

## *H2. Schuster Experiment: Electron Specific Charge $e/m$*

### **I.OBJECTIVE OF THE EXPERIMENT**

We will study the behavior of an electron beam in an electromagnetic field. We will determine the specific charge  $e/m$  of the electron by measuring the radius of the circular motion of the beam in a magnetic field.

### **II. BASIC THEORY**

A particle of charge  $q$ , mass  $m$ , and velocity  $\vec{v}$  in an electromagnetic field feels a force:

$$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B}) \quad (\text{Force de Lorentz}) \quad (1)$$

where  $\vec{E}$  is the electric field, and  $\vec{B} = \mu_0 \vec{H}$  the magnetic induction with  $\mu_0 = 4 \cdot \pi \cdot 10^{-7}$  V · s/(A · m) the magnetic permeability in a vacuum, and  $\vec{H}$  the magnetic field

#### **1. Acceleration of an electron in a uniform electric field**

An electron, or any charged particle, can be accelerated by a uniform electric field  $\vec{E} = -\vec{\nabla}U$  generated between two parallel planes of a capacitor, of voltage  $U$  (fig. 1a). Over the course of the acceleration, the electron acquires a kinetic energy  $E_c$  (conservation of energy):

$$E_c = \frac{1}{2}mv^2 = \int \vec{F} \cdot d\vec{r} = \int q\vec{E} \cdot d\vec{r} = qU \quad (2)$$

#### **2. Motion of the electron in a constant and uniform magnetic field**

In this case, the only acting force is the magnetic force  $\vec{F} = q\vec{v} \times \vec{B}$ . We shall let the students prove that the electron's trajectory in a uniform  $\vec{B}$  field, in the absence of an electric field, is a helix (fig. 2) whose rotational frequency  $\omega$  is given by

$$\omega = \frac{q}{m} B \quad (3)$$

$\omega$  is also called *cyclotron frequency*.

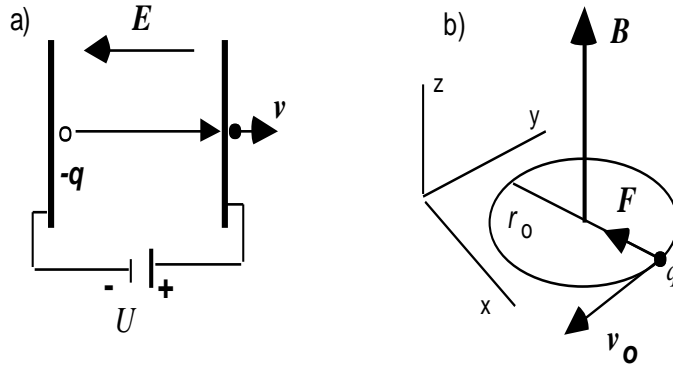


Fig.1 a) Acceleration of a negatively charged particle in a constant electric field  
b) Motion of a charged particle in a uniform and constant magnetic field

**a) Uniform circular motion.** Show that for  $v_z = 0$ , the Lorentz force keeps the electrons on a circular trajectory of radius  $r_0$  at a speed  $v_0$  in the  $xy$ -plane (fig. 1b). The radius  $r_0$  of the circular orbit is given by:

$$r_0 = \frac{mv_0}{qB} \quad (4)$$

From (2), the speed  $v_0$  is determined from the acceleration voltage  $U$ , which yields:

$$v_0^2 = v^2 = \frac{2qU}{m} \Rightarrow r_0^2 = \left(\frac{m}{q}\right) \frac{2U}{B^2} \quad (5)$$

Therefore, we can determine  $\left(\frac{m}{q}\right)$  experimentally by measuring  $r_0 = f(U, B)$ .

**b) Helical trajectory.** If  $v_z \neq 0$ , the motion of the particle is a helix of constant step  $l$  whose axis is parallel to the  $\vec{B}$  field. In this case, we can split the velocity  $\vec{v}$  into its two components:  $\vec{v}_0$  perpendicular to, and  $\vec{v}_p$  parallel to the  $\vec{B}$  field. Interestingly enough, the time of a revolution of the particle is  $T = 2\pi/\omega$  which is independent of the radius  $r$  and the velocity  $\vec{v}_0$  of the particle.

$$T = \left(\frac{m}{q}\right) \frac{2\pi}{B} \quad (6)$$

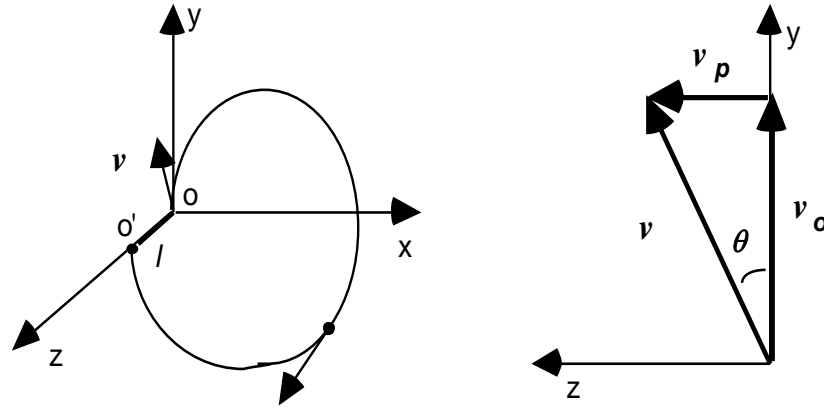


Fig. 2 Helical trajectory of a charged particle in a magnetic field. The rotation direction depends on the sign of the charge. The velocity  $\vec{v}$  can be separated into two orthogonal components

Once we know the period of revolution, we can determine  $q/m$ . However, the measure of  $T$  is rather complicated. It is altogether easier to measure the step of the helix  $l = \overline{OO'}$  which is given by  $l = v_p T$ , and yields:

$$\left(\frac{q}{m}\right) = \frac{2\pi v_p}{B l} \quad (7)$$

therefore, knowing  $\vec{v}_p$  and  $l$  is enough to determine  $q/m$ .

### III. EXPERIMENTAL SETUP

A sphere contains hydrogen at low pressure ( $p \approx 10^{-2}$  Torr), in which we produce a small beam using an electron cannon (fig. 3 and 4). In the cannon, the electrons that are thermally emitted from a hot filament  $F$  are accelerated by a continuous voltage  $U$  between 100–300 V applied from the (conic) anode and the filament. A metallic cylinder  $C$  is brought to a negative voltage in order to focalize the electrons.

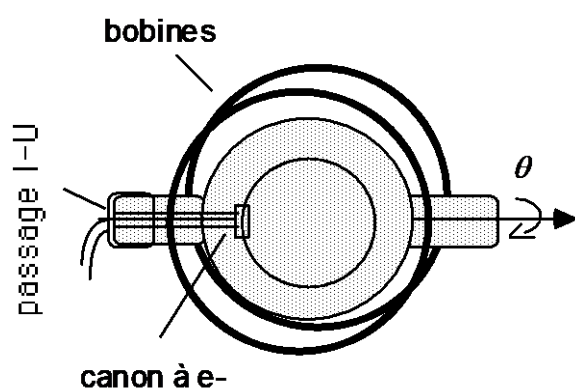
The magnetic field  $\vec{H}$  is generated using Helmholtz coils around the sphere. The property of Helmholtz coils is that the radius of the coils is equal to the distance separating them, which makes the field in the center of the system relatively homogenous. We can show that the field in the centre of the system is equal to:

$$H = \left(\frac{4}{5}\right)^{3/2} \frac{NI}{R} \quad (8)$$

Number of turns:  $N = 130$ ;  
Average radius of the coils:  $R = 0.150 \pm 0.001$  m

However, the induction off of the axis is slightly greater than that in the center. The maker of the coils suggests a correction factor  $f_c$  to better illustrate the observed phenomena:

$$H_c = f_c \cdot H \quad \text{with} \quad f_c = 1.03 \pm 0.01$$



Canon à électrons

Fig. 3 Experimental setup. Helmholtz coils and electron cannon.

Available material

- 1 Glass sphere filled with rarefied hydrogen and an electron cannon, mounted on a stand
- 1 "Leybold" power supply for the acceleration voltage  $U$ .
- 1 "Topward Electric" power supply for the current  $I$  in the coils
- 1 Continuous voltmeter (0 – 250 V)
- 1 Amperemeter (0 – 2 A).
- 1 Optical bench (0.5 m).
- 2 Aiming slits

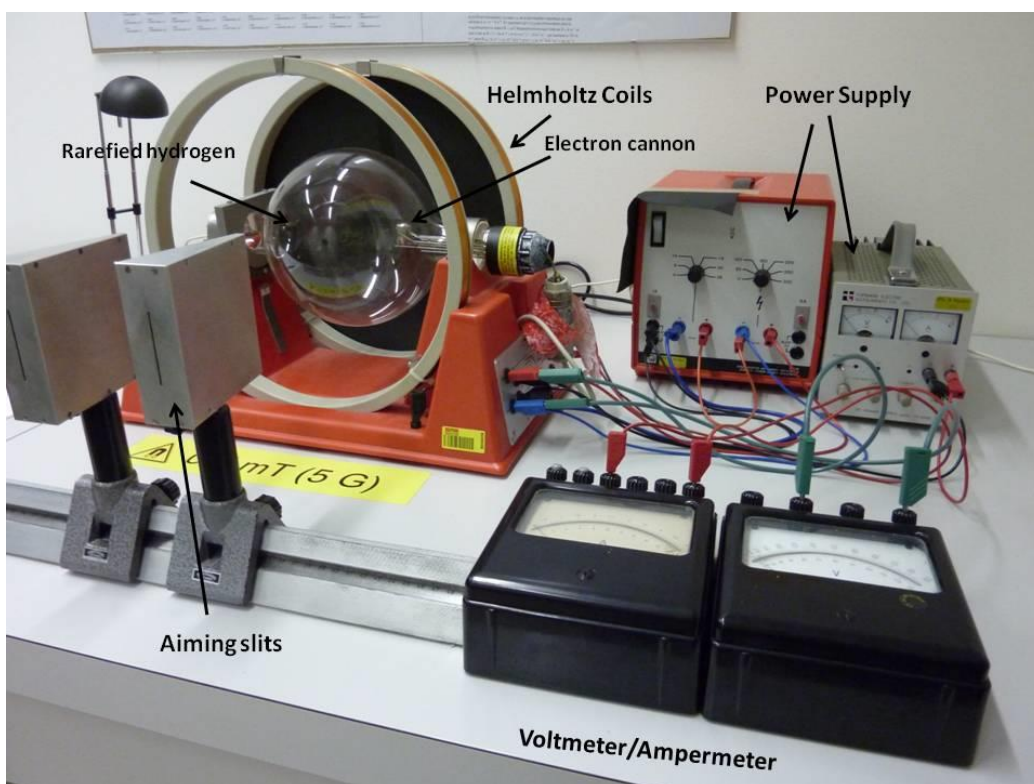


Fig. 4: Image of the setup

Using the setup

- 1) Turn on the “Leybold” power supply. Once the cathode is heated to a light red (approx. 5 min.), increase the voltage  $U$  of the power supply progressively until reaching the electron beam reaches the sides of the tube. Optimal voltage:  $\approx 170 \text{ V}$ . **Never go over 250 V**.
- 2) Turn on the “Topward Electric” power supply. Increase the power supply until the electrons describe a full circle (because of the magnetic field  $H$ ,  $I \approx 1.5 \text{ A}$ ). **Never go over 2 A**.
- 3) Place the optical bench parallel to the circle. The aiming slits allow you to measure the diameter of the circle

Powering the setup down

- 1) Bring all buttons back to their initial position WITHOUT FORCING!
- 2) Turn off all power supplies, and the main plug.

For the lab report, be sure to make an electric diagram of the setup

**IV. SEGGESTED EXPERIMENTS****Observations:**

- i) The magnetic field  $H$  generated by the coils, is joined by the earth magnetic field  $H_T$  whose intensity is  $36 \text{ A/m}$ . Using a compass, determine the direction of the earth's field  $H_T$  with respect to the  $H$  field's direction. Estimate the horizontal component of  $H_T$  as well as its effect on  $H$ . Do these estimates for  $I = 0.8 \text{ A}$  et  $I = 2 \text{ A}$ .
- ii) Turn on the setup by following the above instructions. Rotate the glass tube delicately about its axis ( $\theta \leq 30^\circ$ ). **Don't touch** the end of the tube on which there are high voltages and currents.
  - a) Observe the helical motion of the electrons, and determine the direction of the  $\vec{B}$  field.
  - b) Verify that the step of the helix  $l$  doesn't depend on  $v_p = v \sin \theta$ .

**Determining  $e/m$  :**

Measuring  $e/m$  can be done in specific geometric condition in order to get simple equations.

- 1) **Circular trajectory.** Calculate  $e/m$  using equation (5). For this, orient the axis of the tube in order to launch the electron beam at a velocity  $\mathbf{v}$  perpendicular to the  $\vec{B}$  field. Choose an adequate linear representation, and calculate  $e/m$  from the slope.
  - a) Measure  $r = f(B)$ . For 3 different voltages  $U$  (e.g. 150, 200, 240 V), measure the diameter of 10 to 15 different orbits by varying the current  $I$  in steps of  $\Delta I = 0.1 \text{ mA}$ , starting from  $2 \text{ A}$

- b) Measure  $r = f(U)$ . For a fixed  $H$  field (e.g.  $I \approx 1.2 \text{ A}$ ) do the same measures by varying  $U$  from 140 to 240 V by intervals of  $\Delta U = 10 \text{ V}$ .
- c) Plot all values of a) and b) on a single graph  $r = f(\sqrt{U}/I)$ . Determine  $e/m$  and talk about the expected precision of the measure.
- d) Calculate the precision using a single measure according to (5), and compare it to the precision of a), b) and c).

*Comment:* Do not keep the electron cannon running uselessly for extended periods of time. turn it off during breaks

- 2) **Helix step.** Estimate  $e/m$  from equation (7). in our case, the parallel velocity  $\vec{v}_p$  is obtained by rotation of the electron cannon by a given angle  $\theta$  with respect to  $\vec{B}$ , so its magnitude is given by  $\vec{v}_p = v \sin \theta$  where  $v$  is defined by (2). However, the precise value of  $l$  is harder to calculate.

$l$  can be obtained rather easily, either by measuring both the orbit's radius  $r_0$  for  $\theta = 0$  and the radius  $r_\theta$  of the helical orbit for  $\theta \neq 0$  while keeping  $U$  and  $I$  constant, or also by measuring directly  $r_\theta$  and  $\theta$ . In this case,  $l$  respectively given by

$$l = 2\pi(r_0^2 - r_\theta^2)^{1/2} \quad \text{or} \quad l = 2\pi \cdot r_\theta \cdot \tan \theta \quad (9)$$

equation (7) becomes

$$\begin{aligned} \frac{e}{m} &= \left( \frac{2\pi}{Bl} \right) 2U \sin^2 \theta = \frac{2U}{B^2} \frac{\sin^2 \theta}{r_0^2 - r_\theta^2} \\ \frac{e}{m} &= \frac{2U}{B^2} \frac{\cos^2 \theta}{r_\theta^2} \end{aligned} \quad (10)$$

- a) Fix  $U$  and  $I$  and measure the radius  $r_0$ . rotate the tub in order to see 1 or 2 revolutions (see fig. 2) and measure  $r_\theta$  and the angle  $\theta$ . Calculate  $e/m$  with both methods from (10).
- b) Compare these values, with those found in 1). Compare the experimental values with those indicated in the literature

*Comment:* Do not keep the electron cannon running uselessly for extended periods of time. turn it off during breaks

**Exercises**

- 1) Calculate the electron velocity  $v$  acquired in the capacitor for  $U = 250$  V.
- 2) An electron is in a uniform magnetic field  $B = 15$  Gauss with a speed of  $10^7$  m/s at an angle of  $30^\circ$  with respect to the field. Calculate the radius  $r$  and the step  $l$  if the helical trajectory.

**LITERATURE:** Physique générale II: de Douglas C. Giancoli, ou Alonso Fin  
Cours de physique