C8. "Refreezing" Experiment

I. INTRODUCTION

A metal wire attached to a weight can cut through a block of ice without cutting it. This phenomenon is due to the fusion of ice under the wire, where the pressure is greater, and refreezing of water behind the wire, where the pressure is lower. Bottomley (1872) was the first to discover this effect. He also observed that a wire of low thermal conductivity, for a given diameter and mass, will pass through the ice more slowly than a wire with higher thermal conductivity. A number of experiments have since been produced, but this phenomenon still isn't very well understood.

It's interesting to note that this refreezing mechanism is at the core of the basal motion of glaciers (see appendix)

II. SIMPLIFIED THEORY OF REFREEZING

Nye (1976) suggested a mathematical theory suggesting that the melting-refreezing process is solely responsible for the penetration of the wire. However, later measures have shown that two different modes can be observed: fast and slow refreezing. The difference between these two modes can be up to a factor of 10^3 and the transition from one mode to the other can happen suddenly, when the temperature of the ice approaches the fusion temperature. Let's describe this phenomenon with a simplified theory:

Mechanical equilibrium of the wire in ice

Lets' consider the model of a straight wire of square cross section d caught in a block of ice of width l undergoing stress from a weight Mg due to a mass M attached to the wire (fig. 1).

It seems trivial that the equilibrium of the wire in the block of ice can only occur if there is a pressure differential ΔP between the top and the bottom of the wire, such that:

$$l \cdot d \cdot \Delta P = M \cdot g$$

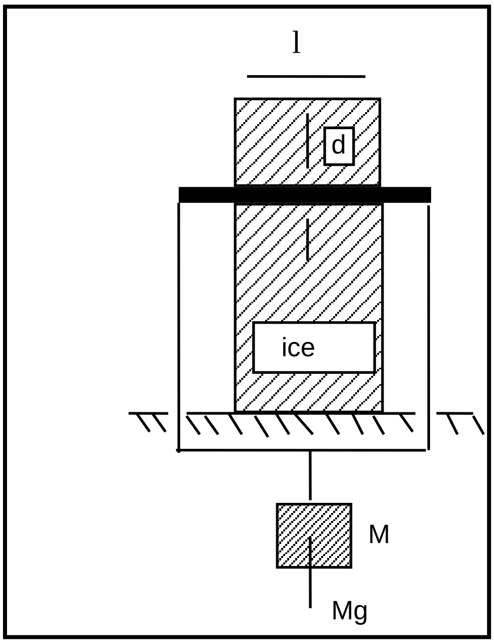

Fig1 : Experiemental model.

The pressure differential between the top and the bottom of the wire is therefore:

$$\Delta P = \frac{M \cdot g}{l \cdot d} \qquad (1)$$

Slow and fast refreezing

The equilibrium between the liquid and the solid phase of water is governed by the thermodynamic Clausius-Clapeyron equation:

$$\frac{\Delta P}{\Delta T} = \frac{L}{T(V_L - V_S)} \qquad (2)$$

in which: P = pressure [Pa]
T = temperature [K]
L = latent heat
V_L and V_S = specific volumes of liquid and solid phases

Ice being one of the few materials whose volume increases when solidifying, therefore $V_L - V_S$ is negative. Therefore, the fusion temperature decreases when the pressure increases (fig. 2), which leads to:

$$\Delta T = -C \cdot \Delta P \qquad (3)$$

with: $C = \frac{(V_S - V_L)\mathrm{T}}{L} \simeq 7.42 \cdot 10^{-5}\ °\mathrm{C/kPa}$

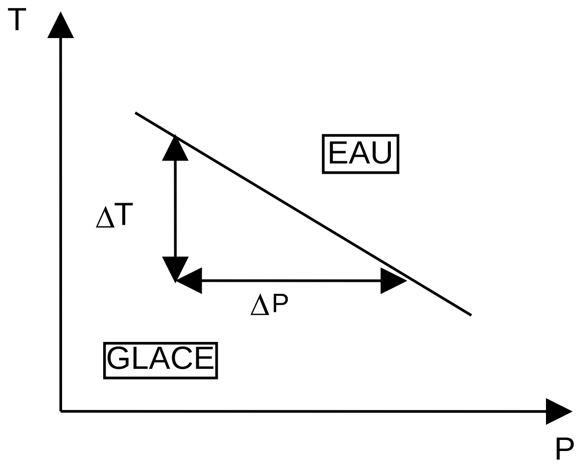

Fig 2 : Phase diagram.

In the wire experiment, this suggests that the fusion temperature of the ice under the wire will be reduced by ΔT according to equation (3) with respect to $0°C$, the fusion temperature at atmospheric pressure.

$$T_F = -C \cdot \frac{Mg}{l \cdot d} \ [°\mathrm{C}] \tag{4}$$

As long as the ice's temperature is below this value, the refreezing will be "slow". However, once the temperature reaches the limit, the ice under the wire will melt, move over the wire, and refreeze.

Simplified model of "fast" refreezing

When the temperature of the ice is close enough to $0°\mathrm{C}$, the system enters a stationary of fast refreezing. The description of this process is shown in figure 3. The mechanical equilibrium forces a pressure differential between the top and bottom of the wire. According to the Clausius-Clapeyron equation, there will still be a temperature difference between the bottom and the top, melting the water at the bottom, and freezing it at the top. The water that refreezes releases heat, which is conducted thermally to the bottom of the wire in order to melt the ice at the bottom of the wire. The mathematical description of this stationary state is rather simple.

Let v be the stationary speed of the wire passing through the ice. The mass of ice melting per unit time is:

$$m_g = \rho \cdot v \cdot l \cdot d \tag{5}$$

And the amount of heat released by the refreezing at the top of the wire per unit time equals:

$$Q = L \cdot m_g = L \cdot \rho \cdot v \cdot l \cdot d \tag{6}$$

where L is the latent heat of ice and ρ the density of ice.

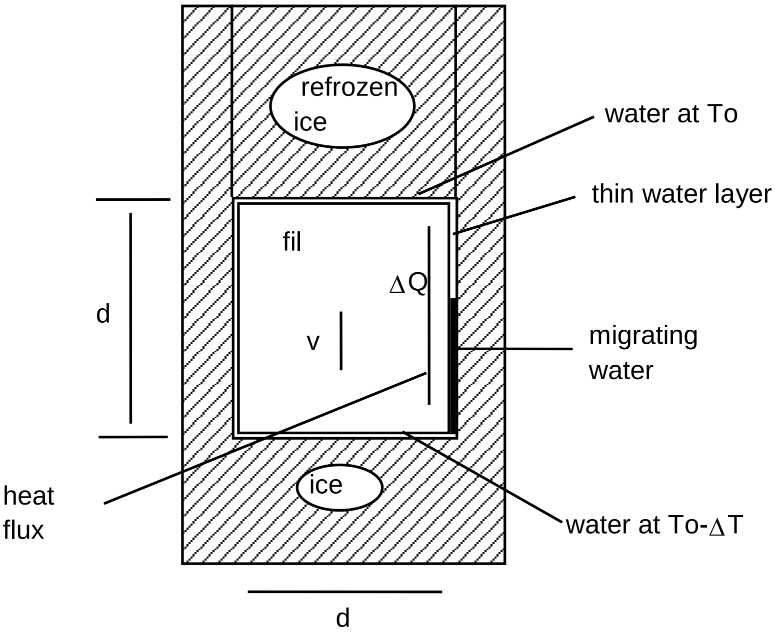

Fig3 : experimental scheme.

The heat conduction through the wire supplies a heat flux J_Q that equals

$$J_Q = \kappa \frac{l \cdot d}{d} \Delta T = \kappa \cdot l \cdot \Delta T \tag{7}$$

However, this flux J_Q must be equal to the heat produced per unit time by the refreezing of water.

$$\kappa \cdot l \cdot \Delta T = L \cdot \rho \cdot v \cdot l \cdot d \tag{8}$$

where κ is the thermal conductivity coefficient of the wire expressed in $[\mathrm{J \cdot m / s \cdot m^2 \cdot K}]$.
In this equation, we can determine the speed v of the wire

$$v = \frac{\kappa}{L \cdot \rho \cdot d} \Delta T \tag{9}$$

and using equations (3) and (1), yields:

$$v = \frac{\kappa}{L \cdot \rho \cdot d} \cdot C \cdot \frac{M \cdot g}{l \cdot d} = \frac{C \cdot g}{L \cdot \rho} \cdot \frac{\kappa \cdot M}{L \cdot d^2} \tag{10}$$

We notice that the speed of the wire is proportional to the wire's thermal conductivity coefficient K and the mass M of the weight, and inversely proportional to the wire's length l in the ice and to the square of its diameter d.

$$v \approx \frac{\kappa \cdot M}{l \cdot d^2} \tag{11}$$

III. OBJECTIVE OF THE EXPERIMENT

Throughout this experiment, we will try to study the penetration speed of different wires, for different diameters and different masses. In parallel, we will attempt, to think of real conditions of temperature and pressure in the bloc, and find a geometry that could alleviate certain issues: inhomogeneity of temperature, heat conduction from the exterior through the metal and the ice, penetration speed faster on the sides than in the center, which modifies the pressure as the wire goes down.

Among the parameters whose study could be interesting, let's state
-*impurities of water:* what difference can we observe between tap water and distilled water? Do we notice a transition when working with distilled water?
-*the mark left behind by the wire:* is it the same for every wire? What does it depend on? How does it come to be?
-*the shape of the wire:* By choosing an appropriate shape of wire, could we get more information about the liquid layer and the viscous flow of water around it?
-*the temperature of the wire:* The temperature of the wire could be recorded by using a few well-located thermal probes.

Comment: The point of this experiment is to study its applications outside of a lab. (The interesting facts can be used to study glaciers for instance), and in its pluridisciplinarity: thermodynamics (Clausius-Clapeyron), chemistry (impurities), mechanics, material sciences and more

IV. EXPERIMENTAL PROCEDURE

The available equipment is made of a set of wire mounted on a frame to which one can attach a weight. The setup can be seen in figure 4. Measurements should be taken every 2 minutes approximately, either by using a webcam (fig. 5) or directly during the experiment. The study of the refreezing phenomenon will be done in 4 steps:

- *(i)* Measure the penetration of the wire as a function of time, by varying a few parameters, such as the type of wire or the mass. Do not forget to measure the width of the bloc of ice before starting.
- *(ii)* From these measures, determine the stationary penetration speeds. Discuss, and try to spot different penetration modes.
- *(iii)* Compare the obtained results to the theoretical ones, depending on the type of wire and the cross section.
- *(iv)* Finally, since we would like this experiment to evolve, suggest a few improvements that could be made to the experiment, based on your experience.

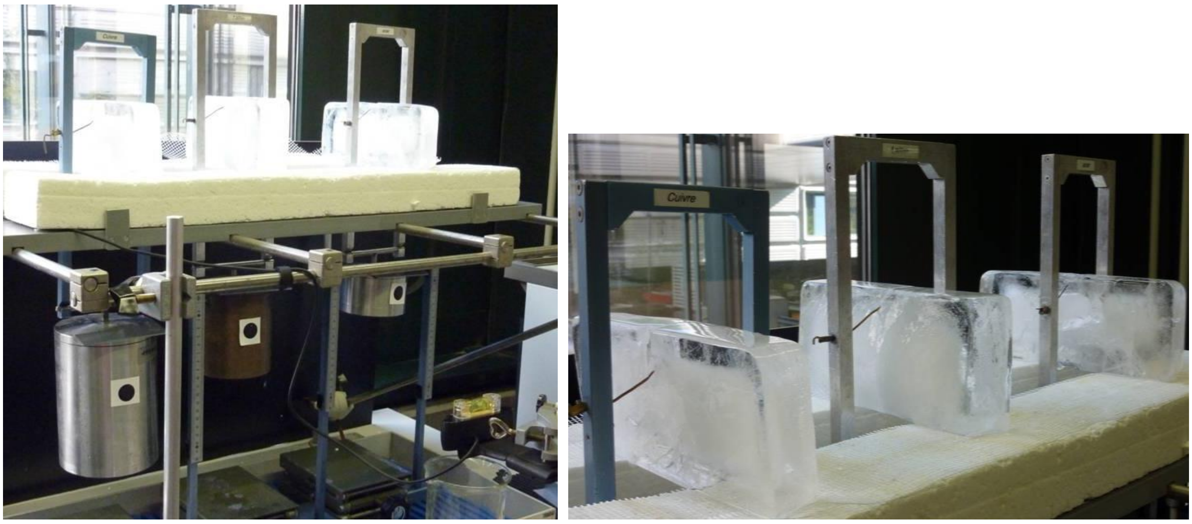

Fig. 4: Images of the experimental setup

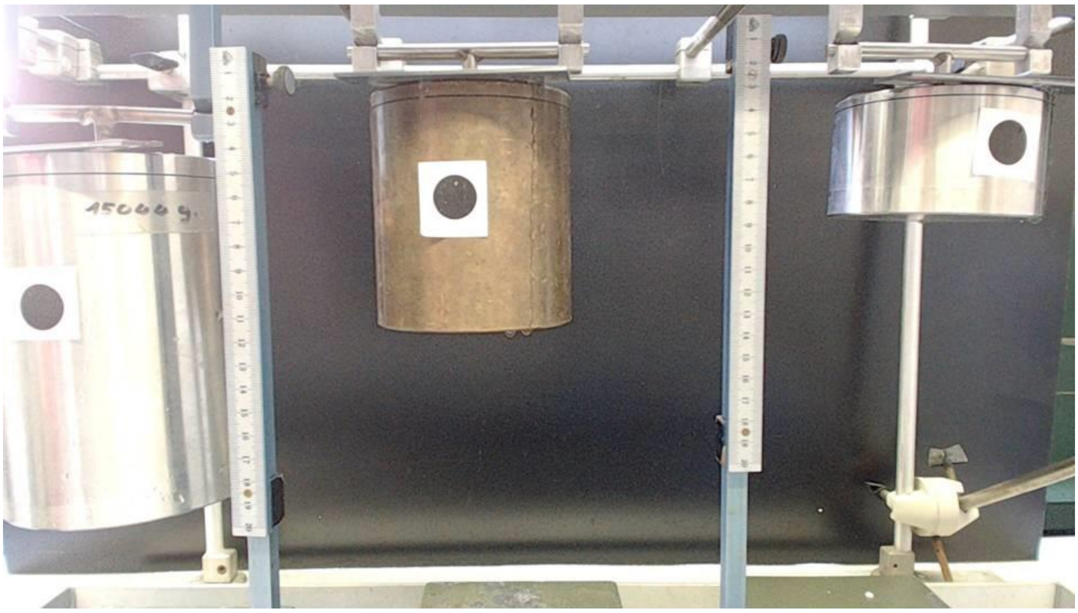

Fig. 5: Image of a measure using the webcam

V) APPENDIX: MOTION OF GLACIERS

Glaciers currently cover 10% of the earth's surface while their extension during glacial periods was about three times as high. On the other hand, many human activities are developing in remote regions of the earth (Alpine dams, pipelines in Alaska, tourist facilities). It is therefore not surprising that our interest in glaciology has significantly increased in recent years.
In addition, the widespread growth (over 50%) of the Alpine glaciers since 1978 offers new opportunities for observation and measurement. Understanding and describing the behavior of glaciers, in correlation with climate, is an extremely complex problem and far from being solved. Answering this question first requires an appropriate modeling of the motion and boundary conditions.

The two components of glacier's movements

The edge of a glacier advances or retreats during the year, the ice mass itself however is always driven by a downstream movement, under the influence of gravitational fields. This movement depends primarily on the mass of the glacier and thus on atmospheric precipitation. The forward motion of the glacier, whether apparent or not, is a result of the annual melting and flowing down.

A warm glacier, in contrast to a cold glacier, is a glacier in which the entire mass is at the melting temperature corresponding to the pressure at that given point, except for a surface layer of a few meters. This is the case for most alpine glaciers. The movement of such a glacier is the combination of two components: ice deformation and basal sliding. We do not yet know whether polar glaciers, whose basal layer is cold, can slide.

In first approximation, the strain rate depends on the height of the ice, the slope of the glacier bed, the shape of the valley through which the glacier flows and the creeping law of ice. It is generally accepted that ice behaves like a body of Glen, i.e. its creep goes like:

$$\varepsilon = A \cdot \tau^n$$ where ε = the effective strain rate
τ = the effective stress

This law is a mix between that of the perfect plastic and viscous Newtonian liquid. The values of A as a function of temperature and n are not well known and vary considerably according to the authors. However, we will admit $n \approx 3$.

Due to the scarcity of direct observations glacier bed and the difficulty of making measurements without disturbing the environment, the problem of slippage is still poorly understood. However the Weertman model can account for a simple way to this phenomenon.

The Weertman model

(i) working hypothesis

The sliding of temperate glaciers can represent up to 90% of the glaciers total motion, which justifies a detailed study. Weertman (1957) developed the first theory. The biggest problem is to explain how the glacier can overcome the bedrock obstacles. Weertman model is based on five hypotheses:

1) the entire ice is at its melting point (it is a temperate glacier).
2) ice and rock are always separated by a thin lubricating film of water and there is therefore no friction.
3) ice contains no impurities and is based on rigid bedrock.
4) ice and rock are separated by a water film (there are no cavities).
5) bedrock is represented as an inclined plane with cubic obstacles. Each obstacle has sides of length a and is separated from its neighbors by a distance λ (fig.6).

Depending on the size of the obstacle, two mechanisms will operate: for large obstacles, the additional stress caused by its presence will cause an increase in the plastic deformation. For small obstacles, the mechanism of melting-freezing will be more effective.

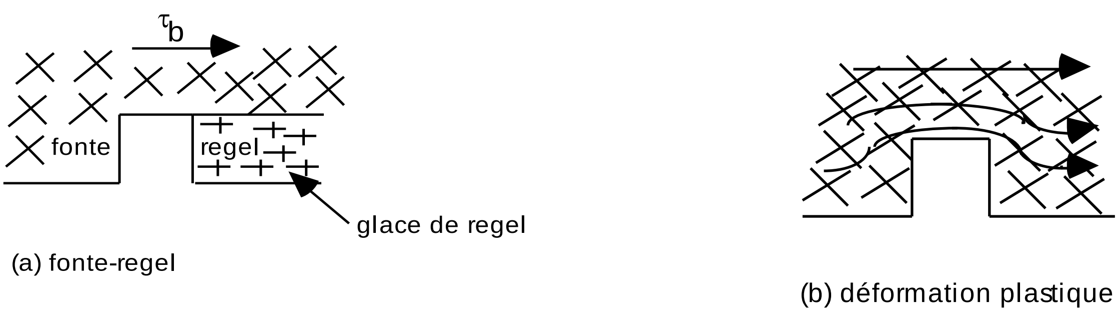

Figure 6: Weertman model

(ii) melting-refreezing mechanism
The equilibrium between the two phases is governed by the Clausius-Clapeyron equation

$$\frac{dP}{dT} = \frac{L}{\Delta V \cdot T}$$

where ΔV is the difference in specific volume between the two phases.

However, ice is one of the few bodies whose volume increases when solidifying. Therefore, when under pressure, the melting temperature decreases.

$$\Delta T = C \cdot \Delta P$$

with $$C = -7.42 \cdot 10^{-5} \; [\mathrm{K/kPa}]$$

In the Weertman model, in the absence of friction, the resistance to sliding is provided entirely by the obstacles. Since there is one cube per surface of area λ^2 (Fig. 5), each cube supplies a mean strength of $\tau_b \cdot \lambda^2 / 2a^2$ and an identical pressure on the opposing face. There will therefore be a total pressure differential given by

$$\Delta P = \tau_b \frac{\lambda^2}{a^2}$$

The upstream melting temperature will thus be lower than the downstream one, enabling the ice to melt upstream. According to the Clausius-Clapeyron equation, the temperature difference between upstream and downstream will be:

$$\Delta T = C\Delta P = C\tau_b \frac{\lambda^2}{a^2}$$

Let's call the sliding velocity due to this mechanism v_l. Per unit time, a volume v_1 a melts. The water then goes around the obstacle, and refreezes at the back, releasing an amount of heat equal to $v_l \cdot a^2 \cdot \rho \cdot L$, where L and ρ are respectively the density and the latent heat of ice. This heat is transmitted through the barrier of thermal conductivity k. We therefore have:

$$v_l \cdot a^2 \cdot \rho \cdot L = k \cdot a \cdot \Delta T$$

from which, by using the previous value of ΔT, and a speed v_1 inversely proportional to a^3 :

$$v_l = \frac{C \cdot k}{L \cdot \rho \cdot a} \tau_b \frac{\lambda^2}{a^2}$$

(iii) a mechanism for plastic deformation
The presence of the obstacle causes additional stresses in the ice, namely compressive stress upstream, and tensile downstream of module $\tau_b \, \lambda^2 / 2a^2$ on both faces. This stress will induce further deformation

$$\varepsilon = A(\frac{1}{2} \tau_b \frac{\lambda^2}{a^2})^n$$

according to the law ice creep shown previously. Supposing that this stress affects a distance approximately equal to the size of the obstacle, we will have:

$$v_2 = (\frac{1}{2})^{\frac{n+1}{2}} \cdot a \cdot A(\frac{1}{2}\tau_b \frac{\lambda^2}{a^2})^n$$

where the velocity v_2 is due to the strain, and the numerical factor appears due to the stress being uniaxial.

(iv) controlling obstacles and sliding speed

Suppose now that the glacier bed is made up of obstacles of different sizes. Small obstacles are easily overcome by the meting-refreezing process, and larger ones by plastic deformation. But there will be an intermediate size for which none of the two mechanisms will work perfectly, and that will be the main slip resistance. This size is given by the equation

$$v_1 = v_2$$

from which we obtain the critical size

$$a_c = 2^{\frac{n-1}{2}} \cdot 3^{\frac{n+1}{4}} (\frac{C \cdot k}{A \cdot L \cdot \rho})^{\frac{1}{2}} \tau_b^{\frac{1-n}{2}} (\frac{\lambda}{a})^{n-1}$$

If the sliding velocity is determined entirely by the size of these obstacles, then

$$v_b = v_1 + v_2 = const \cdot \tau_b^{\frac{n+1}{2}} (\frac{\lambda}{a})^{-n-1}$$

(V) Recent development

Following Weertman's, other theories have been developed, including particularly those of Nye (1970) and Kamb (1970) who used a more realistic bedrock model for their calculations.

On the other hand, measurements of the horizontal velocity of the glaciers showed seasonal and daily variations, which seem to be correlated with variations in height of the glacier surface. To understand this phenomenon, it is necessary to consider the subglacial hydrological network. Drainage of subglacial water seems to be a network of channels, or by a system of cavities connected to each other. These cavities are formed downstream obstacles when the water pressure is sufficient to counterbalance the pressure of the ice. The adaptation of the hydrological network to increased melt water levels requires some time. This explains the variations: in fact, before the water can flow freely again, its pressure increases under the glacier, promoting detachment of the ice and sliding. Lliboutry (1964) was the first to suggest the formation of cavities in his model.

A recent conference (Interlaken 1985) demonstrated the importance of the hydrological network and the lack of precise knowledge in this field. It also helped to highlight the role played by the nature of the ice, rocky bed or not. Recent measures seem to indeed reveal the existence of basal movements in cold glaciers whose bed is deformable. The issue of a glacier sliding on its base is still open. Improving our knowledge requires three steps:

-The development of a simple mathematical model that takes into account new measurements and for experimental verification.

-Measures and direct observations of the glacier bed.

-Laboratory measurements, providing a better understanding of the physical and mechanical properties of ice and the melting-refreezing process.