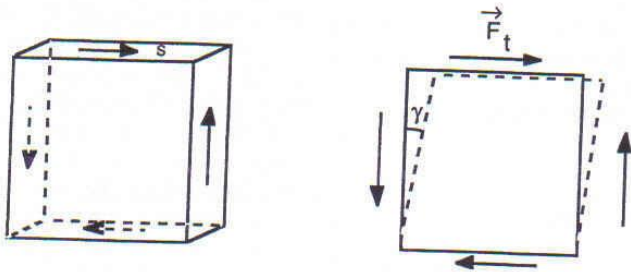


## *B2. Viscosity of Liquids: Viscosity Coefficient*

### I. INTRODUCTION: Phenomenology

Let's consider a shear force experiment on a solid cube. (Fig. 1)



We can easily verify that for the solid not to move or rotate, all forces acting on the cube must be of equal magnitude. Let's define the **tangential stress** by:

$$\tau = \frac{\|\vec{F}_t\|}{s} \quad [\text{N/m}^2] \quad (1)$$

Fig. 1 : Shearing of a cube

If we now look at the more general case of a parallelepiped, and no longer a cube, all four stresses must be equal for the solid not to move or rotate. As a result of these stresses, the solid undergoes an angular deformation  $\gamma$ , and for small  $\tau$ , we have:

$$\gamma = \frac{\tau}{G} \quad [G] = \text{N/m}^2 \quad (2)$$

This is the law of shear stress, where  $G$  is the shear modulus, and depends on the studied material. However, if we pay closer attention to the deformation, we notice that angular deformation  $\gamma$  is a function of time (Fig. 2). The value  $\gamma$  established in equation (2) only represents the asymptotic value of the deformation. Instead, the full equation becomes:

$$\tau = G \cdot \gamma + \eta \cdot \dot{\gamma} \quad \text{avec} \quad [\eta] = \frac{\text{kg}}{\text{m} \cdot \text{s}} \quad (3)$$

$\eta$  is called the dynamic viscosity coefficient. It depends on the studied material as well as other factors, such as temperature.

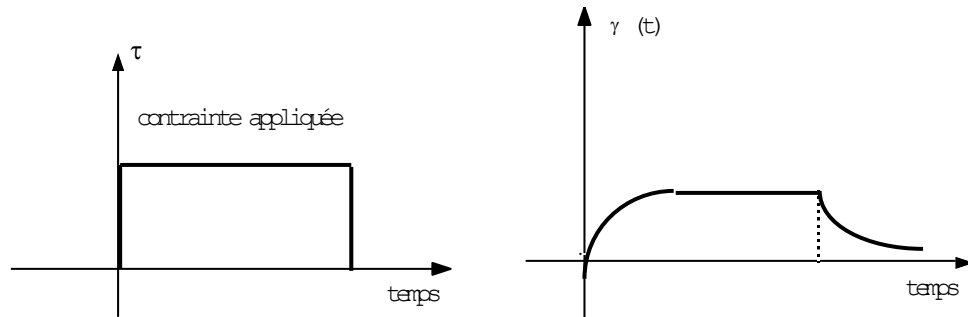


Fig. 2 : Dynamic viscosity

Therefore, viscosity is a property of a body undergoing a deformation. It is a measure of the resistance the neighboring layers present towards sliding on one another.

Viscosity is easiest to achieve and study in fluids. This is because fluids have weaker intermolecular bonds, therefore allow for one element of fluid to move around much more freely than in a solid. This results in the deformation having no asymptotic value. In other words, the shear modulus  $G$  of a fluid is 0. The stress is therefore only linked to the speed of the deformation. Viscosity is often defined by the following experiment.

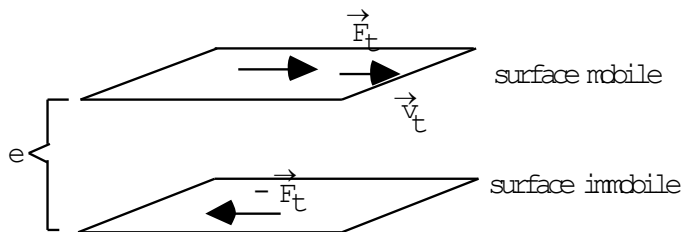


Fig. 3 : Relative motion of the different layers

By maintaining the lower surface stationary and giving the upper surface a speed  $\vec{v}_t$  (Fig. 3), we notice that:

$$\vec{F}_t = \eta \frac{S \vec{v}_t}{e}$$

or  $\tau = \eta \frac{v_t}{e} \quad (4)$

$$\tau = \eta \frac{\partial v_t}{\partial z} \quad (\text{local})$$

The speed profile inside the fluid presented in Fig. 4.

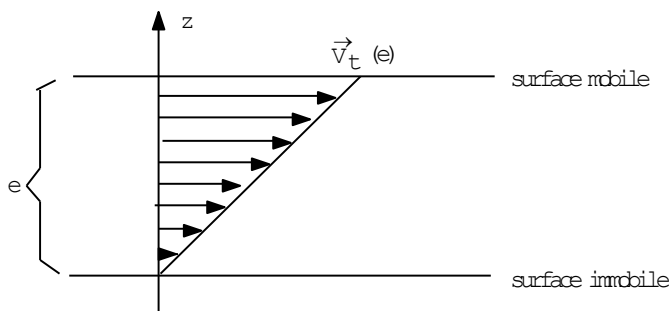


Fig 4: Speed profile inside the fluid.

Comments:

- 1) Equation (4) is coherent with  $\tau = \eta \cdot \gamma$  from (3) for small angles  $\gamma$ .
- 2) Kinematic viscosity is often defined by:  $\nu$  par  $\nu = \frac{\eta}{\rho}$  [ m<sup>2</sup>/s ], where  $\rho$  is the density in [ kg/m<sup>3</sup> ].
- 3) For a pure elastic deformation (i.e.  $\eta = 0$ ), the stored potential energy during the deformation is entirely restored when the deformation is back to zero. However, viscosity implies an irreversible absorption of mechanical energy, that heats up the body through internal friction.
- 4) Elastic compressibility is the phenomenon that connects a isotropic compression to a volume variation. Experience shows that the volume variation isn't instantaneous when the stress is applied. This lets us define a second viscosity coefficient,  $\eta^*$ .

**II. OBJECTIVE OF THE EXPERIMENT**

The objective of this experiment is to determine the viscosity coefficient  $\eta$  of different oils, as a function of temperature, as well as to get familiar with some technical knowledge.

**III. BALL VISCOMETER.**

Although this experiment is fundamentally simple, it isn't technically an experimentally trivial. We will use other simple phenomena to help determine  $\eta$ .

One approach consists in observing a ball falling through a fluid. In a vacuum, this would result in a uniform acceleration, but in a fluid, **drag** forces quickly counteract the acceleration. After a while, the ball reaches a constant speed, and in certain cases, we can use this speed to determine the viscosity coefficient  $\eta$ . That is the principle of **ball viscometer**.

**III.1. A few definitions and results.**

When an object is in motion in a fluid, it undergoes a drag force. The drag force is defined by:

$$T = C_x \cdot S \cdot \frac{1}{2} \cdot \rho \cdot v_\infty^2 \quad (5)$$

$v_\infty$  is the relative speed of the object to the fluid.  $S$  is the area of the projection of the object on a perpendicular plane to its speed, and  $C_x$  is the **drag coefficient**.

This drag coefficient is a function of the “magical” **Reynolds number**,  $Re$ . This dimensionless number is defined by:

$$Re = \frac{\rho}{\eta} v_\infty L \quad (6)$$

where  $L$  is a characteristic length of the object (for example the diameter of a sphere, or the diameter of a cylindrical conduct, ...). Though this is an empirical quantity, it can sometimes be determined theoretically.

For a sphere with a given diameter  $2r$ , we notice the following behavior:

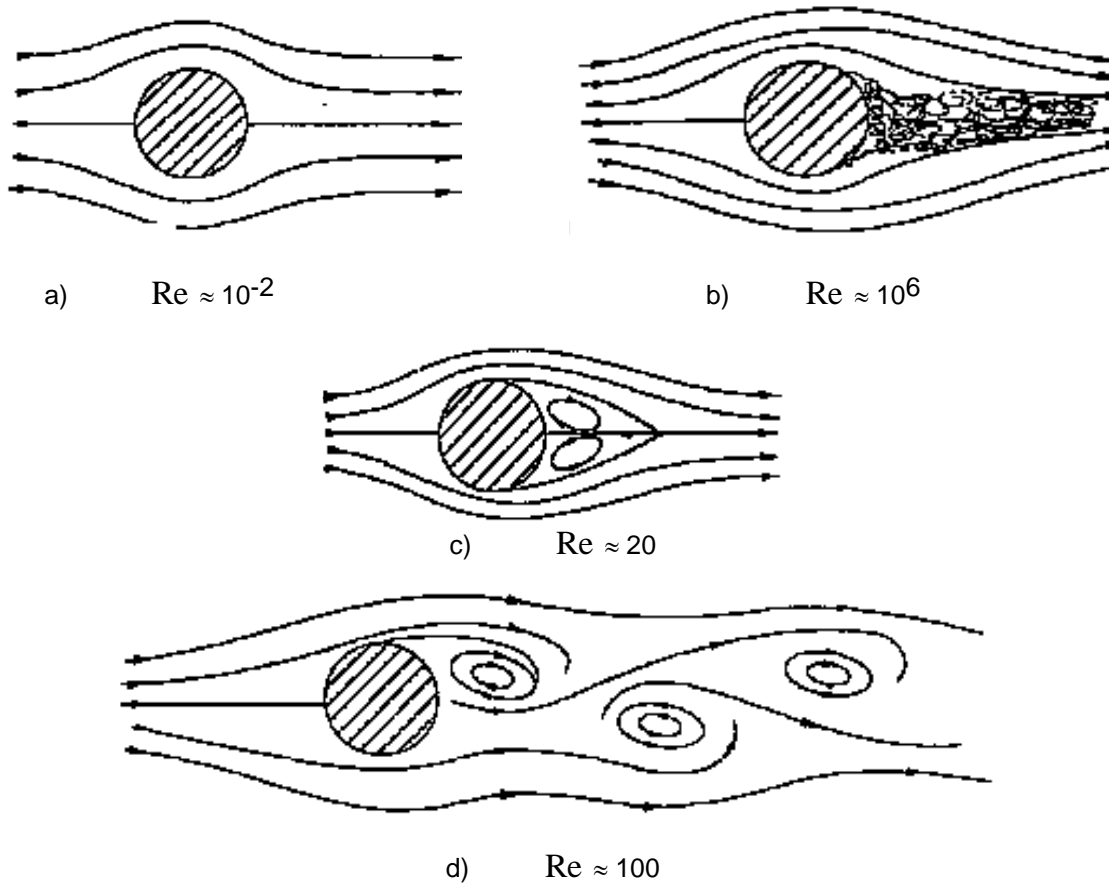


Fig. 5 : Flow around an obstacle

The variation of  $C_x$  as a function of  $Re$  is shown in figure 6.

In the special case of a **permanent flow** (figure 5a), and a small Reynolds number:

$$0 \leq Re < 0.5$$

and by neglecting inertial forces with respect to drag forces, Stokes was able to evaluate  $C_x$  and  $T$ . His calculations yielded:

$$C_x = \frac{24}{Re} \qquad T = 6\pi \cdot \eta \cdot r \cdot v_{\infty} \qquad (7)$$

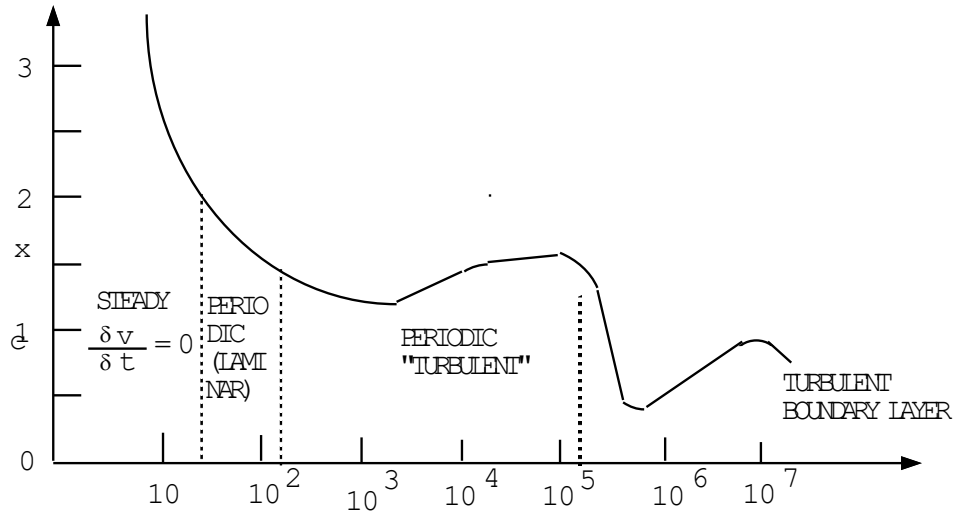


Fig. 6: Types of flow

For a fluid to follow (7), it must be “Newtonian”, i.e. its viscosity is independent of the rate of shear. This is the case of water, acetone, oil, glycerin, and many more, but is not the case for most silicones, and other liquids made up of large chains of molecules. In these liquids, viscosity is anisotropic, and varies with speed. Examples include ketchup, starch suspensions, melted butter.

There exist many other formulae allowing to calculate the drag coefficient, but not all are valid for small Reynolds numbers. We'll mention Oseen's formula, which is applicable for  $0 \leq \text{Re} < 1$  and accounts for inertial forces:

$$C_x = \frac{24}{\text{Re}} + \frac{9}{2} \quad \text{et} \quad T = 3\pi \cdot r \cdot v_\infty \cdot \left( 2\eta + \frac{3}{4} \cdot \rho \cdot r \cdot v_\infty \right) \quad (8)$$

### III.2. Variation of $\eta$ with temperature.

For liquids where the molecules don't form complex groups, “fluidity”, defined by  $1/\eta$ , is calculated by a Boltzmann relation (see statistical mechanics):

$$\frac{1}{\eta} = \frac{1}{A} e^{-\frac{E}{k_b T}} \quad (9)$$

where  $E$  is the activation energy,  $T$  the temperature and  $k_b$  the Boltzman constant.

We can make sense of the equation by looking at the microscopic scale. Microscopically, fluidity is dependent on the momentum transfer between molecules. Therefore, a molecule must be capable of overcoming a potential barrier  $E$  corresponding to the molecules. This allows us to write:

$$\ln(\eta) = \ln(A) + \frac{E}{k_b T} \quad (10)$$

## IV. EXPERIMENTAL SETUP

The experiment consists in letting a rigid ball fall in the liquid we wish to study. We know the characteristics of the ball, as well as the density of the fluid. When the ball reaches a constant speed, the drag force is easily determined by the weight of the ball, and its buoyancy. Therefore, by

measuring the ball's speed, and assuming a small enough Reynolds number, we can determine the viscosity using equation (7).

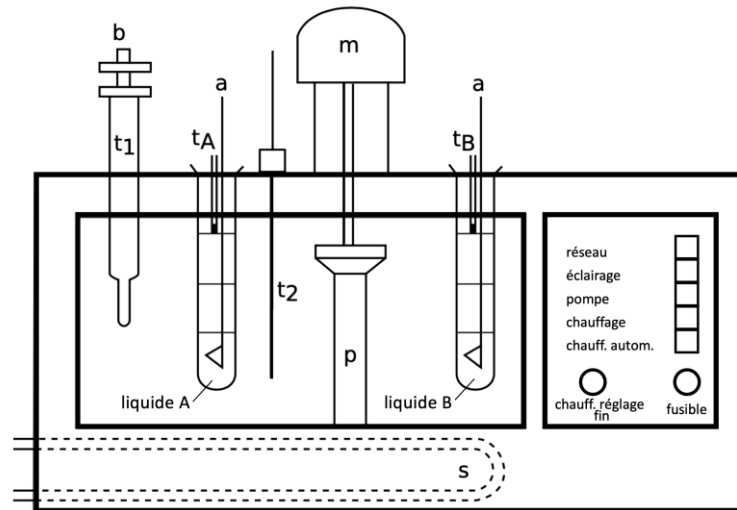


Fig. 7: Scheme of the viscometer

The liquids we wish to study are contained in two glass tubes A and B, inside a basin of water. A heater inside the basin allows us to heat the water. It is operated by a thermostat. the water can also be cooled, by letting tap water flow through the system. An agitator in the water makes sure the temperature stays homogeneous. A digital thermometer indicates the water temperature. The temperatures of the liquids A and B are given by thermometers in their respective tubes. The thermometers are attached to agitators, that make the temperature more homogeneous, and can carry the ball up to the surface.

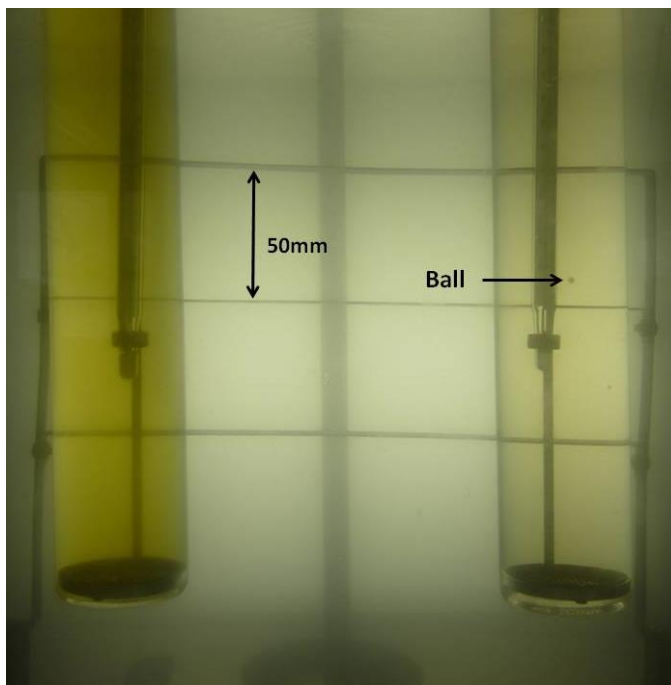


Fig 8: Experimental setup.

### Using the viscometer

Turn the system on by pushing the “main” button. Then push “Illumination” and “Pump” to turn on the light, and the basin’s agitator. To start the heater, hit “Heating” and “Automatic heating up”. It takes approximately 60 minutes to get from room temperature to 80°C. This generally gives the user enough time to take measurements (approximately every 5°C). However, if this is too quick, push the “Automatic heating up” button again to deactivate it and slow up the heating process.

Once the system is at 80°C, deactivate the heater, and turn on the cooling circuit. This is done by opening the tap on the right of the setup, over the sink.



## V. EXPERIMENTS

- 1) a. Determine the viscosity coefficient  $\eta$  for both liquids, between 20°C and 80°C (do not go over 80°C). Do two sets of measurements for each liquid: one during the heating and one during the cooling process.
- b. Plot the values of  $\log \eta$  versus  $T^{-1}$  (don't forget error bars), and determine the constants  $A$  and  $E$  from (10).

Be careful not to create air bubbles when placing the agitators back into the tubes. If the ball comes too close a bubble, the result will be faulty. Likewise, try not to let the ball fall too close to the sides of the tube.

2) The error generated when using the Stokes formula increases with temperature. Please take note of that, and remember that  $\Re$  must not exceed 0.5 ( $\Re$  is calculated from  $\eta$ , equation 6).

3) Please also note that the temperature can vary during a single measurement. This is particularly interesting when comparing the results of  $\eta$  ( $T \uparrow$ ) et  $\eta$  ( $T \downarrow$ ).

4) We can also study the time evolution of the drag force  $T(t) = 6\pi \cdot \eta \cdot r \cdot v(t)$  assuming the initial conditions  $x(t=0) = v(t=0) = 0$ . What is the time constant?

### Comments

Ball diameter:  $(2.00 \pm 0.01)$  mm

Ball mass:  $(4.43 \pm 0.03)$  mg

Distance between bars : 50 mm

The oils densities can be found on a sheet of paper in the drawer.

## VI. APPENDIX : A BIT OF TECHNICAL KNOWLEDGE ABOUT OILS

### 1) Units

The CGS units for dynamic viscosity are the poise :  $1\text{Po} = 1\text{g} \cdot \text{cm}^{-1} \cdot \text{s}^{-1}$  (or the more commonly used centipoises : cPo), and for kinematic viscosity, the stokes:  $1\text{St} = 1\text{cm}^2 \cdot \text{s}^{-1}$  (again, the centistokes is more commonly used : cSt).

In Switzerland, we tend to use conventional Engler viscosity for industrial lubricants. (time it takes for  $200\text{ cm}^3$  of oil to flow divided by the time the same quantity of water takes, at  $20^\circ\text{C}$ ).

### 2) Classification of SAE oils

The most widely used system to classify vehicle oils as a function of their viscosity is that of the "American Society of Automotive Engineers". This system was elaborated to replace unspecific terminology such as "light" or "heavy". SAE 5W, SAE 10W, and SAE 20W all correspond to a give viscosity at  $0^\circ\text{F}$  ( $-17.8^\circ\text{C}$ ). These limits were established to ensure reliable vehicle starting in winter (W stands for Winter) and an effective lubricant at the operating temperature. SAE 20 and SAE 50 oils correspond to specific viscosities at  $210^\circ\text{F}$  ( $98.9^\circ\text{C}$ ). Please note that SAE ratings do not ensure quality, since it is only a viscosity indicator.

SAE specifications being based on only 2 temperatures(  $-17.8^\circ\text{C}$  and  $98.9^\circ\text{C}$ ), we can use an appropriate oil to make an oil that satisfy two or more SAE grades. This is called a "multigrade oil". For instance, a motor oil that has a viscosity of 2'000 cSt at a temperature of  $-17.8^\circ\text{C}$  (SAE 10W) and a viscosity of 15cSt at  $98.9^\circ\text{C}$  (SAE 40) will be graded as an SAE 10W/40 oil, and will therefore cover 4 different SAE ranks: SAE 10W, SAE 20W, SAE 30 and SAE 40, depending on the oil's temperature. This improved viscosity index (low variation of the viscosity with temperature) is generally achieved by adding long chained polymers. The advantage of multigrade oils is that they can be used at a wide range of temperatures.

#### Oil viscosity indices

The viscosity index is a good indicator of the viscosity's variation with temperature. This notion (VI=viscosity index) was created by Dean and Darvis in 1929. They used two different types of oil as a reference: Those that vary a lot with temperature (VI=0) and those that don't (VI=100). The viscosity index is given by comparing the oil with the reference oils. (figure 8)

Supposing we have an oil whose viscosity varies like the dashed line on figure 8, the viscosity index is calculated according to equation (11):

$$\text{VI} = \frac{L - U}{L - H} \cdot 100 \quad (11)$$

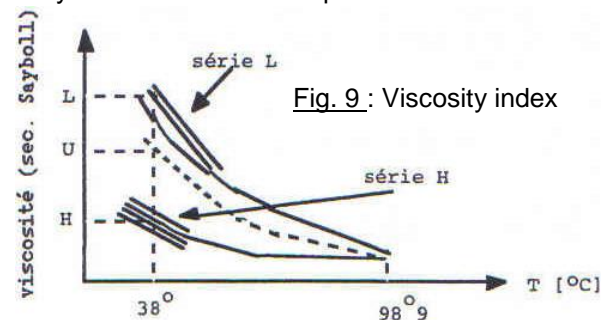


Fig. 9 : Viscosity index

This process allows any oil to be characterized with respect to the two reference oils. The bigger VI, the closer the curve will be to H (strong paraffinic property), and therefore the less it will vary with temperature.