A1. Mechanical Bench

I. INTRODUCTION AND OBJECTIVE OF THE EXPERIMENT

An important part of mechanics relies on a certain amount of measurable quantities that were discovered phenomenologically, before being given a mathematical definition. For instance, the concepts of velocity and acceleration rely on the mathematical definitions of vectors and derivatives. Other concepts, such as forces, potential and kinetic energy, and more, are tied to the previous phenomenological concepts (e.g. friction) or fundamental equations (e.g. Newton's equation).
This experiment will attempt to study a few different properties of motion, such as friction and gravitational force, and look at the elementary equations describing collisions. All of this will be achieved only by measuring speeds of a glider on an air-cushion bench.

II. EXPERIMENTAL SETUP

A glider represents the point mass, and moves on an air-cushion bench (fig. 1). In order to get decent quantitative results, it is necessary to reduce friction on the bench. This is made possible within comfortable limits, by using pressurized air to make the glider hover. The bench's angle is adjusted by turning a screw located at one extremity; one turn of the screw results in an angle variation α of 0.001 radian. The glider's speeds can be measured using two optical gates: every time the glider goes through a gate, it blocks the laser's optical path, and a "Phywe" stopwatch measures the duration of this interruption (fig. 1). Knowing the time t of the interruption, and the glider's length a, we can determine the glider's speed.

$$v = \frac{a}{t}$$

After having launched the glider in position A, with an initial velocity v_0 ,the optical gates 1 and 2 allow us to measure the following crossing times (the stopwatch's button '7' must be placed on position 3):
- In the first direction (*a*), at position B, the time t_{2a} is measured, and thus v_{2a} at gate 2,
- In the direction (a), at position C, the time t_{1a} is measured, and thus v_{1a} at gate 1,
- In the second direction (*b*), after the impact in D, at position C, the time t_{1b} and speed v_{1b} at gate 1,
- In the direction (b), after the impact D, at B, the time t_{2b} and speed v_{2b} at gate 2 are measured.

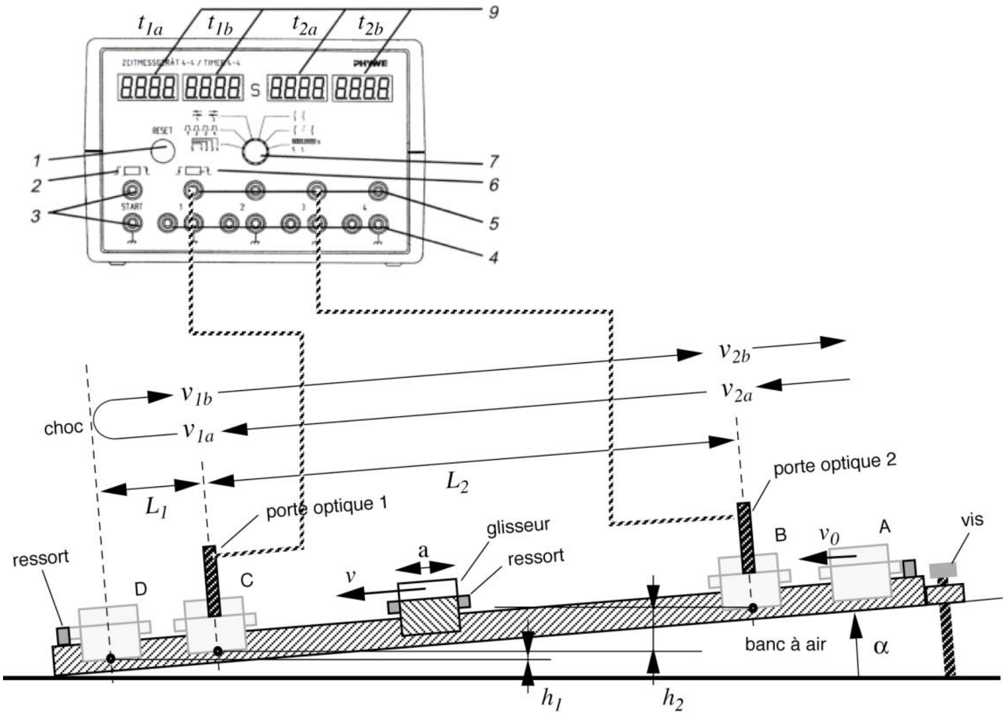

Figure 1: The mechanical air-cushion bench, and the “Phywe” stopwatch.

III. THEORY

III.1. PROPERTIES OF FRICTION

Despite the presence of air-cushions, some friction will still be present. We shall phenomenologically determine the type of friction occurring, by choosing from the following three possibilities:

- **dry friction** $f_{fr}^{\sec}$, indépendante de la vitesse du mobile, mais en général proportionnelle à la force normale $N = mg\cos\alpha$ exercée par le mobile sur son support

$f_{fr}^{\sec} = K\,N = K\;mg\cos\alpha$ où K = le coefficient de frottement sec dynamique (1)

- **viscous friction in a laminar flow** $f_{fr}^{lam}(v)$, that varies linearly with the speed v of the slider

$f_{fr}^{lam}(v) = \beta v$ où β = le coefficient de frottement visqueux laminaire (2)

- **viscous friction in a turbulent flow** $f_{fr}^{turb}(v)$, that varies quadratically with the speed v of the slider

$f_{fr}^{turb}(v) = \gamma v^2$ où γ = le coefficient de frottement visqueux turbulent (3)

Note that the γ coefficient depends on the penetration coefficient of the glider, the density of air ρ, and the apparent area of the glider S, according to the equation $\gamma = C_x \rho S / 2$.

III.2. MEASURING FRICTION

We can measure the glider's energy variation between the points B and C. In the first direction (a), the glider's kinetic energy at point C is equal to that in B minus the energy dissipated through friction between B and C plus the potential energy due to the height difference *h* between the two points. If the average friction in the first direction (a) on a distance L_2 between B and C is given by $\bar{f}_a^{L2}$, we can write:

$$\frac{1}{2}mv_{1a}^{\;2} = \frac{1}{2}mv_{2a}^{\;2} - \bar{f}_a^{L2}L_2 + mgh_2 = \frac{1}{2}mv_{2a}^{\;2} - \bar{f}_a^{L2}L_2 + mgL_2\sin\alpha \qquad (4)$$

Similarly, if is the average friction acting on the distance L_2 between C and B in the second direction (b) is $\bar{f}_b^{L2}$, we have:

$$\frac{1}{2}mv_{2b}^{\;2} = \frac{1}{2}mv_{1b}^{\;2} - \bar{f}_b^{L2}L_2 - mgh_2 = \frac{1}{2}mv_{1b}^{\;2} - \bar{f}_b^{L2}L_2 - mgL_2\sin\alpha \qquad (5)$$

Adding (4) and (5) together yields an expression independent of the angle α

$$\frac{1}{2}mv_{1a}^{\;2} + \frac{1}{2}mv_{2b}^{\;2} = \frac{1}{2}mv_{2a}^{\;2} + \frac{1}{2}mv_{1b}^{\;2} - (\bar{f}_a^{L2} + \bar{f}_b^{L2})L_2$$

from which we can easily deduce an **average friction force** $\bar{f}$ for an for an average round-trip speed $\bar{v}$

$$\begin{cases} \bar{f} = (\bar{f}_a^{L2} + \bar{f}_b^{L2})/2 = \dfrac{m}{4L_2}\left(v_{2a}^{\;2} + v_{1b}^{\;2} - v_{1a}^{\;2} - v_{2b}^{\;2}\right) \\ \bar{v} = \dfrac{1}{4}\left(v_{2a} + v_{1b} + v_{1a} + v_{2b}\right) \end{cases} \qquad (6)$$

III.3. RESTITUTION COEFFICIENT OF A COLLISION

If the shock at point D is not perfectly elastic, part of the energy will be dissipated as heat in the springs. If we define the kinetic energy E_{cin}^a of the slider right before the impact, and the kinetic energy E_{cin}^b right after the impact, we can define the **restitution coefficient of the collision** C as the ratio of E_{cin}^b and E_{cin}^a

$$C_r = \frac{E_{cin}^b}{E_{cin}^a} \qquad (7)$$

The kinetic energies before and after the impact can easily be extrapolated from the kinetic energies measured at the point C

$$\begin{cases} E_{cin}^a = \dfrac{1}{2}mv_{1a}^{\;2} - f_a^C L_1 + mgh_1 = \dfrac{1}{2}mv_{1a}^{\;2} - f_a^C L_1 + mgL_1\sin\alpha \\ E_{cin}^b = \dfrac{1}{2}mv_{1b}^{\;2} + f_b^C L_1 + mgh_1 = \dfrac{1}{2}mv_{1b}^{\;2} + f_b^C L_1 + mgL_1\sin\alpha \end{cases}$$

where f_a^C and f_b^C are the friction forces in the direction (a) and (b) at the point C . The restitution coefficient can be written:

$$C_r = \frac{\frac{1}{2}mv_{1b}^{\ 2} + f_b^C L_1 + mgL_1 \sin\alpha}{\frac{1}{2}mv_{1a}^{\ 2} - f_a^C L_1 + mgL_1 \sin\alpha} \qquad (8)$$

We notice that measuring C_r will be easier if the bench is horizontal since $\sin\alpha = 0$.

III.4. MEASURING EARTH'S GRAVITATIONAL PULL

Subtracting (5) from (4) yields:

$$\frac{1}{2}mv_{1a}^{\ 2} - \frac{1}{2}mv_{2b}^{\ 2} = \frac{1}{2}mv_{2a}^{\ 2} - \frac{1}{2}mv_{1b}^{\ 2} + (\bar{f}_b^{L2} - \bar{f}_a^{L2})L_2 + 2mgL_2 \sin\alpha$$

from which we can determine the **gravitational acceleration** g

$$g = \frac{1}{4L_2 \sin\alpha}\left(v_{1b}^{\ 2} - v_{2b}^{\ 2} + v_{1a}^{\ 2} - v_{2a}^{\ 2}\right) - \frac{1}{2m\sin\alpha}(\bar{f}_b^{L2} - \bar{f}_a^{L2}) \qquad (9)$$

We notice that this measurement depends only weakly on the friction, since only the difference $(\bar{f}_b^{L2} - \bar{f}_a^{L2})$ appears in the expression. However, in order to get an appreciable precision on the obtained result, we will need to determine the phenomenological law governing friction precisely, as well as the corresponding coefficient ($K, \beta\ ou\ \gamma$)

III.5. CONSERVATION OF MOMENTUM DURING A COLLISION

When two objects collide, the total momentum is conserved. We can verify this law by using the air cushion bench in the horizontal position, and launching against one another two gliders A and B, of masses m_A and m_B, and initial velocities v_{A0} and v_{B0} (fig. 2). According to the the law of collisions, the momentum (p_a)before and (p_b)after the impact must be the same, independently of whether the shock is elastic or not. This leads to:

$$p_a = m_A v_{Aa} - m_B v_{Ba} = -m_A v_{Ab} + m_B v_{Bb} = p_b \qquad (10)$$

The speeds of A and B before (v_{Aa}, v_{Ba})and after (v_{Ab}, v_{Bb})can be determined from the speeds $v_{1b}, v_{1a}, v_{2a}, v_{2b}$ measured in the optical gates 1 and 2, using the following set of equations before the impact

$$\frac{1}{2}m_A v_{Aa}^{\ 2} = \frac{1}{2}m_A v_{1a}^{\ 2} - \delta_A f_a^A \quad \text{and} \quad \frac{1}{2}m_B v_{Ba}^{\ 2} = \frac{1}{2}m_B v_{2a}^{\ 2} - \delta_B f_a^B$$

where f_a^A et f_a^B are the friction forces acting on the gliders in the first direction (a) on the gliders A and B, at gate 1 and 2.

and after the impact:

$$\frac{1}{2}m_A v_{Ab}^{\ 2} = \frac{1}{2}m_A v_{1b}^{\ 2} + \delta_A f_b^A \quad \text{and} \quad \frac{1}{2}m_B v_{Bb}^{\ 2} = \frac{1}{2}m_B v_{2b}^{\ 2} + \delta_B f_b^B$$

where f_b^A and f_b^B the friction forces acting on the gliders in the second direction (b) on the gliders A and B, at gate 1 and 2.

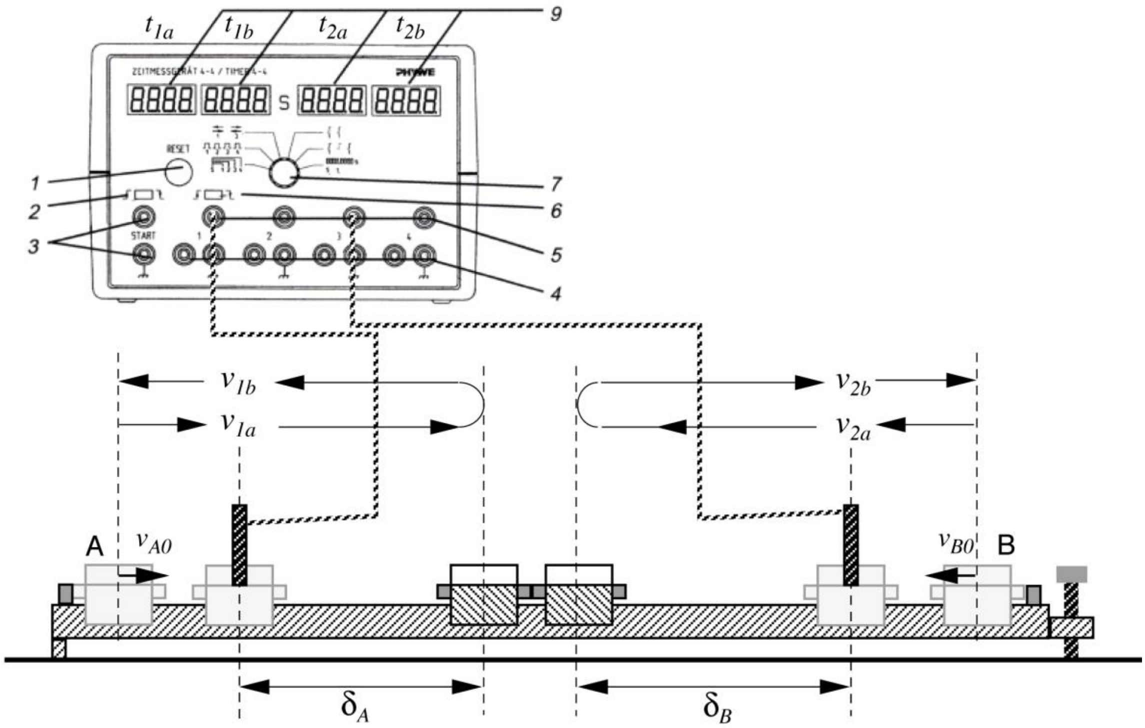

Figure 2- La configuration pour le test de la conservation de la quantité de mouvement

These equations can be approximated as:

$$m_A v_{Aa} = m_A v_{1a}\sqrt{1-\frac{2\delta_A f_a^A}{m_A {v_{1a}}^2}} \cong m_A v_{1a}\left(1-\frac{\delta_A f_a^A}{m_A {v_{1a}}^2}\right) = m_A v_{1a} - \frac{\delta_A f_a^A}{v_{1a}} \qquad \Rightarrow$$

$$\begin{cases} m_A v_{Aa} \cong m_A v_{1a} - \dfrac{\delta_A f_a^A}{v_{1a}} \qquad and \qquad m_B v_{Ba} \cong m_B v_{2a} - \dfrac{\delta_B f_a^B}{v_{2a}} \\ m_A v_{Ab} \cong m_A v_{1b} + \dfrac{\delta_A f_b^A}{v_{1b}} \qquad and \qquad m_B v_{Bb} \cong m_B v_{2b} + \dfrac{\delta_B f_b^B}{v_{2b}} \end{cases}$$

in such a way that the law of collisions (10) can be well approximated by:

$$p_a \cong m_A v_{1a} - \frac{\delta_A \overline{f}_a^{\delta A}}{v_{1a}} - m_B v_{2a} + \frac{\delta_B f_a^B}{v_{2a}} \cong -m_A v_{1b} - \frac{\delta_A f_b^A}{v_{1b}} + m_B v_{2b} + \frac{\delta_B f_b^B}{v_{2b}} \cong p_b \qquad (11)$$

IV. SUGGESTED EXPERIMENTS

IV.1. Friction

Turn on the air bench using the valve attached to the table leg. Set the bench in its horizontal position ($\alpha = 0$). You can adjust and verify the horizontality using a slider. It's very sensitive to any slight angle variation.
Perform ***at least a dozen*** experiments with each one of the sliders, while making sure to change the initial velocity every time, in order to get a wide enough range of measures. Each experiment therefore yields the 4 speeds $v_{1b}, v_{1a}, v_{2a}, v_{2b}$.

- For each on of theses measures, determine the ***average friction*** $\bar{f}$ and ***the average speed*** $\bar{v}$ at which is was measured according to equation (6)***.***
- Plot all results on a graph of $\bar{f}$ versus $\bar{v}$. Is the friction the same for all three sliders? Phenomenologically, is the observed friction dry ($f_{fr}^{\sec} = K N = K\, mg\cos\alpha$), viscous ($f_{fr}^{lam}(v) = \beta v$), or turbulent ($f_{fr}^{turb}(v) = \gamma v^2$) ?
- Determine the equation $f_{fr} = f_{fr}(v)$ by calculating the associated $K, \beta\ ou\ \gamma$ coefficient for each slider.

IV.2. The restitution coefficient

Using the results $v_{1b}, v_{1a}, v_{2a}, v_{2b}$ obtained previously, we can determine the ***restitution coefficient***, knowing that $\alpha = 0$, using (8). Furthermore, having already determined the friction $f_{fr} = f_{fr}(v)$ in IV.1, we can replace the friction forces f_a^C and f_b^C with their expressions $f_a^C \cong f_{fr}(v_{1a})$ and $f_b^C \cong f_{fr}(v_{1b})$, such that

$$C_r \cong \frac{m{v_{1b}}^2 + 2L_1 f_{fr}(v_{1b})}{m{v_{1a}}^2 - 2L_1 f_{fr}(v_{1a})} = \begin{cases} \dfrac{{v_{1b}}^2 + 2L_1 Kg}{{v_{1a}}^2 - 2L_1 Kg} & si \quad f_{fr}(v) = Kmg \\ \dfrac{{v_{1b}}^2 + 2L_1 \beta v_{1b} / m}{{v_{1a}}^2 - 2L_1 \beta v_{1a} / m} & si \quad f_{fr}(v) = \beta v \\ \dfrac{{v_{1b}}^2 (m + 2L_1\gamma)}{{v_{1a}}^2 (m - 2L_1\gamma)} & si \quad f_{fr}(v) = \gamma v^2 \end{cases}$$

- For each of the three gliders, plot C_r as a function of the incident speed v_{1a}. What parameters does C_r mainly depend on?

IV.3. Gravitational acceleration

The bench is placed in an slanted position ($\alpha \neq 0$, of the order of 5×10^{-3} radian). Perfmor ***at least twenty*** experiments with the steel spring slider. Once again, make sure to vary the initial speed every time, in order to get a wide range of measures. Each experiment yields 4 different speeds $v_{1b}, v_{1a}, v_{2a}, v_{2b}$, from which we can determine the ***gravitational acceleration*** *g* using equation (9), by estimating the friction term $(\bar{f}_b^{L2} - \bar{f}_a^{L2})$ using the relation $f_{fr} = f_{fr}(v)$ from paragraph IV.1, i.e.

$$(\bar{f}_b^{L2} - \bar{f}_a^{L2}) \cong \frac{f_{fr}(v_{1b}) + f_{fr}(v_{2b})}{2} - \frac{f_{fr}(v_{1a}) + f_{fr}(v_{2a})}{2}$$

such that

$$g = \frac{1}{4L_2 \sin\alpha}\left(v_{1b}{}^2 - v_{2b}{}^2 + v_{1a}{}^2 - v_{2a}{}^2\right) \begin{cases} +0 & if \quad f_{fr}(v) = Kmg\cos\alpha \\ -\dfrac{\beta}{4m\sin\alpha}\left(v_{1b}+v_{2b}-v_{1a}-v_{2a}\right) & if \quad f_{fr}(v) = \beta v \\ -\dfrac{\gamma}{4m\sin\alpha}\left(v_{1b}{}^2+v_{2b}{}^2-v_{1a}{}^2-v_{2a}{}^2\right) & if \quad f_{fr}(v) = \gamma v^2 \end{cases}$$

- calculate the value g for each experiment, estimate the error made on this value, and indicate the ideal measuring conditions for g.

IV.4. The law of impacts

Using the setup of figure 2, we shall study the law of impacts (10) by verifying equation (11), in which the average friction f_a^A, f_a^B, f_b^A and f_b^B are to be estimated using the law $f_{fr} = f_{fr}(v)$ determined in paragraph IV.1. A good way to proceed is to determine the ratio $P = \left(p_a - p_b\right)/\left(m_A\left(v_{1a}+v_{1b}\right)+m_B\left(v_{2a}+v_{2b}\right)\right)$ experimentally, i.e.

$$P = \begin{cases} \dfrac{m_A\left(v_{1a}+v_{1b}\right)-m_B\left(v_{2a}+v_{2b}\right)+Kg\left(\delta_A m_A\left(\dfrac{1}{v_{1b}}-\dfrac{1}{v_{1a}}\right)-\delta_B m_B\left(\dfrac{1}{v_{2b}}-\dfrac{1}{v_{2a}}\right)\right)}{m_A\left(v_{1a}+v_{1b}\right)+m_B\left(v_{2a}+v_{2b}\right)} & if \quad f_{fr}(v) = Kmg \\ \dfrac{m_A\left(v_{1a}+v_{1b}\right)-m_B\left(v_{2a}+v_{2b}\right)}{m_A\left(v_{1a}+v_{1b}\right)+m_B\left(v_{2a}+v_{2b}\right)} & if \quad f_{fr}(v) = \beta v \\ \dfrac{m_A\left(v_{1a}+v_{1b}\right)-m_B\left(v_{2a}+v_{2b}\right)-\delta_A\gamma\left(v_{1a}-v_{1b}\right)+\delta_B\gamma\left(v_{2a}-v_{2b}\right)}{m_A\left(v_{1a}+v_{1b}\right)+m_B\left(v_{2a}+v_{2b}\right)} & if \quad f_{fr}(v) = \gamma v^2 \end{cases}$$

- measure the speeds $v_{1b}, v_{1a}, v_{2a}, v_{2b}$ for about 10 different experiments with the bench in a horizontal position. Calculate the ratio $\left(p_a - p_b\right)/\left(p_a + p_b\right)$ obtained in each one of these cases, supposing that $\delta_A \cong \delta_B$, and discuss the results.

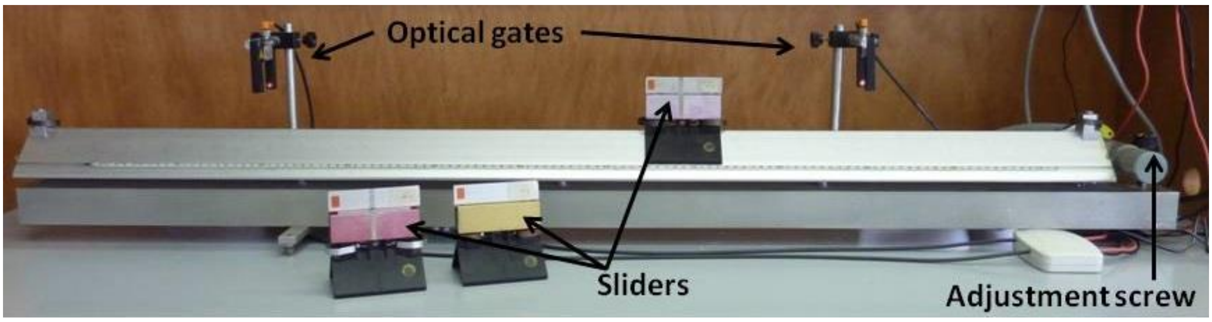

Fig. 3 : Image of the experimental setup