H3. Millikan's Experiment

I. OBJECTIVE OF THE EXPERIMENT

Can electric charge appear in any quantity, or only as an integer multiple of a fundamental unit, or quantum?

Many experiments have been thought up to answer this question. The most classical is that of the American physicist Robert A. Millikan (1869-1953), **the oil drop experiment.**

By observing the vertical motion of charged oil droplets in an electric field, he managed to find the value of the elementary charge, of which all electric charges in nature are multiples. This successful experiment was the reason for his Nobel Prize in 1923.

The objective of this experiment is to recreate Millikan's experiment in order to show the quantification of charges.

II. PRINCIPLES

In 1910, R.A. Millikan successfully proved the quantum occurrence of small amounts of electricity with his famous oil droplet method. He observed charged oil droplets in the vertical electric field of a plate capacitor with plate spacing d. From their radius r and the electric field $E = U/d$ (where U is the voltage applied to the plates), he determined the charge q of a suspended droplet. He found that q was always an integer multiple of an elementary charge e, i.e. $q = n \cdot e$.

If an oil droplet with radius r descends (falls) at a speed $-v_1$ with no electric field E applied, then the earth's gravitational force F_g acting on this oil droplet is countered by Stokes' friction $F_1 = 6\pi \cdot \eta \cdot r_0 \cdot v_1$ (where η is the viscosity of air). Since the speed v_1 is constant (acceleration $a = 0$), the force equilibrium applies:

$$0 = F_g + F_1 \tag{1}$$

If the same oil droplet rises at the speed v_2 in an applied electric field E, then the opposing Stokes' friction force is $F_2 = - 6\pi \cdot \eta \cdot r_0 \cdot v_2$. The force equilibrium still applies, but only under consideration of the electric force $F_E = q_0 \cdot E = q_0 \cdot U/d$ (where q_0 is the charge of the oil droplet). We obtain:

$$0 = q_0 \cdot U / d + F_g + F_2 \tag{2}$$

If we subtract the two equations from each other, the gravitational force F_g is eliminated and we obtain overall:

$$0 = F_E - F_1 + F_2 = q_0 \cdot U / d - 6\pi \cdot \eta \cdot r_0 \cdot (v_1 + v_2) \tag{3}$$

From this we thus obtain:

$$q_0 = 6\pi \cdot \eta \cdot r_0 \cdot (v_1 + v_2) \cdot d / U \tag{4}$$

In order to determine the charge q_0, only the radius r_0 of the observed oil droplet is missing. However, this can be easily deduced from the force equilibrium of its resulting weight force $F_g = V \cdot \Delta\rho \cdot g$ and the

Stokes' friction F_1 when the droplet is descending, where $\Delta\rho$ is the difference in density between oil and air. The following thus holds:

$$0 = F_g + F_1 = (4/3 \cdot \pi \cdot r_0^3) \cdot \Delta\rho \cdot g - 6\pi \cdot \eta \cdot r_0 \cdot v_1 \tag{5}$$

$$r_0 = \sqrt{\frac{9 \cdot \eta \cdot v_1}{2 \cdot \Delta\rho \cdot g}} \tag{6}$$

For a more accurate determination of the charge q (the actual charge of the droplet), one should take into account that the Stokes' friction must be corrected for very small radii r since they are then of the order of magnitude of the mean free path of the air molecules and air can thus no longer be regarded as a homogeneous medium. The corrected formula for the friction force as a function of the air pressure p is:

$$F = \frac{6\pi \cdot \eta \cdot r \cdot v_1}{\left(1 + \frac{b}{r \cdot p}\right)} \tag{7}$$

where b = 80 µm·hPa (constant). Using the short form, $A = b/p$, the corrected radius r is obtained as

$$r = \sqrt{r_0^2 + \frac{A^2}{4}} - \frac{A}{2} \tag{8}$$

and the corrected charge q is obtained as

$$q = \frac{q_0}{\left(1 + \frac{A}{r}\right)^{1.5}} \tag{9}$$

III. THE EXPERIMENTAL SETUP

The experiment setup is shown in Fig. 1.

– To adjust the microscope, remove the capacitor with the cover. Then touch the fixation point with your finger in the centre of the assembly vertically from above (or better use a paper with text) and use the large black wheels at the bottom of the microscope to adjust the focus. See also Fig. 2.
– Rotate the eyepiece to align the scale of eyepiece vertically.
– Reattach the capacitor.
– Turn the voltage regulator on the left side of the Millikan supply unit to the left so that no voltage is applied.
– Connect the voltage output of the Millikan supply unit (upper left) to the capacitor plates, connecting the upper plate to the "+" contact. Use the golden plugs for the capacitor only if using protective ground contacts.
– Caution: voltages of up to 600 V may be present at the capacitor contacts. In this setup, the voltage is not dangerous in case of contact, but may nevertheless be unpleasant in the event of a short circuit due to contact.
– Connect each of the "START/STOP" stopwatch inputs to output 1/2 of the Millikan supply unit. Make sure that both earth conductors are connected to each other.
– Supply voltage to the light of the Millikan apparatus.
– Finally, supply power to the stopwatches and the Millikan supply unit, using the included power adaptors.

Required Numerical Values for Calculations

Distance between capacitor plates: $d = (6.00 \pm 0.05) \cdot 10^{-3}\ \mathrm{m}$.

Density of the droplets: $\rho = (877 \pm 1)\ \mathrm{kg/m^3}$

Air viscosity:

at 0°: $\eta_0 = (1.708 \pm 0.001) \cdot 10^{-5}\ \mathrm{Ns/m^2}$

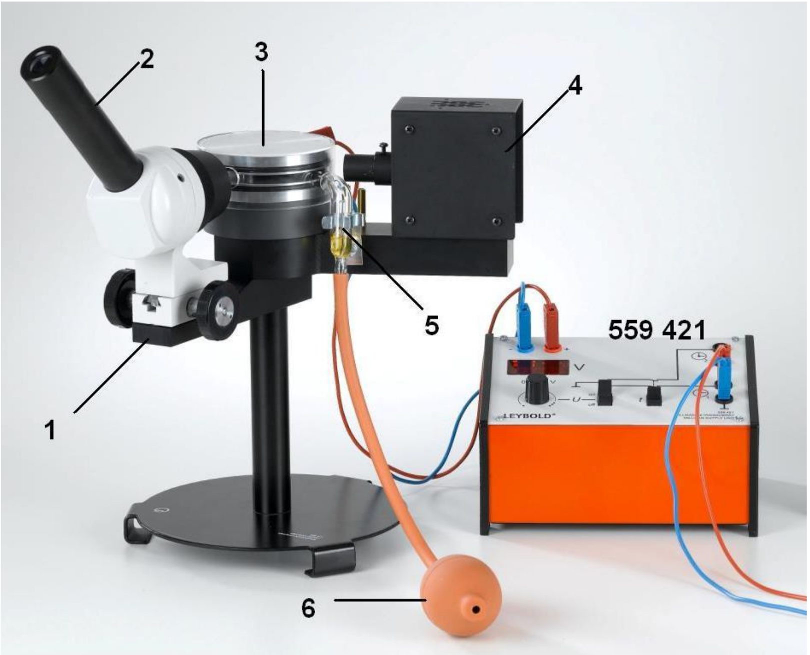
at 18°: $\eta_{18} = (1.837 \pm 0.001) \cdot 10^{-5}\ \mathrm{Ns/m^2}$

at 20°: $\eta_{20} = (1.855 \pm 0.001) \cdot 10^{-5}\ \mathrm{Ns/m^2}$

at 40°: $\eta_{40} = (1.904 \pm 0.001) \cdot 10^{-5}\ \mathrm{Ns/m^2}$

Acceleration of gravity :

$$g = (9.81 \pm 0.01)\ \mathrm{m/s^2}$$



1 Base plate
2 Measuring microscope
3 Plate capacitor
4 Illumination device
5 Oil atomizer
6 Hand pump

Fig. 1 : Image of the setup for Millikan's experiment

Fig. 2 : Adjustment of focus with thumb.

IV EXPERIMENTAL PROCEDURE

This experiment requires intense focus. Caution is also required during the experiment. Do not spill, inhale, or drink the oil. Only use the supplied oil.

– First, switch all switches on the Millikan supply unit to the bottom.

– Disperse the oil by forcefully pressing the hand pump into the capacitor. If there is insufficient oil between the capacitor plates during the experiment, additional oil can be injected at any time.

– Connect the input, E, of the counting device, P, to output 1 of the Millikan supply unit. Connect the input, F, of the counting device, P, to output 2 of the Millikan supply unit. Make sure that both earth conductors are connected to each other.

– Press "MODE" (field top left) twice to set the measurement to "$t_{E,F}$".

– Switch on the voltage in the Millikan supply unit ("U" switch) and roughly select a desired voltage at which rising oil droplets are visible. The display now shows the voltage applied.

– Record the applied voltage in a table.

– On the counter, press the "STOP", "→ 0 ←", and then the "START" buttons (all in the top right field) to pause any measurements in progress, clear the memory, and prepare for a new measurement.

– When the oil droplet crosses a large graduation mark, turn the "t" switch of the Millikan supply unit upwards to start the measurement.

– As soon as the oil droplet has crossed 20 graduation marks, i.e. 2 larger marks (corresponding to 1 mm), switch the voltage switch to measure the descent time on the second channel.

– Carry out the previous step another 3 times.

– To finish the measurement, switch the "t" switch instead of the "U" switch at the lower graduation mark.

– Alternatively, instead of directly switching the voltage at the upper or lower graduation mark, the "t" switch can first be switched down, then the voltage can be switched over. The time measurement can then be reactivated when the graduation mark is passed again. This avoids errors due to delays in the switching moment of the voltage.

– Repeat the above measurement for additional oil droplets. At the beginning of each measurement, make sure that both switches of the Millikan supply unit are at the bottom.

– After the first half of the measurements, switch off the voltage and interchange the contacts on the capacitor; then switch the voltage on again. Now observe oil droplets with the opposite charge. The aim is to have around 30 to 50 droplets measured for the voltage U_1.

– Select a new droplet and make 3 measurements of q for a different voltage. Vary the voltage and repeat the measurement on the same droplet. Should the droplet charge depend on the voltage between the condenser plates?

Points to discuss:

- Compare all values of q_i determined this way, and try to highlight integer multiples n_i of the elementary charge ($e = 1.6022 \cdot 10^{-19}$ C). Idea: Represent all the measured q_i values in a histogram. Discuss the bin size.
- Determine from your experiment your value of the fundamental charge.
- Discuss about the precision of n_i .
- Are there values of n_i for which it isn't possible to show charge quantification q_i ?

Optional:

- Visually estimate the diameter of a droplet, and compare it to the value obtained from equation (8). What do you think?

Comments:

1) Sometimes, the droplet leaves the vertical plane where it is supposed to go up and down. Its sharpness decreases. This can be corrected by adjusting the focus of the microscope

2) Thermal agitation of the air molecules in the capacitor do slightly interact with the system, but shouldn't have too much of a negative impact on the numerical results

V. BIBLIOGRAPHY

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