H10. Black Body Radiation

I. OBJECTIVE OF THE EXPERIMENT

Any body whose temperature exceeds $0\ \mathrm{K}$ emits electromagnetic radiation. The energy lost through this phenomenon can be absorbed by the environment; in particular, another body can receive this energy. The energy transfer from one body to another depends on the geometry (shape as well as relative position), the temperature difference between the bodies, and other properties.

In this experiment, we will look to **measure the power radiated by an oven** at a temperature T using a detector held at room temperature. By setting up a system ingeniously, we can use these measures to verify the **Stefan-Boltzmann law**, and determine the conversion factor of the detector. Using a second experimental setup with a spherical geometry, we will be able to determine the **Stefan-Boltzmann constant**.

II. INTRODUCTION: THERMAL RADIATION

In the history of physics, the "black body experiment" played a very important role: it's one of the experiments that questioned classical physics around the turn of the century, and took part in defining the basic quantum hypotheses.

What is a black body?

A black body is an ideal object whose property is to **absorb all incident thermal radiation**, independently of their wavelength. Likewise, it is also the **best radiation emitter**; its spectrum is continuous, and for a given temperature, no body will emit a more intense radiation.

A real body whose emitted spectrum is continuous, but whose radiation intensity is lower than expected, is called a grey body

Basic theory

Towards the end of the 21st century, Rayleigh and Jeans studied the theoretical black body, using classical statistical physics as their basis. Their results lead to the following relationship between the energy density U and the emitted frequency υ

$$U_{RJ}(\upsilon) = \frac{8\pi \cdot k \cdot T}{c^3}\upsilon^2 \qquad (1)$$

where U is the energy density, T the black body's temperature, c the speed of light, k the Boltzmann constant and υ the frequency.

However, the experimental result $U_{\exp}(\upsilon)$ did not follow this equation (Fig. 1).

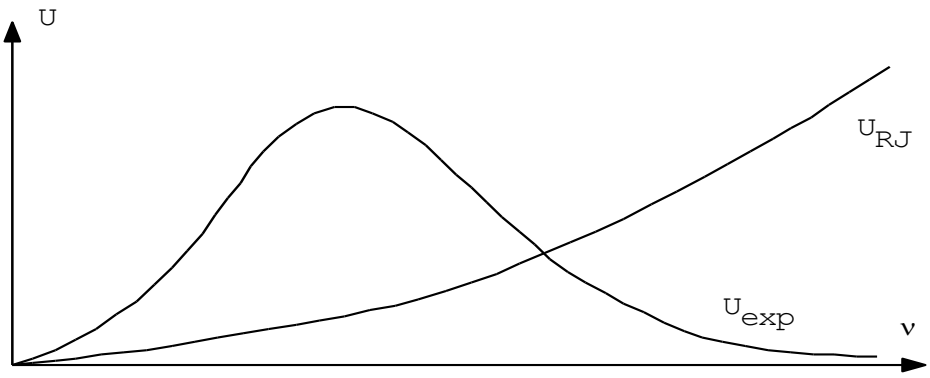


Fig.1 Comparison between Rayleigh and Jeans' calculations (U_{RJ}) and the measured energy density (U_{exp})

In particular, $\upsilon \to \infty$ yields $U_{exp} \to 0$, but $U_{RJ} \to \infty$

This was called the "ultraviolet catastrophe".

Planck repeated the study of black body radiation, with the added hypothesis that the energy of the emitted or absorbed radiation of frequency ν is **quantized**, and equal to:

$E = h\upsilon$ where $h = 6.625 \cdot 10^{-34}\ J \cdot s$ is Planck's constant.

This leads to the following black body spectral distribution: $U(\upsilon) = \frac{8\pi h}{c^3} \frac{\upsilon^3}{e^{h\upsilon/kT} - 1}$ or rather, expressed as a function of the wavelength $\lambda = c/\upsilon$

$$U(\lambda) = \frac{8\pi hc}{\lambda^5} \frac{1}{e^{hc/\lambda kT} - 1} \tag{2}$$

which corresponds perfectly to the experimental value.

Integrating this expression over all wavelengths yields a radiated power per unit surface of the body P_N given by:

$$P_N = \frac{2\pi^5 k^4}{15 h^3 c^2} \cdot T^4 = \sigma \cdot T^4 \quad \text{in } W/m^2 \tag{3}$$

where T is the temperature of the emitting surface in Kelvin.

This equation is called **Stefan-Boltzmann law**. Please note that the **Stefan-Boltzmann** constant σ, depends only on three physical constants k, h and c, and is equal to:

$$\sigma = 5.67 \cdot 10^{-8} \quad Wm^{-2}K^{-4} \tag{4}$$

III. DESCRIPTION OF THE EXPERIMENTAL SETUP

III.2 ORTHOGONAL GEOMETRY

We would like to measure the power emitted by the rear surface of a ceramic oven. In order to do so, the available tools must be set up correctly. The **geometric configuration can greatly alter the measurements**. The radiation detector must be exactly aligned with the oven (Fig. 2).

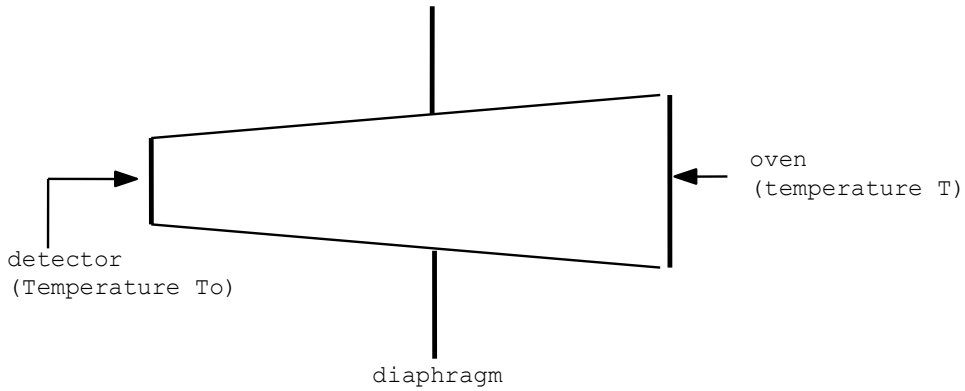


Fig.2 Orthogonal geometry setup

The diaphragm makes sure that the radiation received by the captor comes exclusively

Calculating the power recorded by the detector (Fig. 3)

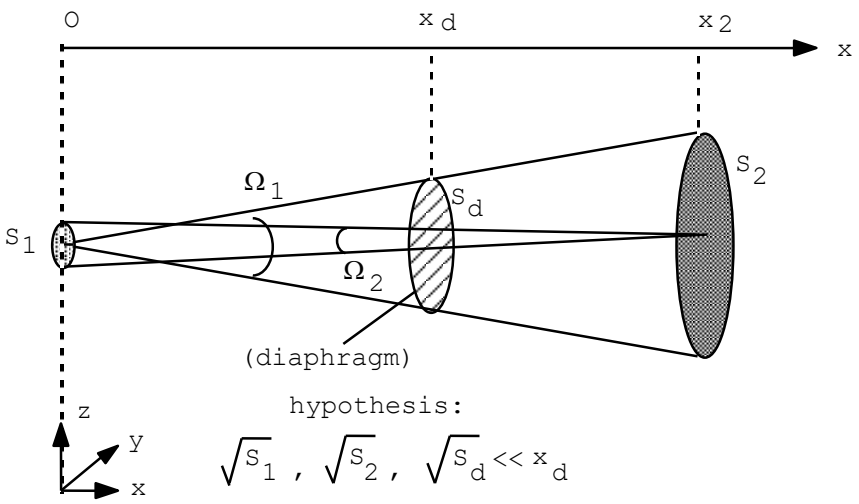


Fig. 3: Definition of solid angles and surfaces

In the half space defined by the emitting surface and the surface of the detector, the oven's emitted power equals $P_{Four} = S_2 \sigma T^4$. The power emitted by the oven (at a temperature T) that is received by the detector is contained in the solid angle Ω_2. so:

$$P_{Four \to Det} = \frac{\Omega_2}{2\pi} \cdot S_2 \sigma T^4$$

Using an analogous reasoning, we find that the power radiated from the detector (at a temperature T_0) to the oven $P_{Det \to Four}$ is contained in the solid angle Ω_1 :

$$P_{Det \to Four} = \frac{\Omega_1}{2\pi} \cdot S_1 \sigma T_0^{\,4}$$

In addition to that, the surface S_1 (detector) receives radiation from:

- objects and walls at a temperature T_0 in the half space $x > 0$, out of the solid angle Ω_1: $P_{Obj \to Det}$
- the lamps and people around the experiment: P_{Obj}.

on the other hand, the surface S_1 also emits radiation to all surrounding objects with x > 0, out of the solid angle Ω_1: $P_{Det \to Obj}$.

If we consider all bodies to be black bodies, and since the walls and objects are all at room temperature T_0, we can state $P_{Obj \to Det} = P_{Det \to Obj}$. We are left with:

$$P_{Total} = P_{Four \to Det} + P_{Obj} - P_{Det \to Four} \qquad (5)$$

Finally, defining the solid angles:

$$\Omega_1 = \frac{S_2}{x_2^2} = \frac{S_d}{x_d^2}, \Omega_2 = \frac{S_1}{x_2^2} \Rightarrow \Omega_1 S_1 = \Omega_2 S_2 \text{ , yields:}$$

$$P_{Total} = \frac{S_1 S_d}{2\pi x_d^2} \sigma \left(T^4 - T_0^4 \right) + P_{Obj} \qquad (6)$$

Detection. The detector converts the received power P_{Total} in a voltage V_T, according to a conversion factor C_{Det} such that $V_T = C_{Det} \cdot P_{Total}$ i.e.:

$$V_T = C_{Det} \cdot P_{Total} = C_{Det} \cdot \frac{S_1 S_d}{2\pi x_d^2} \sigma \left(T^4 - T_0^4 \right) + V_{Obj} \qquad \text{in volts}$$

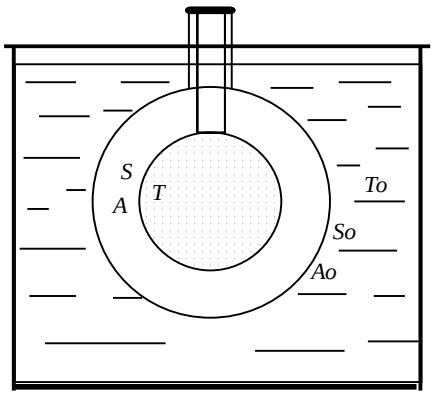
where	$V_{Obj} = C_{Det} P_{Obj}$	Offset due to lamps and people
	S_1	Surface of the detector
	S_d	Surface of diaphragm opening
	x_d	Diaphragm – detector distance
	C	Detector conversion factor
	T	Oven temperature
	T_0	Room temperature

III.2 SPHERICAL GEOMETRY

For two concentric opaque spheres (Fig. 4), of surface S and S_0, and temperatures T_0 and T, with average absorption factors A_0 and A close to 1, the effective radiated power is given by (to be shown by student):

$$P_T^{emis} = P_{in\to ext}^{emis} - P_{in\to ext}^{reflechit} - AP_{ext\to in}^{emis} = AA_0 S\sigma\left(T^4 - T_0^4\right) \tag{7}$$

Therefore, is we know $A_0 = 0.96$ and $A = 0.91$, then measuring the emitted energy as a function of time is enough to determine the Stefan-Boltzmann constant.



Experimental conditions:

i) the outer sphere is kept in a thermal reservoir (water basin), so T_0 is constant (room temperature)

ii) the inner sphere (filled with water) is at a slightly higher temperature T, with $T = T_0 + \theta$, with $\theta \ll T_0$, to make calculations easier.

Fig.4 Experimental setup: two concentric glass spheres

a) Evacuated container

If a vacuum is kept between both spheres, the thermal energy transfer is solely due to radiation, so the thermal energy variation is given by:

$$\kappa\frac{d\theta}{dt} = -P = -\alpha S\sigma\left(T^4 - T_0^4\right) \tag{8}$$

where αAA_0 and $\kappa = \kappa_{eau} + \kappa_{verre}$ is the heat capacity of the internal sphere.

If we approximate $\theta \ll T_0$, equation (8) becomes (2nd order expansion)

$$\kappa\frac{d\theta}{dt} \approx -\alpha S\sigma\left[4T_0^3\theta\left(1 + 1.5\frac{\theta}{T_0}\right)\right] \tag{9}$$

in which we introduce the following dimensionless: $\Phi = \frac{T - T_0}{T_0} = \frac{\theta}{T_0}$ et $\Gamma = \frac{4\alpha S T_0^3}{\kappa}\sigma$

in order to get:

$$\frac{d\Phi}{dt} = -\Gamma\Phi\left(1 + 1.5\Phi\right) \tag{10}$$

The solution to this differential equation is elementary (verify!):

$$\ln\left(\frac{\Phi}{1 + 1.5\Phi}\right) = -\Gamma t + B \tag{11}$$

where B is an integration constant defined by $t = 0$.

b) Non-evacuated container.

If the space separating the spheres contains air rather than a vacuum, and extra heat transfer occurs due to the air's thermal conductance χ. Consequently, equation (8) becomes:

$$\kappa\frac{d\theta}{dt} = -P = -\alpha S\sigma\left(T^4 - T_0^4\right) + \chi\theta \tag{12}$$

By limiting ourselves to the 1st order expansion of $\left(T^4 - T_0^4\right) \approx 4T_0^3\theta$, we get

$$\frac{d\theta}{dt} = -\left(\Gamma + \frac{\chi}{\kappa}\right)\theta = -\Gamma_{air}\theta \tag{13}$$

whose solution is given by:

$$\theta(\mathrm{t}) = \theta_0 \exp\left(-\Gamma_{air} t\right) \tag{14}$$

IV. SUGGESTED EXPERIMENTS

A) Oven's radiated power

The setup (fig. 5) is made of:

- an oven supplied by an autotransformer (Variac)
- a detector and voltmeter
- a diaphragm
- an optical bench
- a thermometer and it's measuring probe
- a cover.

The experimental procedure is as follows:

- Set up the oven, detector and diaphragm on the optical bench, and make sure the oven can only receive radiation from the oven.
- Measure the room temperature T_0 and the captor's voltage at room temperature V_0
- Slowly heat the oven up to $1000\ °\mathrm{C}$, while measuring the detector's voltage V every $50\ °\mathrm{C}$. Be sure to hide the captor with the cover between measures, so that the captor doesn't heat up.
- Measure T and V as the oven cools as well.

A1) Supposing the T^4 dependency, determine $<C_{Det}>$up and $<C_{Det}>$down (< > = "averaging over all points", up/down indicate heating or cooling phase)

A2) Verify the Stefan-Boltzmann law. Using $C_{Det} = <C_{Det}>$up plot $\log(\mathrm{V}-\mathrm{V}_0+\mathrm{cst}\cdot\mathrm{T}_0^4)$ versus $\log(T)$. Experimentally determine the T dependency. Repeat this step for C_{Det} = $<C_{Det}>$down.

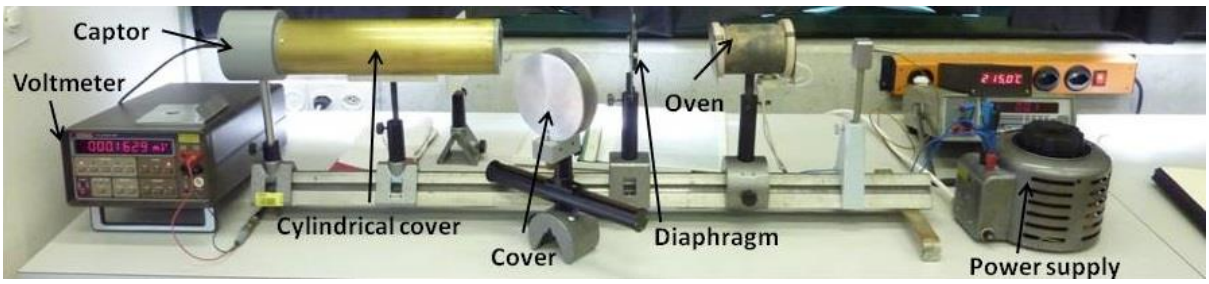


Fig. 5 : Image of the orthogonal geometry setup

B) Determining the Stefan-Boltzmann constant

For this experiment, we will use three identical containers with an internal volume of $280\ \mathrm{cm}^3$ and glass $1.5\ \mathrm{mm}$ thick: an *evacuated* container, a *non-evacuated* container, and an *evacuated and silver* container. In order to measure the Stefan-Boltzmann constant, we will use the evacuated container. The non-evacuated container (filled with air between the sides), will be used to evaluate the impact of air on the thermal transfer. Finally, the evacuated and silver container will be used to determine the α_{Ag} coefficient of a Dewar. The radiated thermal exchange is significantly reduced when using silver walls, since for metals $A(\lambda) < 1$.

Measure the temperature T_0 of the heat bath in which the three containers are placed, and fill them with $280\ \mathrm{ml}$ of water heated to about 10 to 12 °C more than T_0. Be sure to measure the precise volume of water poured into each of the containers. Let the temperature stabilize for 5 to 10 minutes. Take note of the temperature T of the internal containers every 5 minutes, and do so for approximately 1h1/2 (collect 15 to 20 data points)

B1) Determine Γ and σ from equation (11)

B2) For the non-evacuated container, estimate the air's thermal conductance χ from equation (14).

B3) For the silver container, determine α_{Ag}.

Useful parameters: $\kappa = cM$

$$c_{water} = 4.184\ \mathrm{kJ/(kg\cdot K)} \qquad c_{glass} = 0.837\ \mathrm{kJ/(kg\cdot K)}$$

$$\rho_{water} = 10^3\ \mathrm{kg/m^3} \qquad \rho_{glass} = 2.5\cdot 10^3\ \mathrm{kg/m^3}$$

V. BIBLIOGRAPHY

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