F4. Optical Instruments

I. INTRODUCTION

Optical instruments are used to improve the observation of objects by replacing them by images. These can be divided into two different categories:

- Those that supply real images (telephoto lens, projectors, photo camera, … any instruments that can project the image of the object onto something)
- Those that supply virtual images that can only be seen by the eye (mirrors, telescope, classes, periscope)

The purpose of this experiment is to build several optical instruments that project real images (projection microscope, …) on an optical bench, and then to assess their principal characteristics.

II. BACKGROUND THEORY

II.1. Geometric optics

Geometric optics studies the propagation of light, without considering its wavelike nature. It introduces the concept of light rays, and is based on the following postulates:
1. Propagation of light in a homogeneous and isotropic environment is rectilinear.
2. Reversibility of light rays. One ray will follow the same path in both directions.
3. Independence of light rays. Each beam of light can propagate independently of the surrounding ones, that have no influence on one another.

When a light beam hits the surface of two different environments (diopter), it will generally separate into two: one reflected beam that stays in the first environment, and one refracted beam, that penetrates the second environment after changing directions
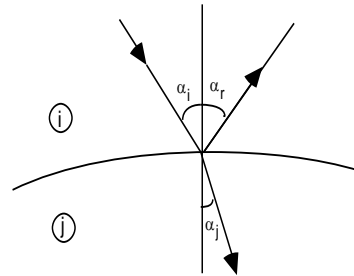

Fig. 1: Reflection and refraction of light

The geometry of the light rays follows the Snell-Descartes laws:

1) The incident, reflected and refracted beam are all in the same plane (fig. 1)

2) Incident angle α_i and reflection angle α_r are equal and of opposite signs.

$$\alpha_i = -\alpha_r \tag{1}$$

3) For any monochromatic ray, the incident angle α_i and refracted angle α_j are connected by:

$$\frac{\sin \alpha_i}{\sin \alpha_r} = n_{ji} = \frac{u_i}{u_j} \tag{2}$$

where n_{ji} is a constant called the <u>refraction index</u> of an environment j with respect to an environment i for which the light. u_i and u_j are the speeds of light in the environments i and j. When the environment i is a vacuum, we have:

$$\frac{c}{u_j} = n_j \tag{3}$$

n_j being the <u>absolute refraction index</u> of the environment j and c is the speed of light in a vacuum, which is a universal constant ($c = 2.99793 \cdot 10^8 \ m/s$)· $c \geq u$.

We can easily see that: $\qquad\qquad\qquad n_{ji} = \frac{n_j}{n_i} \tag{4}$

And according to the 2nd postulate: $\qquad n_{ji} = \frac{1}{n_{ij}} \tag{5}$

Equation (3) is generally written in the form:

$$n_i \cdot \sin \alpha_i = n_j \cdot \sin \alpha_j = n_k \cdot \sin \alpha_k = ... = const \tag{6}$$
for a given beam.

The indices vary with the frequency of the considered light. Here are a few values.

air (at $0 \ °C$ and $760 \ mm$ de Hg)	1.000294
water (at $20 \ °C$)	1.33
glass (Crown)	1.52
glass (Flint)	1.6
diamond	2.4

<u>OPTICAL SYSTEMS</u>

An optical system is made of a collection of surfaces, either separations between different transparent environments (dioptric systems), or reflecting surface (catoptric system), or both (catadioptric systems), preferably simple, in order to simplify the practical implementation. The geometric nature of the surface assigns names to the diopters. In the event of a spherical diopter, the line connecting the center of the sphere to the selected side of the diopter is called the principle axis. Any plane containing this axis is called principle cross section.

An optical system defines a correspondence between two spaces that can overlap: The <u>object space</u>, and the <u>image space</u>. Geometric optics precisely define the nature and span of both spaces for a given optical system.

II.2. Lenses

An optical system made of a transparent refracting space limited by spherical surfaces is called a lens. Generally, lenses are made of flint glass (refractive index: 1.6 to 1.7) or crown glass (refractive index 1.6). There are two types of lenses: converging lenses and diverging lenses.

We will only consider **thin lenses** (we shall neglect the thickness of the lens) and light rays slightly inclined to the main axis, which allows first-order approximations. The properties of thin lenses are:

> *1. Any light ray passing through the optical center of a thin lens suffers no deviation*

> *2. Any light ray parallel to the principal axis of a lens refracts by passing through the focal point, whether in real or in virtual space.*

The image of an object made by a lens is the locus of the image points that make up the objects

Let's build the image made of a point A by a converging lens (Fig. 2a). We know the trajectories of three light rays from point A. The intersection of two of these paths is sufficient to determine the position of the image point A '. The AO ray passing through the optical center O undergoes no deviation. The ray parallel to the optical axis passes through the focus point F, after refraction. The ray passing through the focus point F ', becomes parallel to the optical axis after refraction through the lens. The point A' which is the intersection of the refracted rays, is the image of the point A, and the image of the arrow AB is the arrow A'B '.
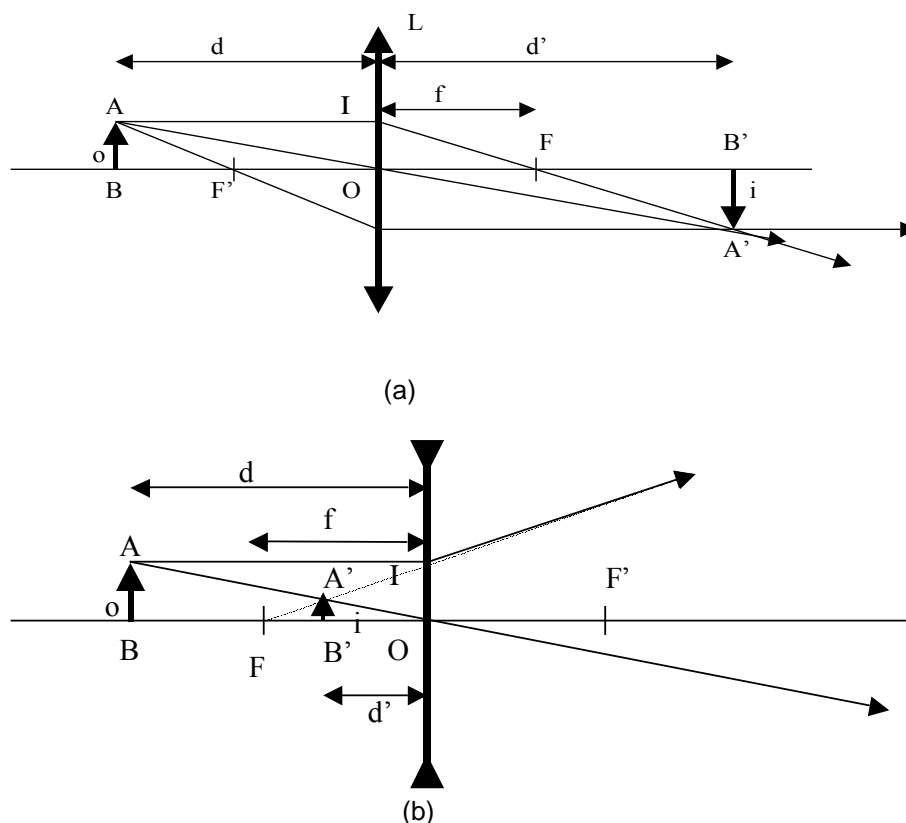

(a)

(b)

Fig. 2: Image of an object generated by converging (a) and diverging (b) lenses

The geometrical construction of the image of a point formed by a diverging lens is similar to that of the converging lens, however, notice that the extensions of the refracted rays converge at the image point, and not the refracted rays themselves (Fig. 2b). In Fig. 2b, the image AB is virtual (cannot be projected onto the screen).

To simplify the notation, we denote by o the size of the object AB, by i the size of the image A'B', by d the distance of the object OB to the lens, by d' the distance of OB' to the lens and by f the focal length OF of the lens.

Considering the similar triangles A'OB' and AOB, yields the following magnification γ.

$$\gamma = \frac{i}{o} = \frac{d'}{d} \tag{7}$$

Now, looking at the similar triangles A'FB' and IFO, we get:

$$\frac{i}{o} = \frac{d'-f}{f} \tag{8}$$

Using (7) and (8), yields:

$$\frac{1}{d} + \frac{1}{d'} = \frac{1}{f} \tag{9}$$

Equations (7) and (9) are used to determine the position and the properties of images made by lenses provided that the parameters d, d' and f are positive in the case of real objects, images and foci and negative if objects, images and foci are virtual. Since the lenses refract the light, the parameters d, d' have a conventional sign depending if they are on the opposite sides or on the same side of the lens.

According to these conventions:

f > 0	converging lens		f < 0	diverging lens
d > 0	real object		d < 0	virtual object
d' > 0	real image		d' < 0	virtual image

II.3. Stigmatism.

The theoretical and generally biunivocal correspondence between the object and image spaces can only be studied in practice. To each object point M, there corresponds in reality a *caustic surface* that can only in degenerate into a single image point M' in certain cases. When this happens, it is called *stigmatism*. However, it is impossible for this stigmatism to happen to all points of space. We must therefore satisfy ourselves with *apparent stigmatism* for certain parts of space, by doing our best to reduce the caustic surface of each point, until we reach satisfying results with respect to the imperfections of the receptor (eye, photograph,…)

Centered optical systems

Experiment and theory both show that to get stigmatism for two points with a single diopter, it needs to be of cylindrical symmetry around the line connecting both points. Therefore, optical instruments will be a succession of cylindrically symmetric diopters around an axis that will be called optical axis of the centered system. The medium separating these diopters are supposed transparent, isotropic, and entirely defined by their refraction index.

Limits of rays

The quality of images supplied by centered systems is conserved as long as we stay in the paraxial domain which is the portion space where we can neglect calculations of second-order or higher, with respect to the angle of the rays on the optical axis. In this region any object in a perpendicular plane to the optical axis will cast an image perpendicular to the optical axis. This gives a triple use to diaphragms: 1) they fix the lighting of the image; 2) they limit the dimensions of the image; 3) they stop unwanted light from entering the system.

II.4. Characteristics of an optical instrument

1) **Magnification** G **:** it's the ratio of the viewing angle α' when watching the object through the optical instrument, to the viewing angle α when watching the object without the optical instrument

$$G = \frac{\alpha'}{\alpha} \tag{10}$$

2) **Separating power:** it's a measure of the perceptibility of details. It can be limited by chromatic or geometric aberrations, diffraction phenomena, the granular structure of the screen or captor,…

For an ideal optical instrument, images of points are actually circular diffraction spots. Figure 3 shows what these diffraction profiles tend to look like. First in the case of a single point, then in the case of two neighboring points.

Fig. 3 : Aspect of light points supplied by instruments

The separating power depends on the opening u (fig. 4) in the object space, the opening u' in the image space, the cone of light rays coming from a point S and going to a point S'. It also depends on the n and n' index of the object and image space, and on the wavelength λ of the light.
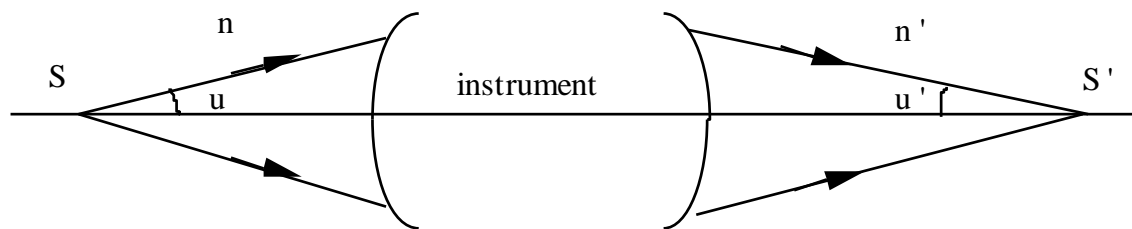

Fig. 4: Opening angle of the incoming and outgoing rays.

The R' ray of a diffraction spot is:

$$R' = \frac{0.61 \cdot \lambda}{n' \cdot \sin u'} \tag{11}$$

Therefore, the separation condition for two image points is:

$$\overline{M'N'} \geq R' = \frac{0.61 \cdot \lambda}{n' \cdot \sin u'}$$

and two object points

$$\overline{MN} \geq = \frac{0.61 \cdot \lambda}{n \cdot \sin u} \tag{12}$$

3) **The Field** is defined by the portion of the object space for which the instrument can provide a clear image.

4) **Clarity** is a measure of the apparent luminosity of the objects, when observed through the instrument versus without the instrument

It is defined by the ratio: $C = \frac{\phi'}{\phi}$ (13)

ϕ' and ϕ respectively designate the light flux penetrating the eye with and without the optical instrument, C can be greater than 1 (star watched through a telescope)

III. OPTICAL INSTRUMENT EXAMPLES

III. 1. Projection microscope

In principle, the direct vision microscope is made of an objective lens, of short focal length (a few mm), that gives a first magnified real image of the object. This image is then examined through a second lens, of focal length of the order of 1cm, that serves as a magnifying glass for the eye of the observer. In reality, both lenses are made of a more complex set of lenses, that aim to increase the performance of the instrument (less aberrations, separating power, clarity, …)
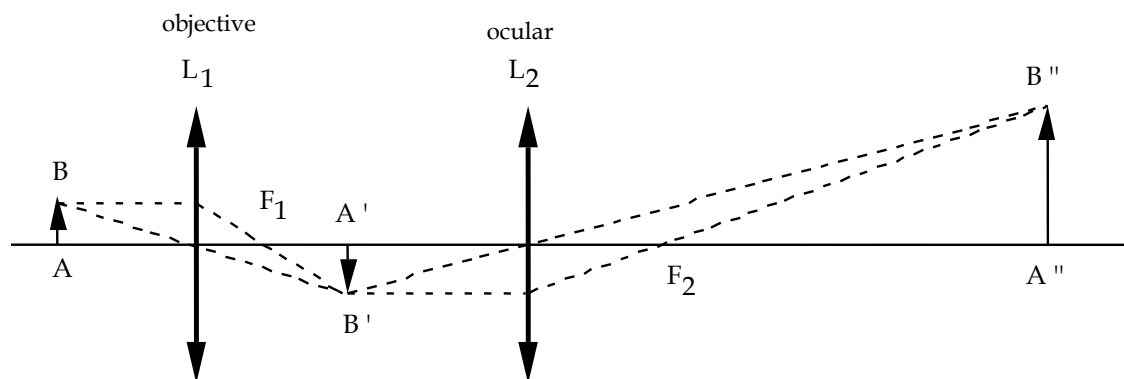
Fig. 5: Diagram of a projection microscope.

If we place the ocular lens L_2 farther than the image A'B', at a distance greater than the focal length (Fig. 5), we get a real image that can be projected on a screen or photographic captor.

III. 2. Telephoto lens.

The telephoto lens is made of two lenses: one converging lens L_1 and one diverging lens L_2 (Fig. 6). The lens L_1 makes an image A'B' of the object placed at infinity. The image is located on the focal point (F_1). This real image is a virtual object of which the diverging lens makes a real image A''B'' on the captor. The dimensions of A''B'' can be modified by moving L_1 and L_2 along the axis: zoom
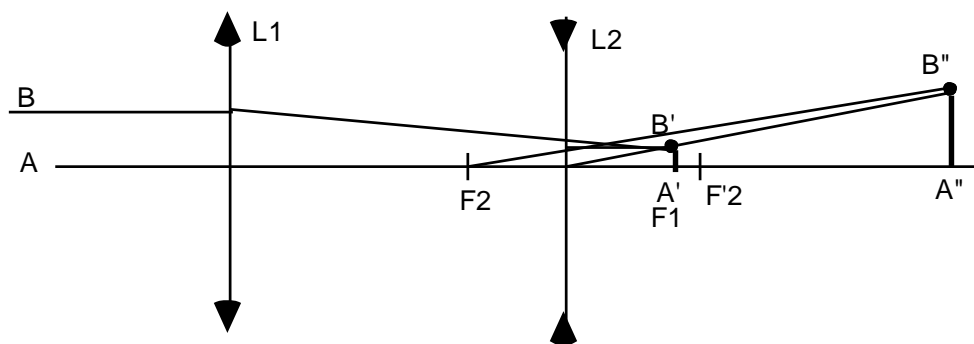
Fig. 6: Diagram of a telephoto lens

In order to study the interaction of a converging and a diverging lens on an optical bench, we will build a similar instrument, as seen in figure 7
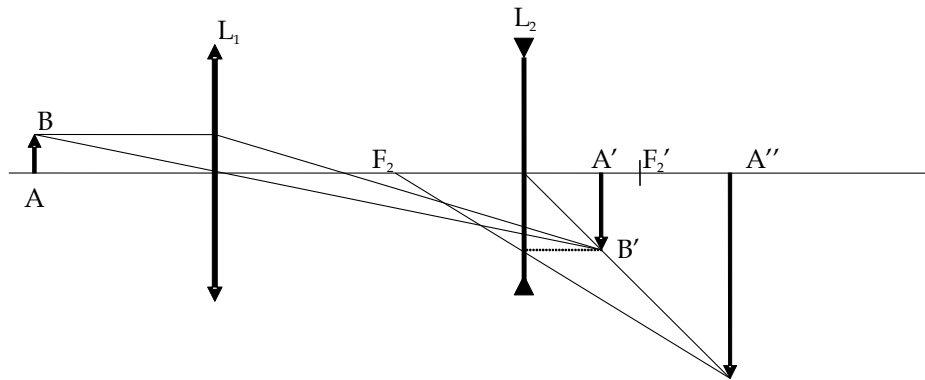
Fig. 7: Diagram of telephoto lens with object at finite distance.

IV. SUGGESTED EXPERIMENTS

Study the two types of real image optical instruments explained above: projection microscope and telephoto lens. Choose performances according to your project (e.g. magnification 50x to 70x). An example of the experimental setup can be seen in figure 8.

Let's start with the projection microscope:

1. Using the lens formulae, calculate the position of the different images with respect to the object.

2. Do a scaled model of the entire setup (lenses, light rays, …)

3. Create the instrument on the optical bench, and verify the laws of geometric optics experimentally.

4. List the performances of your optical instrument: magnification, resolutions, field, and clarity. Discuss your results

Repeat steps 1 through 4 with a telephoto lens with object at finite distance (Fig. 7). We can also simulate an object at infinite distance by adding a converging lens after the light source, while making sure that the light source is one the focal point of said lens, in order to get parallel rays (fig. 8)
In order to simplify, we will try to get the same magnification as the projection microscope. That way, we can keep the lens L_1 and only replace the lens L_2 by a diverging lens. You must then calculate the position of L_2 and the screen in order to get the desired magnification

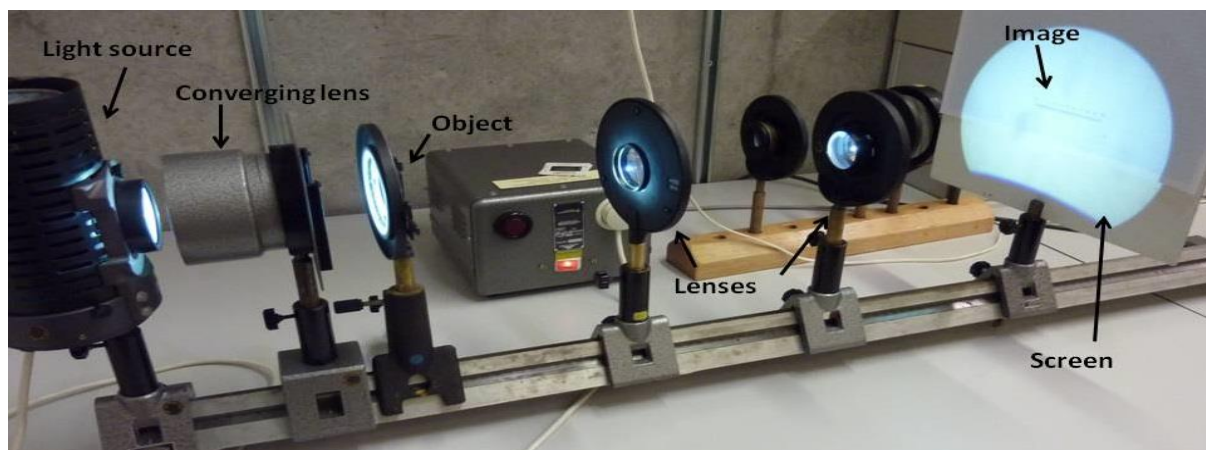**Comment:** In order to turn on the light source, turn on the main switch, and hold the red button pressed for about 10 seconds, until the light goes on.

Fig. 8: Image of the experimental setup.

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