C5. Heat Transfer

I. OBJECTIVE OF THE EXPERIMENT

We would like to study heat transfers in different systems, and measure the thermal conductibility coefficients of several materials.
We will also look into different thermal isolation possibilities.

II.INTRODUCTION: HEAT EXCHANGE

Heat exchange occurs between two systems that are at different temperatures. This exchange can happen in three different ways:

a) Radiation

All bodies, solid as well as liquid, emit energy through their surface in the form of electromagnetic radiation. The value of the energy thereby emitted is given by the Stefan-Boltzmann law:

$$\varepsilon_T = \sigma T^4$$

where ε_T is the surface power density of the thermal black body radiation $[\mathrm{W/m^2}]$ and

$\sigma = 5.67 \cdot 10^{-8}\,\mathrm{W/m^2K^4}$ is the universal Stefan-Boltzmann constant.

b) Convection

Heat exchange via convection can happen when certain macroscopic parts of a fluid move around with respect to one another, or to the solid with which they are in contact with. The heat exchange happens by reciprocal motion of the particles at different temperatures.

c) Conduction

The thermal conductibility phenomenon occurs, in solids and liquids, when there is a temperature gradient. In the one-dimensional stationary case, it is described by the Fourier equation:

$$J = \frac{dQ}{dt \cdot dS} = -\lambda \frac{dT}{dx} \tag{1}$$

where J = heat flux
dQ = amount of heat transported through a unit surface element over a time interval dt
dT/dx = temperature gradient
λ = thermal conductivity

Supposing a heat conservation law

$$\frac{dq}{dt} = -\frac{dJ}{dx} \quad \text{with} \quad dq = c \cdot dT \tag{2}$$

where c = specific heat and "dq" the heat variation per unit mass ($dQ = m \cdot dq$)

This lead to the one-dimensional heat equation:

$$\frac{\partial T}{\partial t} = \frac{\lambda}{c}\frac{\partial^2 T}{\partial x^2} \tag{3}$$

In the easiest stationary state, we suppose that the internal and external temperature T_i and T_e are those at the surface of the solid. Supposing this condition, and using (1), we get:

$$J = \lambda \frac{T_i - T_e}{d} \tag{4}$$

where d is the thickness of the wall separating the inside from the outside.

However, this simple edge condition isn't verified experimentally, and we notice that the temperatures on the surfaces are slightly different from T_e and T_i. If $T_i > T_e$, then $T_{si} < T_i$ and $T_{se} > T_e$ (cf. fig.1). Equation (4) becomes:

$$J = \lambda \frac{T_{si} - T_{se}}{d} \tag{4}$$

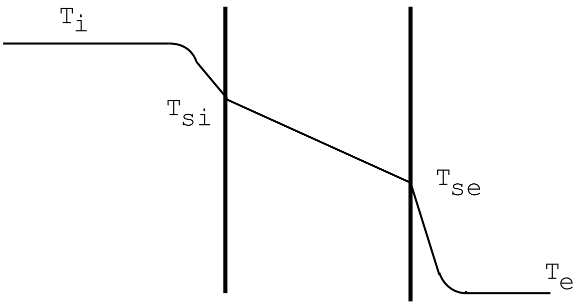


Figure 1: Temperature profile of a wall separating two environments at temperatures T_e and T_i with $T_i > T_e$.

The *edge conditions* can thus be written empirically using an exchange coefficient defined by:

$$J_i = \alpha_i (T_i - T_{si}) \tag{5}$$

and

$$J_e = \alpha_e (T_{se} - T_e)$$

Since flux is conservative in the stationary state, we must verify:

$$J_i = \alpha_i (T_i - T_{si}) = \alpha_e (T_{se} - T_e) = \lambda \frac{T_{si} - T_{se}}{d} = k(T_i - T_e) \tag{6}$$

Where k is defined as the transmission coefficient

From equation 6, we get:

$$k = \frac{1}{\frac{q}{\alpha_i} + \frac{1}{\alpha_e} + \frac{d}{\lambda}} \tag{7}$$

Using a thin metallic plate (e.g. 1 mm of Cu), the difference T_{Si} - T_{Se} ($d/\lambda \ll 1/\alpha_i + 1/\alpha_e$) is often negligible with respect to $T_i - T_e$. From this we can deduce that the heat flux going through the plate depends very little on the plate's thickness. Also, the thickness of the bottom of a sauce pan has very little effect on the time needed for the water to boil, and the only effect of a better conductor only results in a more uniform temperature on the inner surface of the pan. The most determining factor in the time required for water to boil is the contact between the heater and the pan. Equation (7) can be generalized to the case of multiple layers by:

$$k = \frac{1}{\frac{1}{\alpha_i} + \frac{1}{\alpha_e} + \sum_{j=1}^{n} \frac{d_j}{\lambda_j}}$$

where λ_j is the thermal conductivity coefficient, and d_j the thickness of the j-th layer.

Thermal conductivity is a molecular property. Thermal conduction thus only takes place in materials. It is, however, not related to the movement of the materials along a path, but only related to energy transfer due to molecular impacts. It thus represents an exchange of kinetic energy from one molecule to the next. In the case of metals, thermal conduction takes place, along with molecular oscillations, also due to electron flows that increase the conductance. In metals not all electrons are bound in a fixed place. The so-called conducting electrons wander about in the lattice structure (somewhat similar to the molecules in a gas) and can therefore easily absorb kinetic energy. This is the reason why electrical conductors have significantly higher thermal conductivity than electrical insulators. The thermal conductivity at 20°C is approximately:

$$\lambda_{20°C} \approx \frac{2.45 \cdot \chi_e \cdot T}{10^8} \text{ in } \left[\frac{\mathrm{W}}{\mathrm{m \cdot K}}\right]$$

where $\chi_e \left[\frac{1}{\Omega \cdot \mathrm{m}}\right]$ is electrical conductivity and T $[\mathrm{K}]$ is absolute temperature.

III. SETUP DESCRIPTION

III. a) Setup for plate samples (building materials)

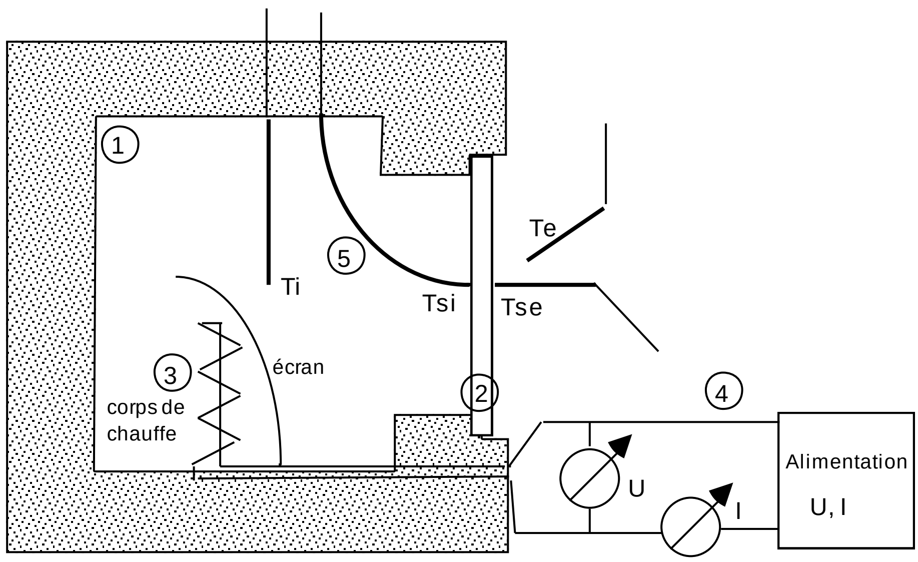
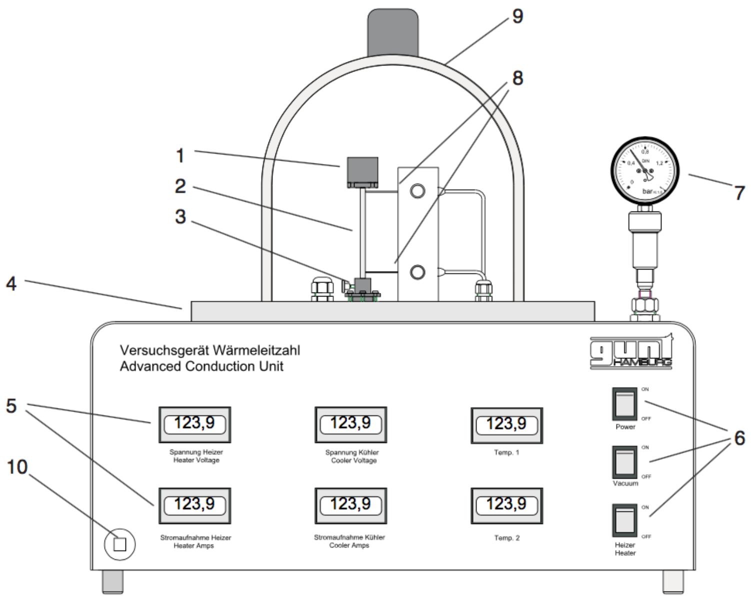


Figure 2: Measurement of the thermal conductivity of plates.

The setup is made of the following elements (fig.2):
A well isolated measuring chamber (1) with a $13.5 \times 13.5\ \text{cm}^2$ window to place the materials to study (2). Inside the chamber is a heating body (3) with a power supply on the outside (4). The different temperatures T_i, T_{si}, T_{se} and T_e are measured using thermocouples (5).

III. b) Setup for metals

In order to measure the thermal conductivity of metals, metal bars of comparatively small diameter are used as specimens. To suppress erroneous measurements due to the surrounding air, the specimen chamber can be evacuated using a vacuum pump.



1 Heater (Heat Source)
2 Specimen
3 Cooler
4 Aluminium Plate (as heat sink)
5 Displays
6 Switches
7 Vacuum Gauge with Venting Valve
8 Temperature Sensors
9 Bell jar
10 Measured Data Connection for PC

Figure 3: Measurement of the thermal conductivity of metals under vacuum: setup scheme

The experimental unit (fig. 3) comprises a switch housing that has all the necessary **displays for measured values** (5) on its front panel; all the **switches** (6) are also fitted here. The items for the experiment are on the top of the unit.
To generate a temperature gradient for the experiment, heat must be both generated and also drawn away, that is cooling is required. A Peltier element is used as the **cooler** (3). This is permanently fitted to a large **aluminium plate** (4) that is also used as a heat sink for the hot side of the cooler. The cooler has a mounting in which a **test bar** (2) can be inserted vertically. The electrically operated **heater** (1) is not fixed. It is fitted to the top of the test bar using a mounting. To measure the temperature gradient on the test bar established by the temperature difference between the heater and cooler, two **thermocouples** (8) are used. These are inserted in the corresponding holes on the test bar using thermally conductive paste.
To exclude the effects of ambient air on the experiment, a **bell jar** (9) can be placed over the experimental arrangement. This bell can be evacuated for the experiment using the vacuum pump installed in the experimental unit. On the **vacuum gauge** (7) the magnitude of the vacuum can be monitored. This experimental unit is equipped for PC data acquisition. The unit can also be operated without limitations even if not connected to a computer.

IV. SUGGESTED EXPERIMENTS

1. Determine the thermal transmission coefficient of several materials (ceramic, glass, polystyrene,...).

Procedure:

Put the pane of material to study in the window of the isolating chamber (fig. 2) and place the thermocouples in a straight line perpendicular to the window in order to measure a one-directional heat flux. T_{si} et T_{se} are in contact with the plate on either side, T_i and T_e are approximately $1\,\mathrm{mm}$ away from the pane.

Turn on the electric supply, and set the current on $1\mathrm{A}$. Measure the electrical power.

In order to use the aforementioned formulae, the temperatures T_i, T_{si}, T_{se} and T_e must be stable. Therefore, take note of the temperature every 30 seconds, until the temperature differences are practically stable (~ 20 minutes). At that point (stationary state) we can suppose that the power supplied to the heating body is entirely dissipated through the sample.

Knowing the surface through which the heat is transported (S), the power (dQ/dt), the thickness of the sample (d) and the temperatures T_i, T_{si}, T_{se} et T_e, we can determine k, λ and possibly α_i and α_e.

2. Study the phenomenon of double glazing. Repeat the whole procedure in 1, once for two glass panes in contact, and once for two glass panes separated by a small air gap

3. Determine the thermal transmission coefficient of several metals (copper, aluminium, stainless steel, brass)

Procedure:

- Remove bell jar (fig. 3) and place on one side
- Insert a test specimen into the mounting on the cooler (fig. 4)
- Loosen knurled bolts on the retaining brackets for the thermocouples
- Apply some thermally conductive paste to the measuring end of the thermocouples and insert in the measuring openings on the test bar. Then fix the thermocouples in place by tightening the knurled bolts again
- Place heater on the free end of the test bar
- Place bell jar over the measuring arrangement (if necessary slightly lubricate the seal with petroleum jelly or grease first)
- Switch on heater and cooler using switch
- Place valve behind the vacuum gauge in the position „**Evacuate**“. Then activate the vacuum pump using switch
- After a certain time, a steady state condition will be established. This means that the temperatures no longer change.
- Then calculate the heater power output from the measured values for the heater voltage and current
- Using eq. (1) calculate the value of the thermal conductivity λ of the related specimen material (for $\dot{Q}$ insert the heater output power in the equation).

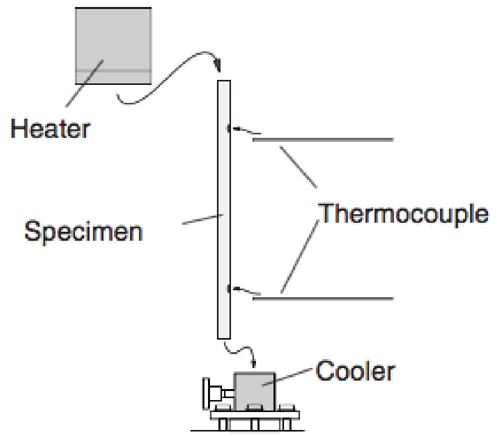


Figure 4: Fitting a specimen

V. BIBLIOGRAPHY

All theories discussed in here can be found, with full development in:
"Heat Transfer", J.P. Holman, International Student Edition, Mc Graw-Hill, Kogakusha, Ltd.

Numerous problems, with explained solutions, can be found in :
"The Heat Transfer Problem Solver" Research and Education Association New-York.

Experiment instructions WL 375 Advanced Conduction Unit