Testing out the Fluctuation-Dissipation Theorem in granular matter

Tom Vadot, Matteo Veneziano, TA: Thomas Hagger

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Motivation

→ **Fluctuation-dissipation theorem** (FDT) generalises the notion of temperature [1].

→ Granular media are **complex non-equilibrium systems**, FDT can be used to study their collective behaviour.

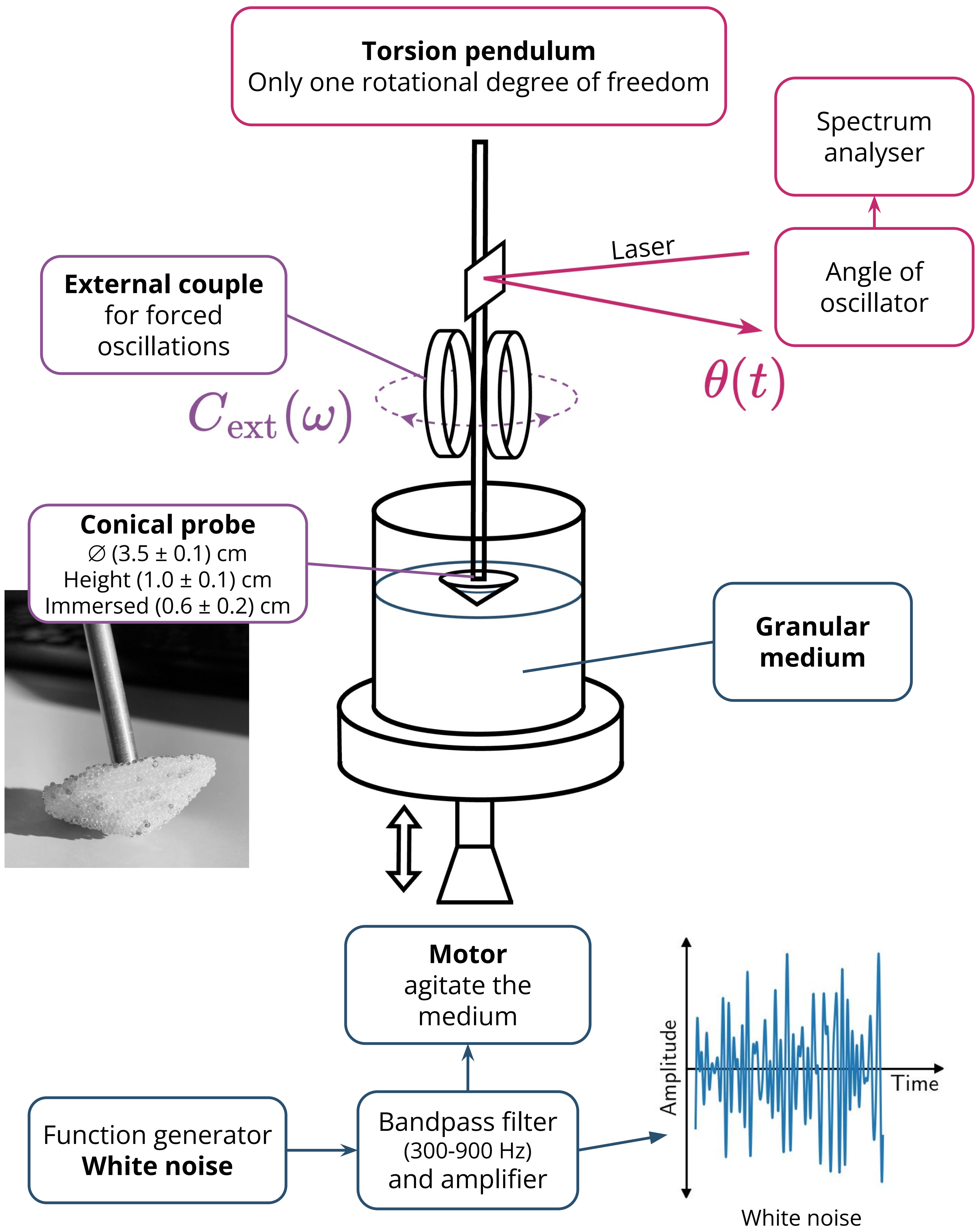
→ Study of mechanical interactions with granular materials have applications in **space exploration**: rovers have to move on diverse fine sands on Mars [2].

Objectives

→ Show the **fluid-like** behavior of agitated granular matter.

→ Verify whether the system has reached **thermodynamic equilibrium**.

Experimental Setup

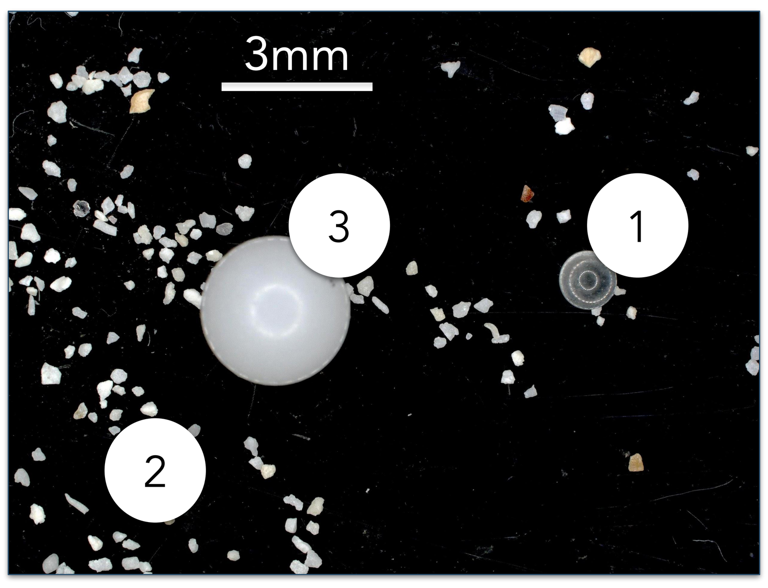


→ System described by **Langevin's equation**

$$I\langle\ddot{\theta}(t)\rangle + \alpha\langle\dot{\theta}(t)\rangle + I\omega_0^2\langle\theta(t)\rangle = C(t)$$

I : probe moment of inertia
α : friction coefficient
ω_0 : resonance frequency

→ Three granular materials studied



Microscopy image of the granular materials

Material	Diameter [mm]	Mass [mg]
(1) Glass	1.09 ± 0.08	1.9 ± 0.1
(2) Sand	0.34 ± 0.05	0.20 ± 0.05
(3) Plastic	2.9 ± 0.1	19.8 ± 0.01

Conclusions

→ Behaves **fluid-like** when strongly agitated, $\alpha \propto \text{Amplitude}^{-1}$, as in [3].

→ **Friction reaches a minimum**. Possibly due to long chains of microscopic interactions being unable to form under strong vibrations.

→ Thermodynamic equilibrium is not reached ⇒ still dependent on history. Probe was **possibly not immersed enough** [3].

→ Observed **resonance** due to interactions with rough surface of probe [3].

Friction in granular medium

Susceptibility

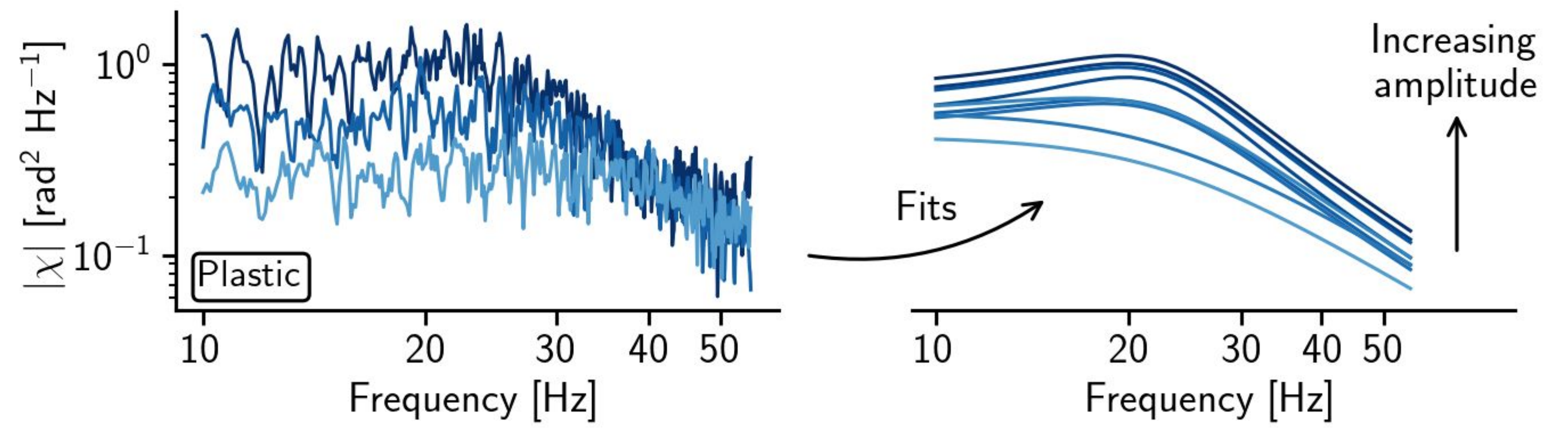
→ When the system is subject to an external couple C, its response is given by the **susceptibility function** χ, defined in Fourier space by

$$\tilde{\theta}(\omega) = \tilde{\chi}(\omega)\tilde{C}(\omega) \qquad \tilde{\chi}(\omega) = \tilde{\chi}'(\omega) - i\tilde{\chi}''(\omega)$$

→ Taking the Fourier transform of Langevin's Equation then yields

$$|\tilde{\chi}(\omega)| = \frac{1}{\sqrt{I^2(\omega^2-\omega_0^2)^2+\alpha^2\omega^2}} \qquad \tilde{\chi}''(\omega) = \frac{\alpha\omega}{I^2(\omega^2-\omega_0^2)^2+\alpha^2\omega^2}$$

→ Obtained data was fitted to find Langevin's equation parameters

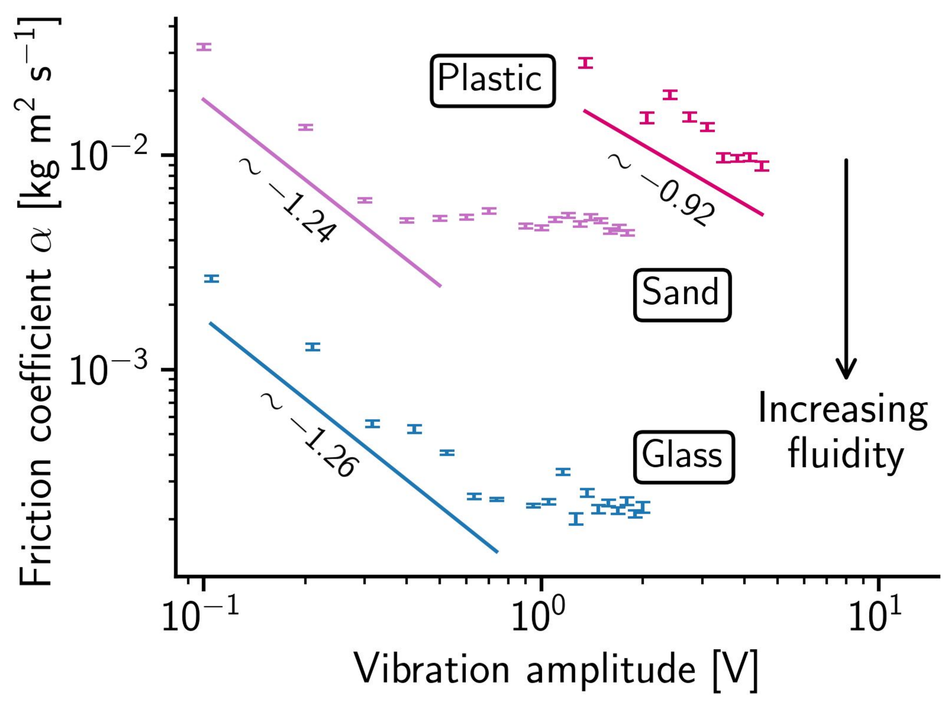


Friction

→ Assume relation between applied voltage and maximal acceleration is linear.

→ **Friction inversely depends on vibration amplitude ⇒** viscosity of "fluid" lowers.

→ **Plateau**: minimal friction reached for this medium. No plateau reached for the plastic medium, due to its large mass and size.



Fluctuation-Dissipation

More than one peak ⇒ resonance

Power Spectral Density (PSD) $S(\omega)$

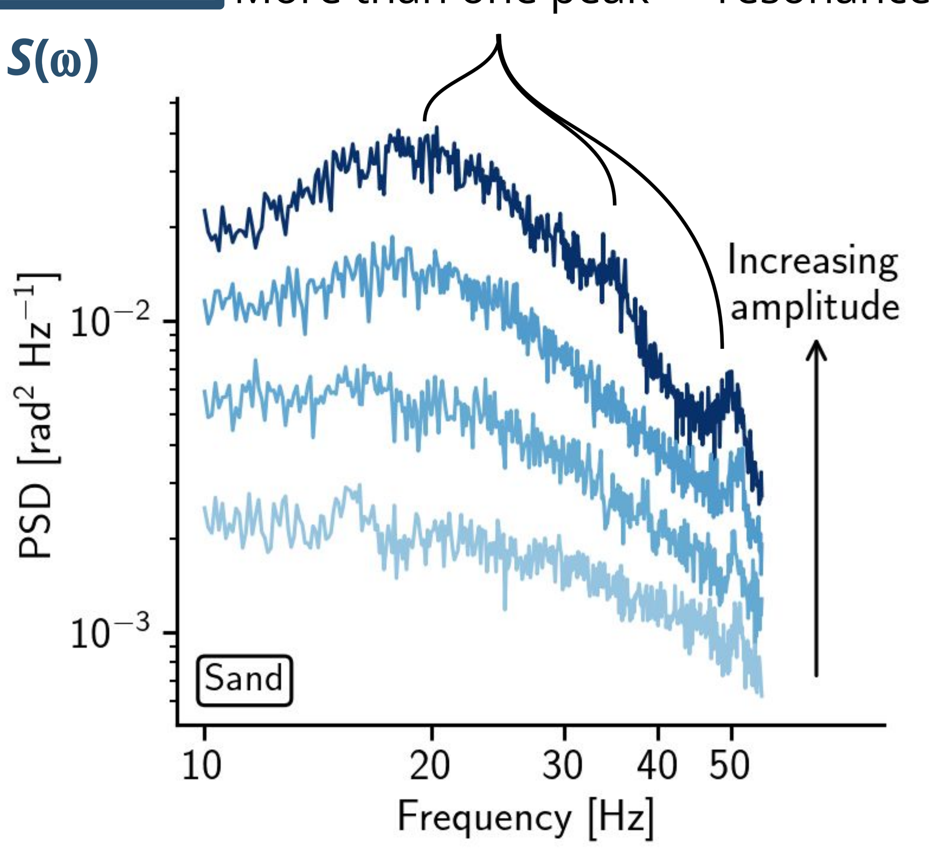
$$S(\omega) = \lim_{\tau\to\infty}\frac{2}{\tau}\left\langle\left|\tilde{\theta}_\tau(\omega)\right|^2\right\rangle$$

$$S(\omega) = \frac{2q}{I^2(\omega^2-\omega_0^2)^2+\alpha^2\omega^2}$$

with q the amplitude of the random microscopic forces.

→ The PSD is greater with larger pendulum fluctuations.

→ Measure of the influence of the **agitation of the medium** surrounding the probe.



Fluctuation-Dissipation Theorem

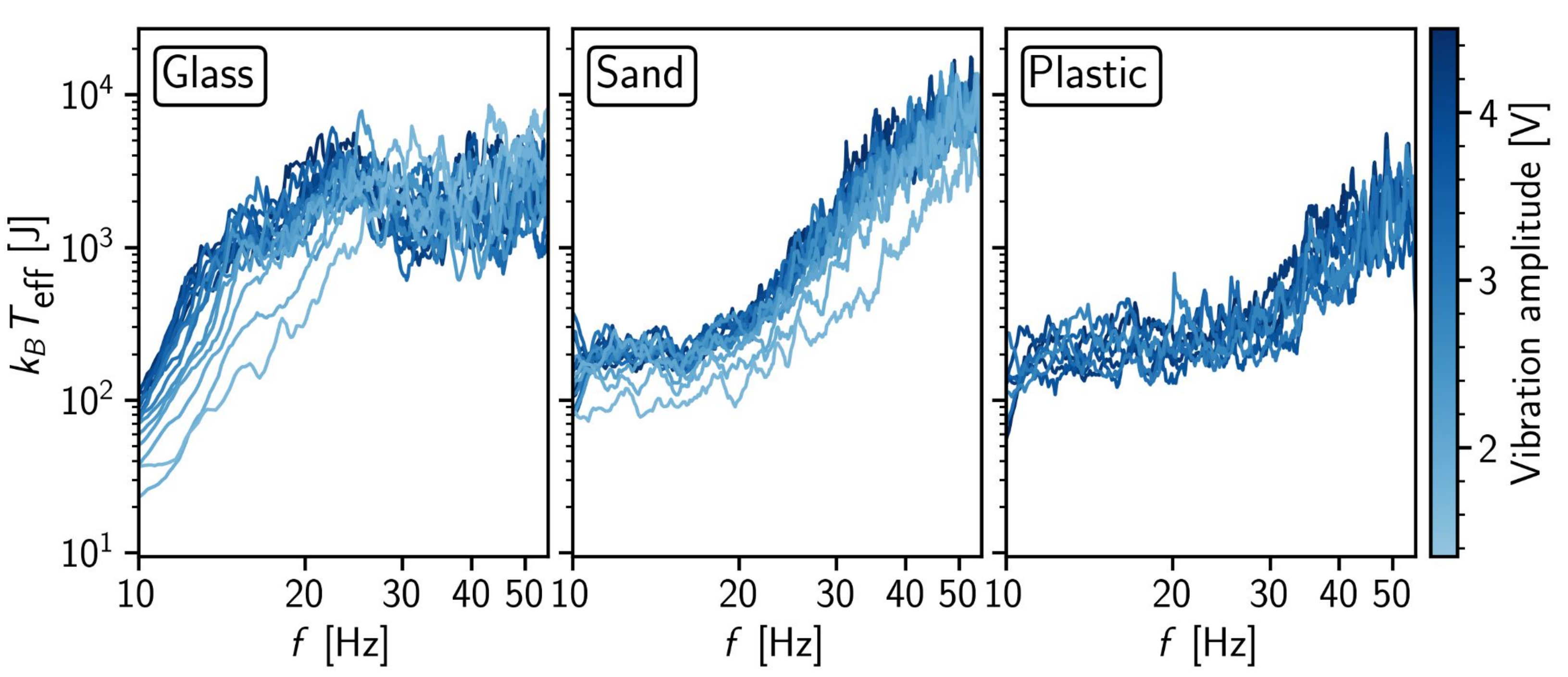
$$\frac{S(\omega)\omega}{4\tilde{\chi}''(\omega)} = k_B T_{\text{eff}}(\omega)$$

→ Generalised for non-equilibrium systems by defining frequency-dependent effective temperature T_{eff}.

→ If the theorem is satisfied, T_{eff} is constant.

→ Measures reveal **non-constant effective temperature** between 10-55 Hz.

→ Slight **dependence on vibration amplitude**.



[1] Travaux Pratiques de Physique, "Physique des milieux granulaire", EPFL
[2] R. A. Wilkinson, R. P. Behringer, J. T. Jenkins and M. Y. Louge, "Granular Materials and the Risks They Pose for Success on the Moon and Mars", AIP Conference Proceedings, 2005
[3] P. Mayor, "Fluid and glassy phases of vibrated granular matter studied with a torsion oscillator", Ph.D. thesis, Lausanne, EPFL, 2005