

B1. Aerodynamics

I. INTRODUCTION

This *Fluid Mechanics* experiment attempts to determine air flows on different objects placed in a wind tunnel. We will be looking into *laminar* and *turbulent flow*, depending on the corresponding Reynolds number, as well as a few quantities related to turbulent flow, such as the *penetration coefficient* C_x , the *drag coefficient* C_w and the *lift coefficient* C_F , all of which have interesting technological and aerodynamic implications.

II. BACKGROUND THEORY

Laminar and turbulent flow, Reynolds number.

A flow is characterized by a velocity vector field \vec{v} and a pressure scalar field p . There are two different types of flow:

- *Laminar flows*, for which the vector function $\vec{v}(\vec{r}, t)$ is univocally determined by \vec{r} . We can recognize them by their current lines that never cross.
- *Turbulent flow*, for which the vector function $\vec{v}(\vec{r}, t)$ isn't univocal, which implies that the current lines do overlap, so much so that the only meaningful quantity we can express about a given point is the time-averaged velocity at that point.

When the fluid's flowing velocity – be it in a pipe or around an obstacle – exceeds a certain critical value, the laminar flow that was observable at low velocities becomes unstable and becomes turbulent. This regime is determined by vortexes that arise from the viscous friction. The transition from laminar to turbulent flow is quantized by the Reynolds number Re . It is a dimensionless number that characterizes a given flow, and is given by:

$$Re = \frac{\rho l v}{\eta}$$

where ρ is the fluid's density, v is the characteristic velocity of the flow, η is the fluid's viscosity coefficient and l is a characteristic length of the obstacle or pipe.

The Reynolds number corresponding to the critical velocity v_c describing the transition from laminar to turbulent flow is called the *critical Reynolds number* Re_c . This number depends mainly on the geometry of the flow. For instance, a flow in a cylindrical pipe with smooth surfaces has a critical Reynolds number of the order of $Re_c = 2300$, which means that any flow satisfying $Re < Re_c$ will be laminar. However, if $Re > Re_c$ the flow is turbulent.

The Reynolds number is also crucial in *similarity theory*. Among other things, this theory solves the technical difficulty of studying characteristics of aerodynamic objects using scaled down models in wind tunnels. For two flows to be physically equivalent, the requirements are:

- Geometric similarity between the pipe/obstacle and its scaled down equivalent
- Both Reynolds numbers must be equivalent.

Resistance force on an obstacle in a viscous flow.

If a fixed obstacle (solid object) is placed in a viscous flow, the fluid will apply a *resistance force* \vec{R} on it (Fig. 1).

For a flow at low velocity, ($Re < Re_c$), the flow is laminar, and the resistance is essentially due to the friction at the surface of the obstacle.

The resistance force \vec{R} depends on the fluid's viscosity η , the obstacle's geometric dimension l , and the flow's velocity v_∞ (this velocity is always to be measured far away from the obstacle). For example, a solid sphere placed in a laminar flow will undergo a resistance force given by *Stokes formula*.

$$\vec{R} = 6\pi \cdot \eta \cdot a \cdot \vec{v}_\infty$$

where a is the sphere's radius and v_∞ is the speed at infinite (long far).

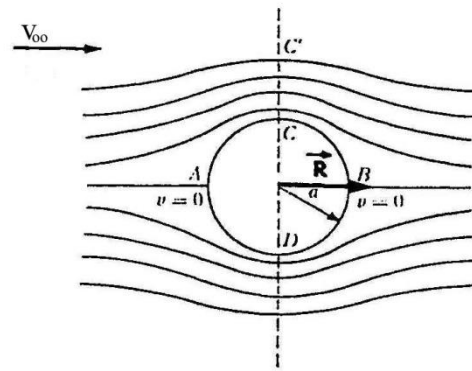


Fig 1 : Resistance force in case of laminar flow

For a high velocity flow ($Re > Re_c$), the flow is turbulent (Fig. 2), and the resistance force is essentially due to the inertial forces associated with the generation of vortices behind the obstacle, in such a way that \vec{R} no longer depends on the fluid's viscosity η .

The resistance force \vec{R} for a turbulent flow depends on the obstacle's geometrical dimensions, represented by the apparent area S of the obstacle's surface (i.e. the area of the projection of the obstacle on a plane perpendicular to the flow). It also depends on the square of the flow's velocity v and on the fluid's density ρ and on a dimensionless *penetration coefficient* C_x , according to the formula

$$|\vec{R}| = C_x \frac{\rho v^2}{2} S$$

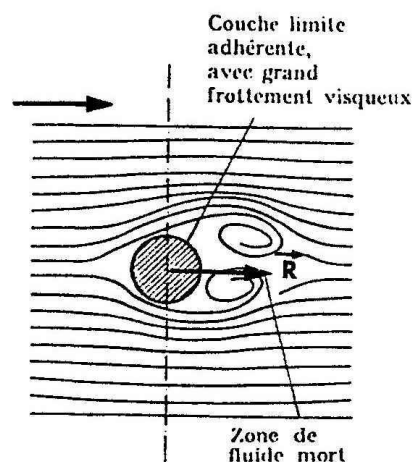


Fig 2 : Turbulent flow

The penetration coefficient C_x depends on the geometrical function of the obstacle, and is a slow-varying function of the Reynolds number. For relatively large intervals of R ($5'000 < R < 50'000$), the penetration coefficient C_x can be considered constant for a given geometric form. A few examples of C_x can be found in Fig. 3, for obstacles of cylindrical symmetry.

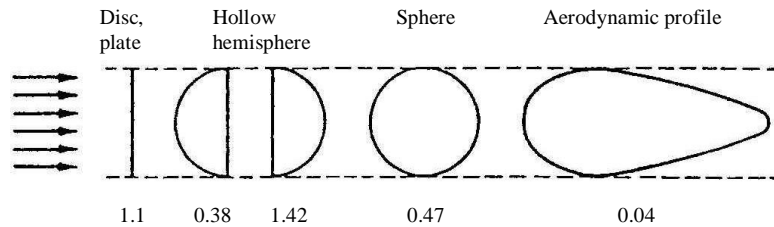


Fig 3 : Penetration coefficient for different cylindrical symmetry.

Lift and drag forces on a wing

Due to the asymmetry of a wing - particularly the sharp rear edge – and viscous friction on the wing's surface, a vortex forms at the back of the wing (Fig. 4). This vortex simultaneously generates a drag force \vec{W} (horizontal resistance) and a lift force \vec{F} (vertical force). Both forces are of the following form:

$$|\vec{W}| = C_W \frac{\rho v^2}{2} A \quad \text{et} \quad |\vec{F}| = C_F \frac{\rho v^2}{2} A$$

where C_W et C_F are the *lift* and *drag coefficients* respectively, and $A = l \cdot t$ is the lifting surface of the wing.



Fig 4 : Illustration of forces on a wing a wing.

The drag and lift coefficients C_W and C_F of a given wing depend on its geometric properties, in particular its cross-section (thickness and asymmetry), its length/depth ratio l/t , as well as its *angle of attack* γ , i.e. the angle between the wing and the flow's direction.

For a given wing, the angle of attack γ is therefore an adjustable parameter, allowing to adjust the lift and drag coefficients C_W and C_F . A *polar Lilienthal diagram*, is used to determine the optimal angle of attack. It is a graph of the lift coefficient C_F versus the drag coefficient C_W for different angles of

attack. The optimal angle of attack is defined as the angle for which the lift to drag ratio $\frac{|\vec{F}|}{|\vec{W}|} = \frac{C_F}{C_W}$ is maximal.

In a Lilienthal diagram (Fig. 5), the tangent to the curve that goes through the origin determines the maximal $\frac{C_F}{C_W}$ ratio, and optimal angle of attack

is given by the touching point.

Please note that a wing's maximal $\frac{C_F}{C_W}$ ratio

also depends on the wing's geometry. In particular, this value increases for larger l/t ratios (i.e. length/depth). This explains why gliders have very long and thin wings.

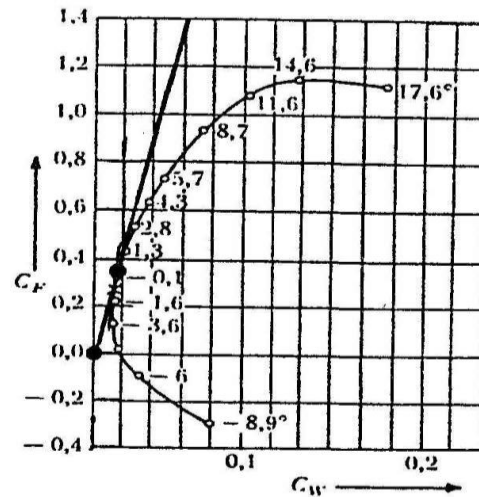


Fig 5: Lilienthal diagram.

III. EXPERIMENTAL SETUP

The wind tunnel you will use has: (see Fig. 6) :

- An aerodynamic transparent rectangular tunnel, with access through the bottom. It is connected to a fan whose throughput can be adjusted electronically.
- A mounting mechanism on which different objects can be attached, in order to study their penetration coefficients C_x . A wing can also be attached in order to study the lift and drag coefficients C_W et C_F . Finally, one can also attach a model of a racing car to determine the penetration coefficient. When studying the racing car, don't forget to place a stand beneath the car to emulate the road.
- The mounting mechanism includes two micrometers that allow the objects to be aligned vertically and horizontally, and for the wing, adjust the angle of attack.
- The mounting mechanism also includes two detectors that measure the horizontal and vertical force on the considered object. The electronics used to retrieve values from these captors feature a set of zeroing buttons, (coarse and fine), and the measure is done via low-pass filters to eliminate fluctuations due to turbulent flow. Please note that due to these filters, the response time is delayed, and it takes a little while for a measure to stabilize.
- A propeller anemometer is used for the digital measure of the flow's velocity in the wind tunnel. It is placed near the exit of the ejection pipe, after the fan and a homogenization segment. The anemometer can be moved up and down in order to measure a velocity profile of the flow.

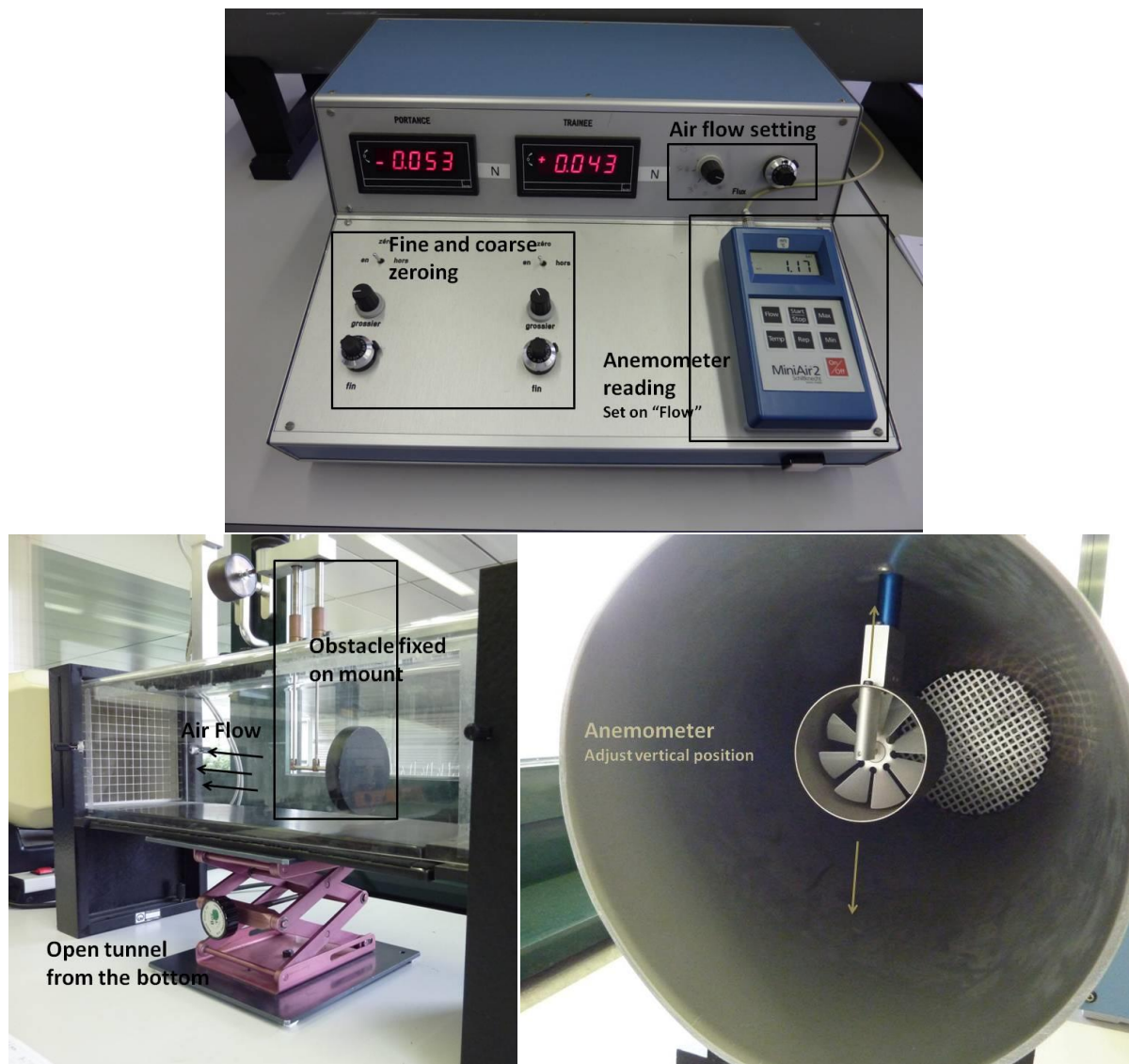


Fig 6: Experimental setup.

IV. POSSIBLE EXPERIMENTS

Calibration of the flow velocity

For a given velocity in the wind tunnel, measure the velocity profile at the exit of the ejection pipe, i.e. the air's velocity as a function of the height of the anemometer. Assuming a cylindrical symmetry of the flow, and using the hydrodynamics continuity equation,

$$\bar{v} \cdot S_1 = \oint_{S_1} \vec{v}_1 \cdot d\vec{S} = \oint_{S_2} \vec{v}_2 \cdot d\vec{S}$$

S_1 = cross-section of the wind tunnel

\bar{v} = average flow velocity in the tunnel

\vec{v}_2 = ejection velocity profile

S_2 = cross-section of the ejection pipe

determine a correction factor C_α allowing to deduce the flow's mean velocity \bar{v} in the tunnel, by measuring only the output velocity v_c at the center of the ejection pipe, i.e. find C_α such that $\bar{v} = C_\alpha v_c$.

Penetration coefficient of shapes with cylindrical symmetry

Measure the resistance force C_x of different shapes, as a function of the flow's average velocity \bar{v} in the tunnel. Talk about the effect of the object's shape and cross-section, as well as the flow's mean velocity \bar{v} on the penetration coefficient C_x . Verify the v^2 dependency of the drag force.

Drag and lift coefficients of a wing

For a given mean velocity, \bar{v} in the wind tunnel, measure the drag and lift of the wing as a function of the angle of attack. Plot the Lilienthal diagram $C_F(C_W)$ of the wing for a wide range of angles between -10° and $+20^\circ$. The zero angle is defined as the angle for which the lift vanishes.

Verify the v^2 dependency of the drag and lift force, for example, by plotting a second Lilienthal diagram for the same wing, using a very different value for \bar{v} , the mean velocity of the air flow.

Using the Lilienthal diagram, determine the maximal C_F/C_W ratio, and give the optimal angle of attack.

Penetration coefficient of a model car

Measure the resistance force and plot C_x versus the mean velocity \bar{v} . When taking this measure, be sure to place a stand under the car, to emulate the road. Using the micrometers, place the car 1/2 mm over the road to avoid friction.

Simultaneously measure the vertical force on the car as a function of \bar{v} and talk about the sign of the force.

Calculate the respective Reynolds numbers of the scaled down model and a real racing car, knowing that a real car can reach 300 km/h. Using these Reynolds numbers, talk about the validity of the C_x measured on the model in comparison with that of a real car. What would you need to change in order to get comparable results?