\pi \), where \( \pi \) is an integer. Also, when the beam is set in motion, it will start vibrating according to a motion that can be decomposed into its eigenmodes. The corresponding amplitudes depend on the type of vibration. If we choose to represent the amplitudes on the y-axis and the corresponding frequencies on the x-axis, we get what we call a spectrum. As another example, let’s mention light and optical spectra. The light emitted from any lamp or object can be decomposed using a prism or grating spectroscope. Another example is radio waves, where we use an RCL circuit to filter out the undesired frequencies.

II. PHENOMENOLOGY AND OBJECTIVE OF THE EXPERIMENT

The experiment shows that the propagation of light has a wavelike and particle behavior. For a wave on the surface of water, the height of water is the oscillating amplitude. In the case of light, the oscillating quantities are the electromagnetic fields.

When in presence of two coherent (i.e. same frequency and non-random phase shift) point sources, we observe areas of space where the intensity is higher (maximum) and lower (minimum). We call this interference.

For example, for two independent coherent light sources, the diagram of figure 1 shows the interference process.

\( S_1 \) and \( S_2 \) are two point sources separated by a distance \( g \). The observer \( P \) located in \( r_1 \) and \( r_2 \) with respect to \( S_1 \) and \( S_2 \). If \( |r_1| - |r_2| \) is exactly equal to a wavelength \( \lambda \) or an integer multiple \( n \) of it, we can easily convince ourselves that the two waves arriving in \( P \) will add up to a maximum, whereas for a distance of \( \frac{\lambda}{2}, \frac{3\lambda}{2}, \frac{5\lambda}{2} \), etc., the waves will cancel out to a minimum.
Fig. 1: Schematic representation of the interference between two coherent light sources $S_1$ et $S_2$.

If $r_1$ and $r_2 >> g$, we have $|r_1| - |r_2| I \approx g \sin \theta$, where $\theta$ is the angle under which we see the sources.

The maxima condition is therefore: $g \sin \theta = n \lambda$

On the screen, we see an intensity distribution.

$$\cos^2 \left[ \frac{2 \pi g \sin \theta}{\lambda} \right]$$

We see that no interference take place if $g < \lambda$, or if $g >> \lambda$ since the maxima and minima are very close together. We therefore need $g$ to be larger, but of the same order of magnitude as $\lambda$.

The resolution, i.e. the thinness of the maxima can be considerably increased by using a grating, a collection of coherent sources separates by $g$ (fig. 2)

Similarly, we get:

$$g \sin \theta = n \cdot \lambda \quad \text{with an increased resolution}$$

Knowing the lattice parameter $g$, we can measure $\lambda$ and vice-versa. The grating allows us to select the desired wavelength from several different wavelengths. The atoms of matter can already be considered a grating ($g \approx$ a few $\text{Å}$) vis-à-vis X-rays ($\lambda \approx 1 \text{Å}$), and as was already previously mentioned, the described phenomenon can occur for any type of wave (stress, strain, acoustic, ...) A grating is called a spectroscope, and the whole process is called spectroscopy.
Fig. 2: The resolution increases with the number of coherent light sources

We generally call light, any electromagnetic wave of wavelength between $\lambda = 10^6 \text{ nm}$ (infrared) to $\lambda = 10 \text{ nm}$ (ultraviolet). The visible spectrum is somewhere between 380 nm and 760 nm. From a practical standpoint, and optical grating is a screen with a lot of thin equidistant slits (distance $\sim \lambda$) or better, a glass sample engraved with thin equidistant lines.

Using spectroscopy, A.J. Angstrom observed the hydrogen light of the sun in 1861, and noticed that they were made of 4 distinct lines. A careful study of this showed that the wavelengths of these rays followed a mathematical progression. In 1855, JJ Balmer, a mathematician from Basel, expressed the empirical law:

$$\lambda = \text{cst} \cdot \left( \frac{n^2}{n^2 - 2^2} \right) \quad n > 2$$

which explains the 4 lines that were found ($n = 3, 4, 5, 6$) and were used to predict a few more ($n>6$). He also predicted other series of lines by replacing 2 with another integer.

Balmer's law:

$$\frac{1}{\lambda} = R_H \cdot \left( \frac{1}{n_1^2} - \frac{1}{n_2^2} \right) \quad \text{avec} \quad n_1 \geq 1 \quad \text{et} \quad n_2 > n_1$$

$R_H = 109677.576 \pm 0.012 \text{ cm}^{-1}$ is the Rydberg constant for hydrogen.
This is one of the discoveries of the discontinuous characteristics of the emission spectrum of atoms that lead to the introduction of quanta and wave-particle duality.

The object of the experiment is to get familiar with grating spectroscopy: determine the grating constant, analysis of the emission spectra of different gases, measure of the Rydberg constant

**Experimental setup**

The grating $R$ is placed on a stand at a $L = 50$ cm of the measuring ruler $M$ (see Fig. 3). The origin of the ruler is placed on the light source $S$ that can be turned on with the button $B$. The lines are observed in the $\theta$ direction. Measuring $I$ and knowing $L$, we can determine $\theta$.

**III. SUGGESTED EXPERIMENTS**

1. Determine the grating parameter $g$ (calibration of the instrument (Fig. 4)) by using the mercury vapor source (wavelengths $\lambda_i$ indicated by the table of lines (see below). Calculate $g$ for each line. Compare the obtained results and calculate the average

<table>
<thead>
<tr>
<th>Element</th>
<th>$\lambda$ (nm)</th>
<th>color</th>
<th>intensity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hg</td>
<td>579.0</td>
<td>yellow</td>
<td>average</td>
</tr>
<tr>
<td>Hg</td>
<td>577.0</td>
<td>yellow</td>
<td>average</td>
</tr>
<tr>
<td>Hg</td>
<td>546.0</td>
<td>green</td>
<td>high</td>
</tr>
<tr>
<td>Hg</td>
<td>435.8</td>
<td>blue</td>
<td>high</td>
</tr>
<tr>
<td>Hg</td>
<td>407.8</td>
<td>purple</td>
<td>low</td>
</tr>
<tr>
<td>Hg</td>
<td>404.7</td>
<td>purple</td>
<td>high</td>
</tr>
</tbody>
</table>

2. Identify rays emitted by other sources (argon, neon, helium, etc.) and calculate their wavelengths using the average value of $g$ previously obtained. Compare these values with the table (Handbooks of physics and chemistry)

3. Identify the emission lines of hydrogen, and calculate their wavelengths. Then, calculate the Rydberg constant $R_H$ using Balmer’s law ($n_1 = 2$). For the red line $n_2 = 3$, for the green one $n_2 = 4$, and for the blue one $n_2 = 5$. Compare your results

4. Give a simple explanation of the separation of colors through a grating, as well as the order of interference $n$. How do you explain the colored band spectrum of the molecules? How do you explain the different values that one obtains for the same color, but different tubes? Give daily or technical applications of spectroscopy.
5. Determine the composition of a gas contained in a high pressure bottle. We will take a small amount of gas from a tube in order to perform a spectral analysis. The tube will have first been emptied using a pumping mechanism (fig. 5).

Procedure:

1) Place the tube (6) on the pumping mechanism.
2) Pumping out the tube: turn on the vane pump, open the valves (V1) and (V3). Pressure is measures using a bourdon and Pirani gauge (TM2, of the measuring unit) empty the circuit up to the gas bottle. Close the microvalve (5).

*Caution: the microvalve is closed when it can turn freely.* Do not force

When the pressure drops below \(5 \times 10^{-2} \text{ mbar}\), turn on the turbo-pump (START button on the control panel, the pump needs to accelerate before reaching a stationary state of 40'000 rpm).

3) After approx. 30 min, the vacuum is of the order of \(10^{-5}\) mbars, close the plate valve (V3) and turn off the turbo-pump with the STOP button. Using the microvalve (5) introduce approximately \(1 \text{ mbar}\) of gas in the “tube for spectral analysis” (pressure is measured using the Pirani gauge (TM2). Close the valve V1, turn off the vane pump.

*Caution:* Never let the turbo-pump run on an atmosphere greater than \(5 \times 10^{-2} \text{ mbar}\). For this purpose, we can use the by-pass mechanism (valve V2). Think before acting, or ask your TA.

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**Fig. 4:** Photo of the experimental setup

**Fig. 5:** High vacuum setup
ANNEXE: Série de Fourier: quelques rappels

Soit \( f : \mathbb{R} \rightarrow \mathbb{R} \) une fonction périodique de période \( T \):
\[
f(t + T) = f(t), \quad \forall t.
\]
On se pose la question suivante: est-il possible de décomposer \( f(t) \) en une somme de fonctions simples admettant la même période? Par fonctions simples, on entend
\[
\sin\left(\frac{2\pi}{T} nt\right), \cos\left(\frac{2\pi}{T} nt\right).
\]
ces deux fonctions sont périodiques, de période \( T \).
On se pose alors la question suivante: existe-t-il des coefficients \( a_n, b_n \) \((n \in \mathbb{N})\) tels que
\[
f(t) = b_0 + \sum_{n=1}^{\infty} \left[ a_n \sin\left(\frac{2\pi}{T} nt\right) + b_n \cos\left(\frac{2\pi}{T} nt\right)\right]
\]
(1)
Il est possible de montrer que:
\[
a_n = \frac{2}{T} \int_{T/2}^{T/2} f(t) \sin\left(\frac{2\pi}{T} nt\right) \, dt \quad \text{(2)}
\]
\[
b_n = \frac{2}{T} \int_{T/2}^{T/2} f(t) \cos\left(\frac{2\pi}{T} nt\right) \, dt
\]
\[
b_0 = \frac{1}{T} \int_{T/2}^{T/2} f(t) \, dt
\]
En utilisant les identités suivantes:
\[
\forall d, \int_{-T/2}^{T/2} \sin\left(\frac{2\pi}{T} t\right) \sin\left(\frac{2\pi}{T} t\right) \, dt = \frac{T}{2} \delta_m
\]
\[
\forall d, \int_{-T/2}^{T/2} \cos\left(\frac{2\pi}{T} t\right) \cos\left(\frac{2\pi}{T} t\right) \, dt = \frac{T}{2} \delta_m
\]
\[
\forall d, \int_{-T/2}^{T/2} \sin\left(\frac{2\pi}{T} t\right) \cos\left(\frac{2\pi}{T} t\right) \, dt = 0
\]

Les \( a_n \) et \( b_n \) sont appelés les coefficients de Fourier de \( f \) et la série (1) série de Fourier.

**Remarque:** Contrairement aux coefficients de Taylor, ceux de Fourier ont un caractère non local: la connaissance de l’un quelconque des \( a_n, b_n \) nécessite celle de la fonction \( f \) sur tout l’intervalle compris entre \(-T/2 \) à \(+T/2 \) (c’est-à-dire partout puisque \( f \) à la période \( T \)).

Le lecteur pourra se convaincre que a) si \( f \) est paire, tous les \( a_n \) sont nuls, b) si \( f \) est impaire, tous les \( b_n \) sont nuls. Il pourra, à titre d’exercice, calculer les coefficients de Fourier de \( f(x) = \cos^2(x) \) ou ceux de la fonction "crâneau".

Si \( f \) est une fonction de période \( T \), l’ensemble de ses coefficients de Fourier constitue son spectre. On le représente graphiquement:

```
   \[ a_n \]
   \[ b_n \]
   n
```

Les termes correspondant à \( n=1 \) sont dits *fondamentaux* et pour \( n>1 \) on parle d’*harmoniques*. 