Determination of the global variables of a vibrated granular medium : An application of the fluctuation-dissipation theorem

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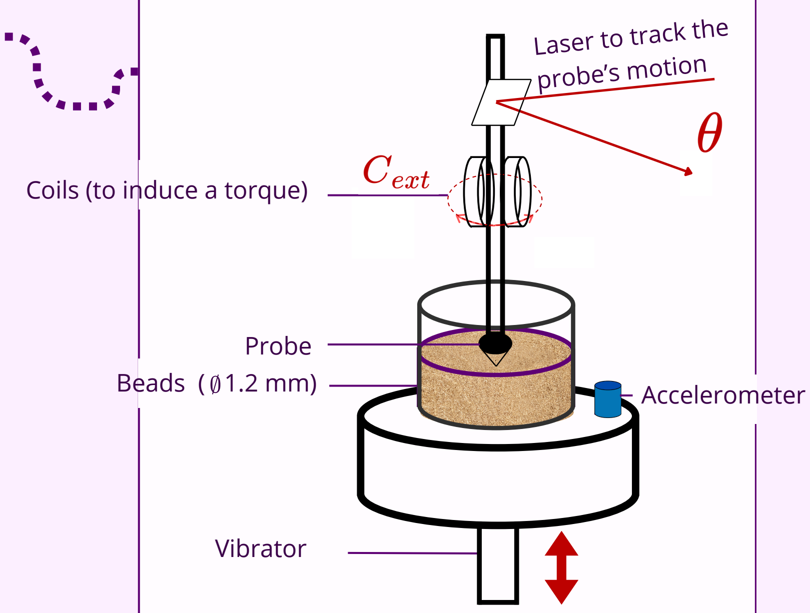
Introduction

The fluctuation-dissipation theorem (FDT) is a central result of statistical physics that, in particular, allows to predict the response of a system given the thermodynamics fluctuations. Therefore kowing the response allows to find back global variables such as the thermodynamic temperature of a fluid.

Suppose a granular medium made of macroscopic beads, one could make an analogy to a fluid, in an attempt of finding out about global variables of the medium such as the temperature or friction coefficient, given the response of the system. Their definition would then be proper to the granular medium, in the same way the thermodynamic temperature is proper to a fluid.

Experimental setup

Torsion oscillator immerged in a granular medium



- The vibrator imposes a white noise onto the beads, inducing a random white noise torque $\xi(t)$ onto the probe.
- The coils induce a periodic torque $C_{\text{ext}}(t)$ onto the probe

The vibration's intensity directly influences $\xi(t)$, and is given by :

$$\Gamma = \frac{\text{peak acceleration}}{g}$$

Characterizing the response of the medium

The probe then follows the **motion equation** : $I\ddot{\theta}(t) = -G\theta(t) - \alpha\dot{\theta}(t) + \xi(t) + C_{\text{ext}}(t)$

It follows that the **linear response (susceptibility)** of the system and its imaginary part are :

$$\tilde{\chi}(\omega) = \boxed{\frac{\tilde{\theta}(\omega)}{\tilde{C}_{\text{ext}}(\omega)}} = \frac{1}{(\omega^2 - \omega_0^2)I + i\alpha\omega} \qquad \tilde{\chi}'' = \frac{\alpha\omega}{(\omega^2-\omega_0^2)^2 I^2 + \alpha^2\omega^2}$$

which can thus be measured.

Any **power spectral density (PSD)** is computed using : $S(\omega) = 2\,\langle|\tilde{\theta}(\omega)|^2\rangle$

Therefore, one can measure $S_\theta(\omega)$, and using the linear response one can compute $S_\xi(\omega)$

which by considering the **Langevin equation** ($I\ddot{\theta}(t) = -G\theta(t) + \alpha\dot{\theta}(t) + \xi(t)$) can be derived to be :

$$S_\xi(\omega) = \frac{4\alpha k_B T}{(\omega^2-\omega_0^2)^2 I^2 + \alpha^2\omega^2}$$

where T is defined by applying the **equipartition theorem** to the probe's random torque $\xi(t)$ stemming from the excited beads

FDT

The **fluctuation-dissipation theorem** then yields

$$\frac{S_\xi(\omega)\cdot\omega}{4\tilde{\chi}''(\omega)} = k_B T$$

allowing to **experimentally find** T

Parameters of the experiment

The aim is to find :

- T through the FDT
- α the friction of the beads (damping of the system) through a fit of $S_\xi(\omega)$

One can change :

- The depth of the probe
- The shape of the probe
- The material of the beads : **glass, metal**

Experiment variations :

Varrying depth

Great depth

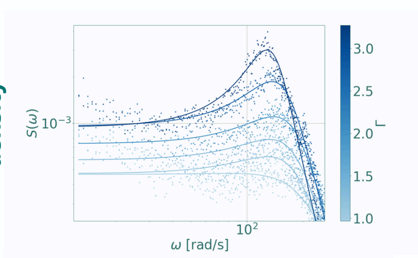
→ Negative PSD
→ Unphysical results given the model

Low depth

→ Exploitable results
→ Minimal depth is the way to go!

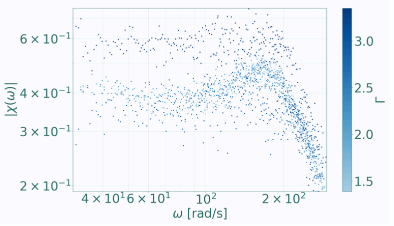
Results with glass, high conical probe

Power spectral density



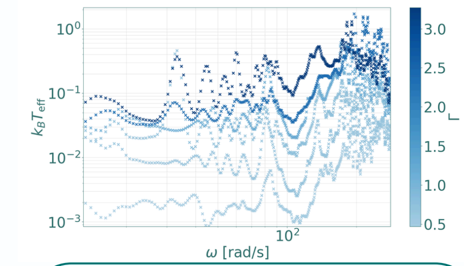
→ Fit works well, **model is valid**
→ Expected trend :
Large excitation → Great power spectral density
→ Bump at the mechanical resonance frequency of the setup

Susceptibility



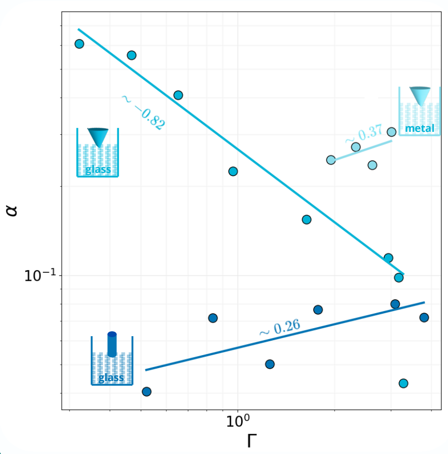
→ Increased linear response with excitation

→Bump at the mechanical resonance frequency of the setup



→ Lot of noise
→ Some flat domains where the FDT is valid.
→ Higher Γ is correlated with higher temperature
Large excitation → Large T

Friction α versus Γ



→ Friction decreases and **medium fluidifies**

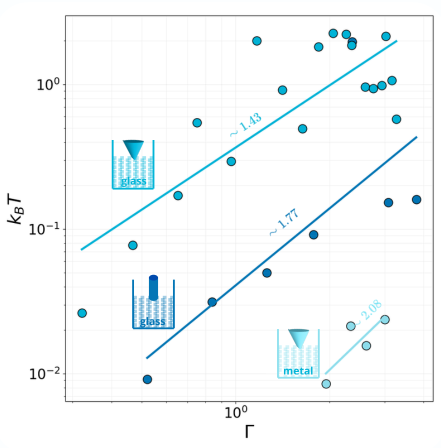
→ Friction does not change much.

→ Cylinder **not able to capture the motion of the beads** as their excitation is parallel to it.

→ No conclusive trend

→ Friction much higher than glass at the same excitation

Energy $k_B T$ versus Γ



→ $k_B T \sim \Gamma^{1.43}$

→ **Energy grows with** Γ, as expected

→ $k_B T \sim \Gamma^{1.77}$

→ **Smaller energy** as it is harder to excite the cylinder

→ $k_B T \sim \Gamma^{2.08}$

→ **Smaller energy as the grains are heavier.** Less motion for the same excitement

Conclusion

→ Model fits the data well
→ Barely imerged probes are better
→ FDT can be applicable to find T
→ Energy varies linearly to quadratically with Γ
→ Glass medium fluidifies when strongly vibrated
→ Metal beads have a higher friction
→ Cylindrical probe is highly inefficient

References

- Traveaux pratiques 3, "Physique des milieux granulaires", EPFL
- P. Mayor, "Fluid and glassy phases of vibrated granular matter studied with a torsion oscillator", Ph.D. thesis, Lausanne, EPFL, 2005