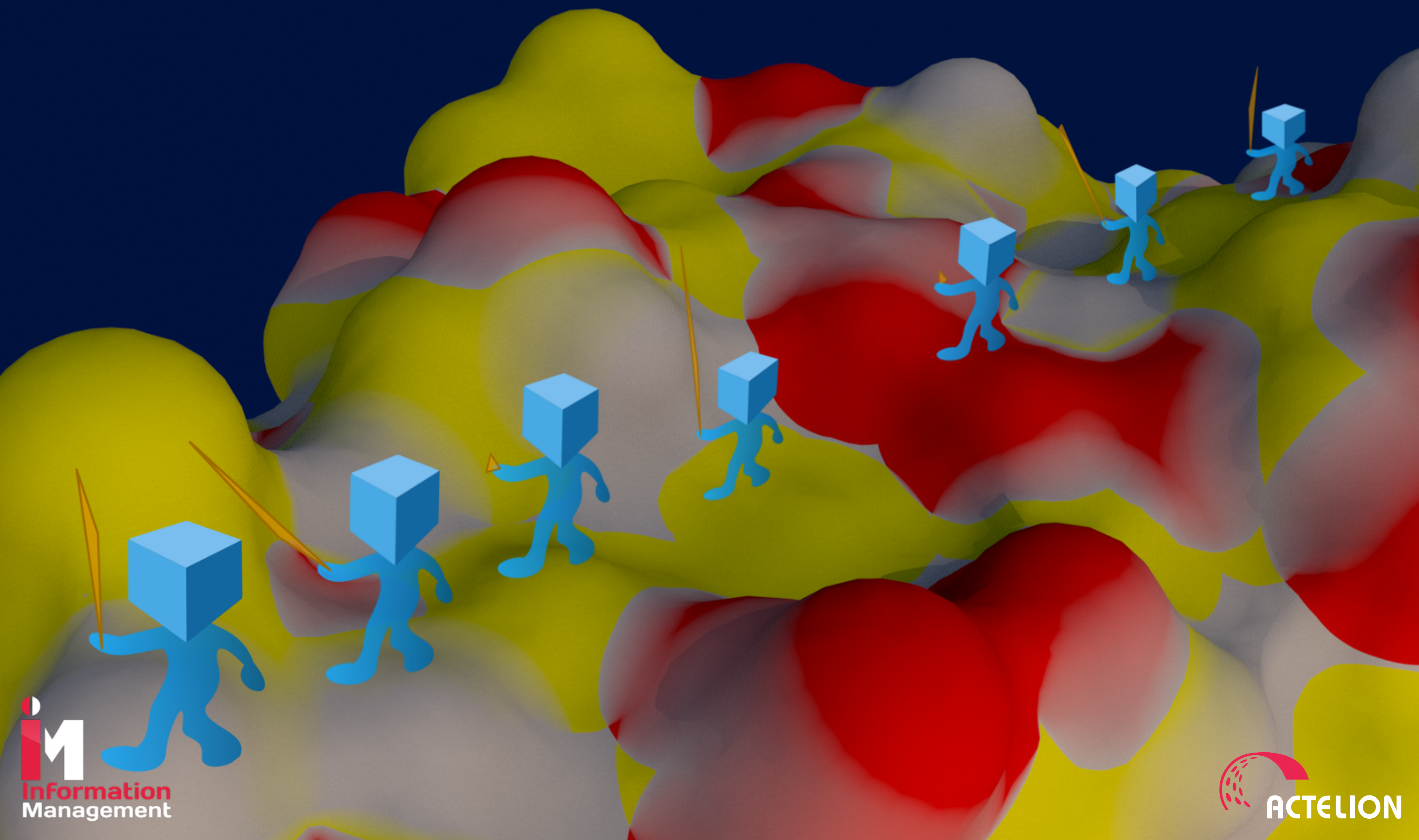


# Marching Cubes Without Skinny And Tiny Triangles

## And How To Employ Them To Create Smooth Molecular Surfaces

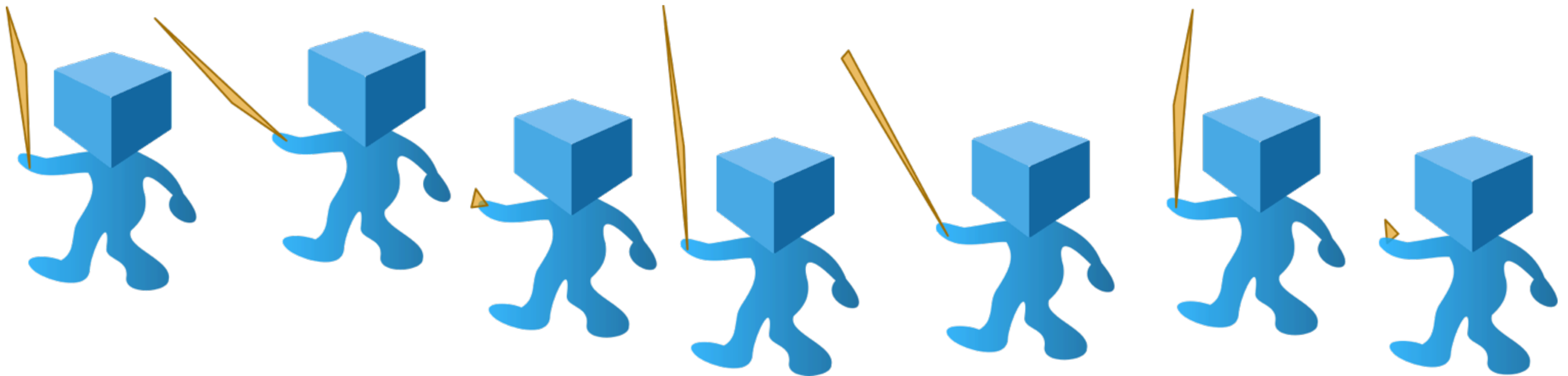
Thomas Sander, Sep 2016





# TABLE OF CONTENTS

- What are Marching Cubes?
- Why do we care about them?
- Why do they carry small and skinny triangles?
- How to make them relinquish these triangles?



# The Marching Cubes Algorithm

## *Iso-Layer Triangulation from Voxel Data*

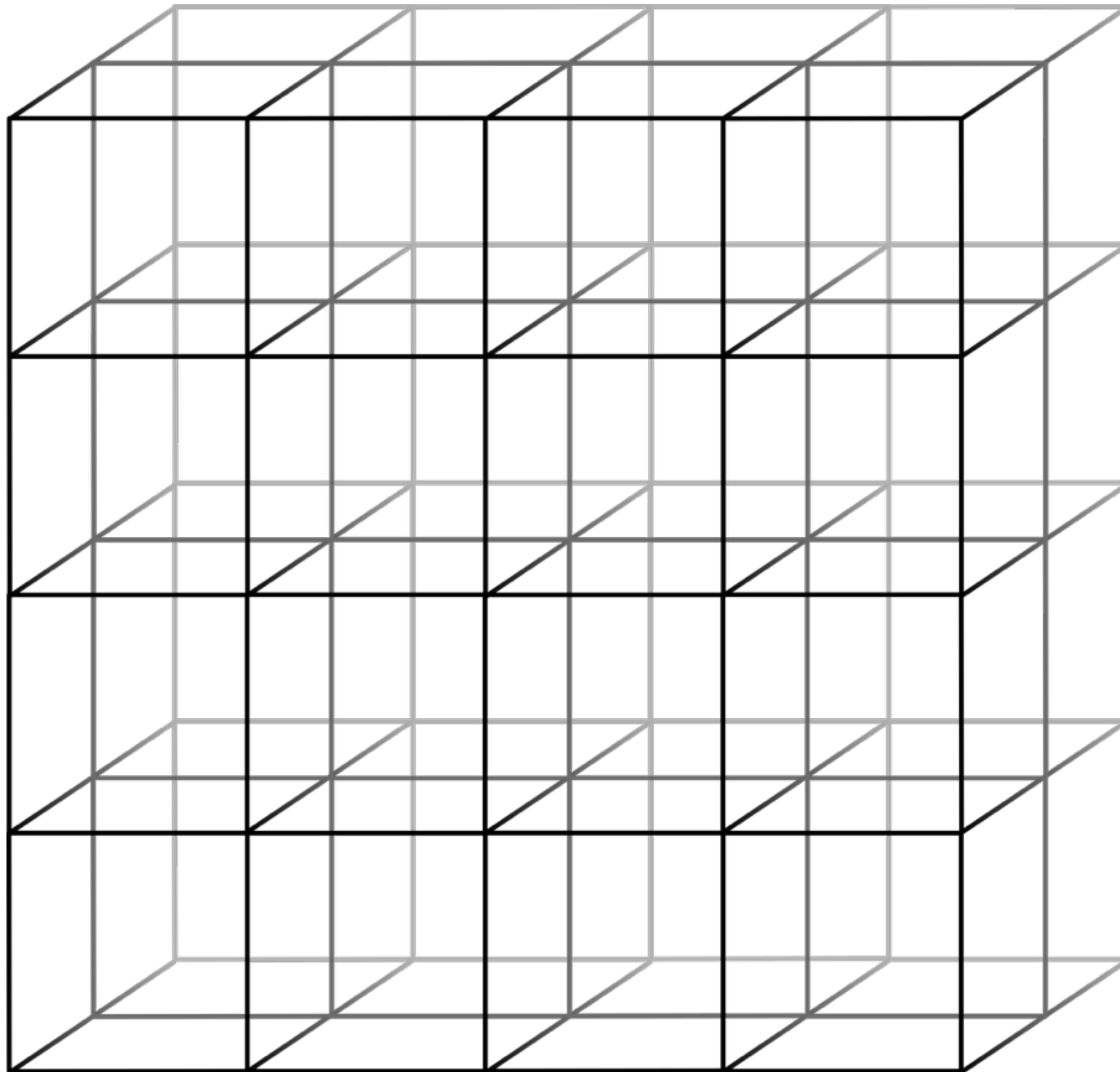
- **Paper published in the 1987 SIGGRAPH**

**Lorensen, W. E.; Cline, Harvey E. (1987). "Marching cubes: A high resolution 3d surface construction algorithm". ACM Computer Graphics. 21 (4): 163–169.**

**DOI: 10.1145/37402.37422**

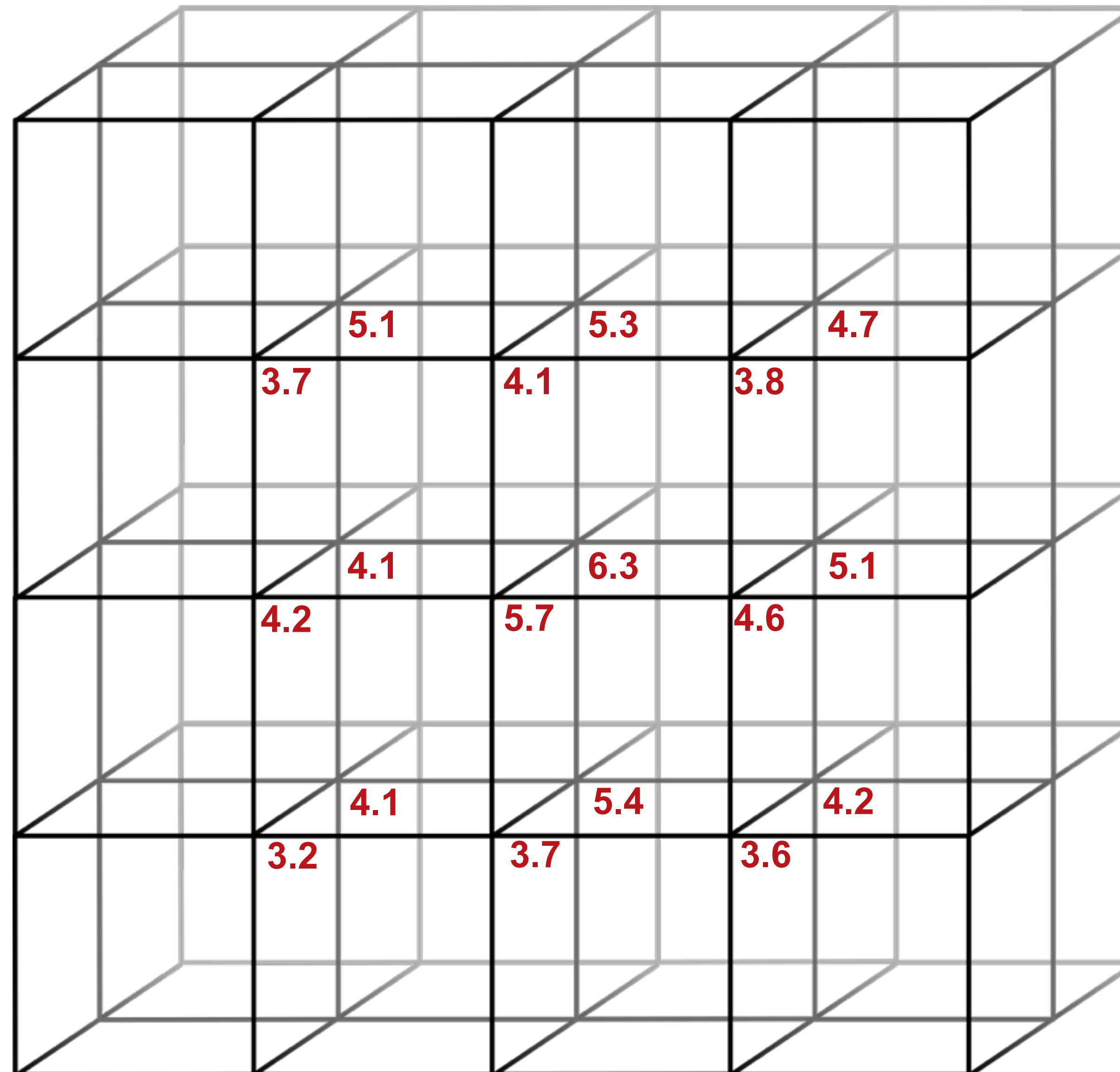
- **One of the most frequently cited papers in computer graphics field**
- **Patent expired in 2005**
- **Applied for CT, MRI, Molecular Modelling**

# 3D-Voxel Data





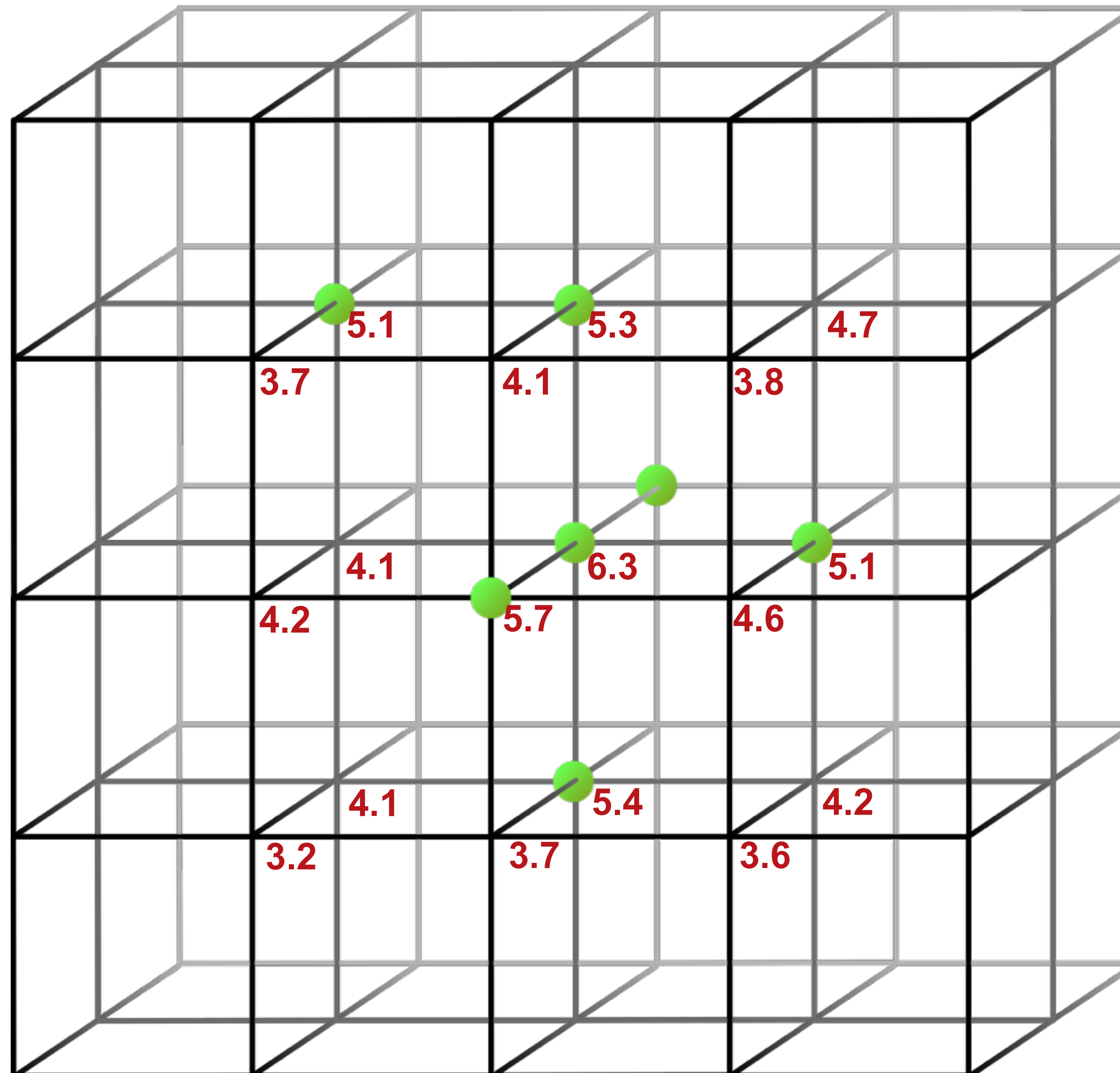
# 3D-Voxel Data



Iso-surface at 5.0

# 3D-Voxel Data

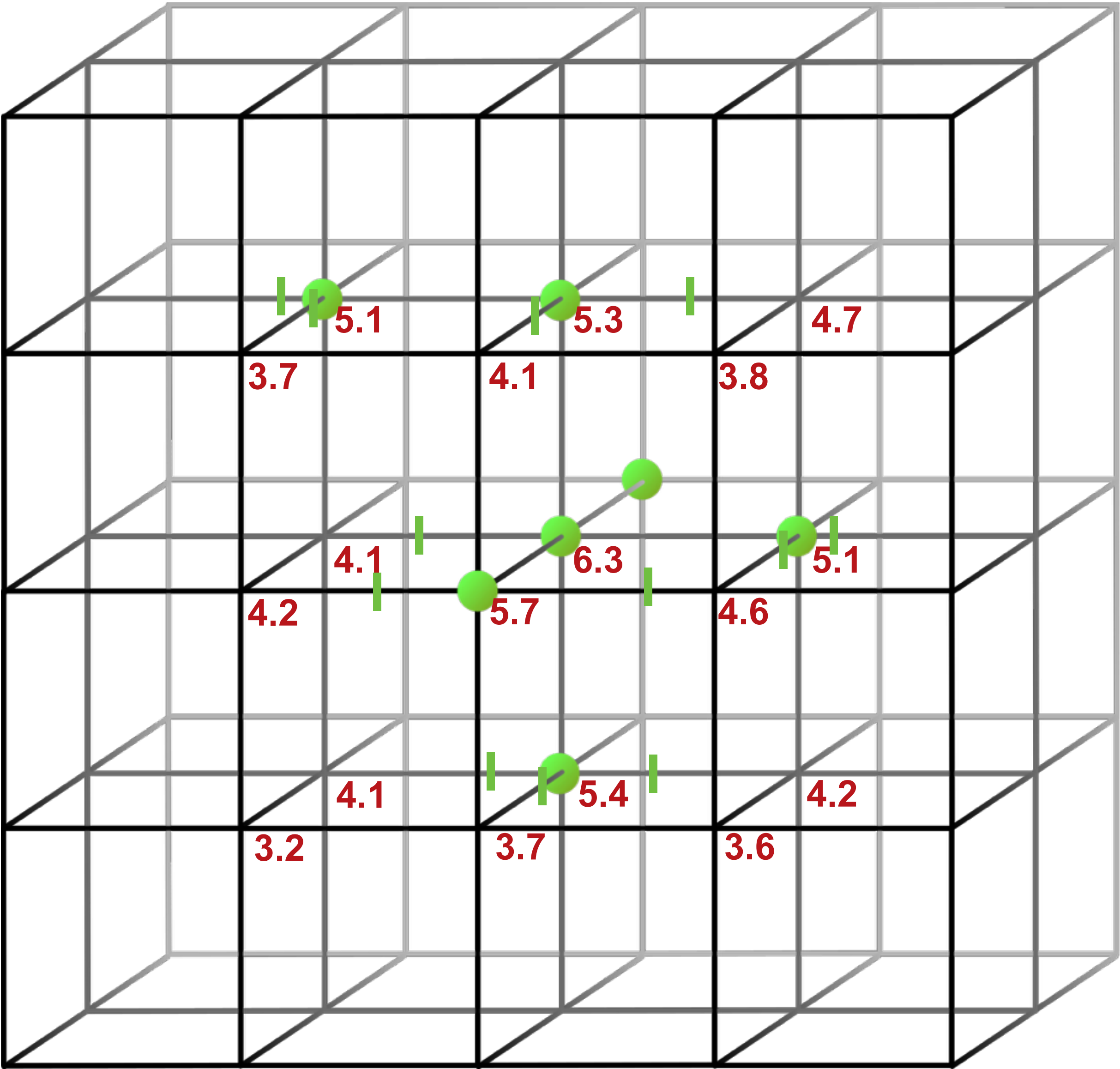
Iso-surface at 5.0



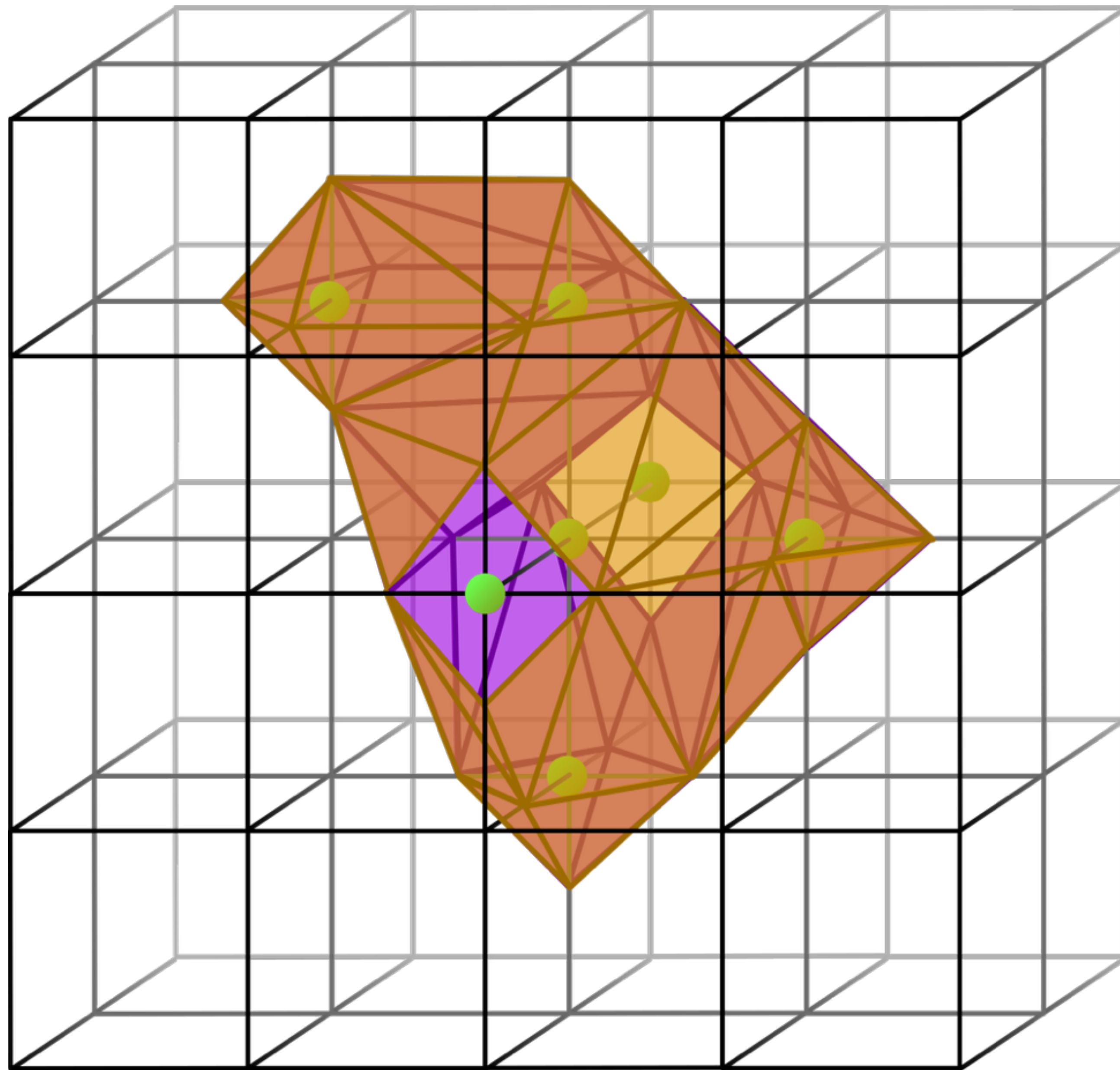


# 3D-Voxel Data

Iso-surface at 5.0

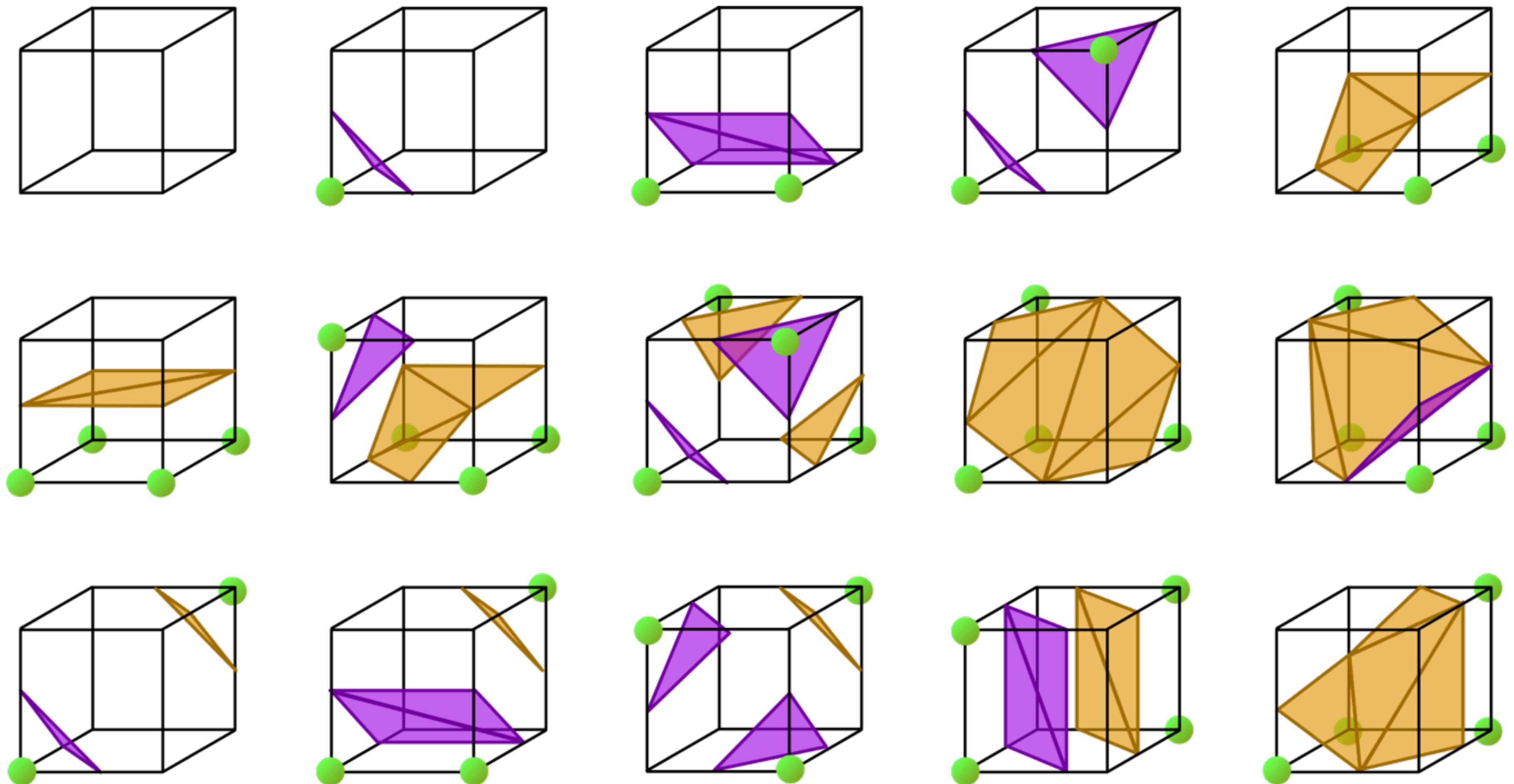


# Individual Cube's Triangles Connect to Surface





# 256 Cubes with Triangles Derived From 15 Base (+8) Cube Families

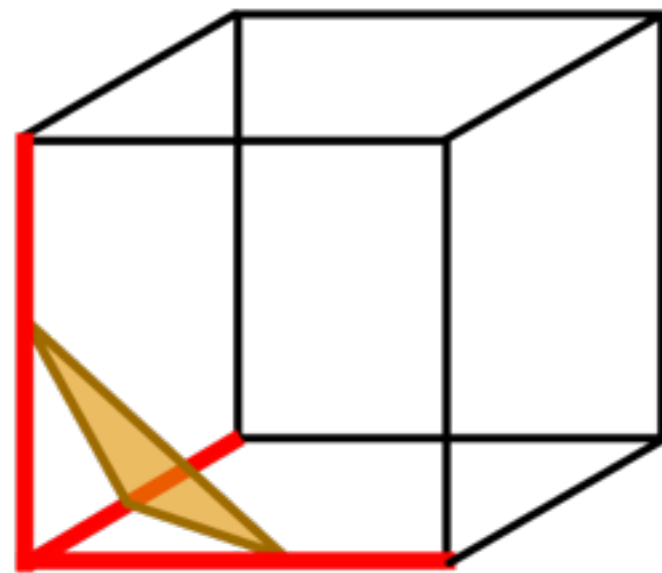


**inside**

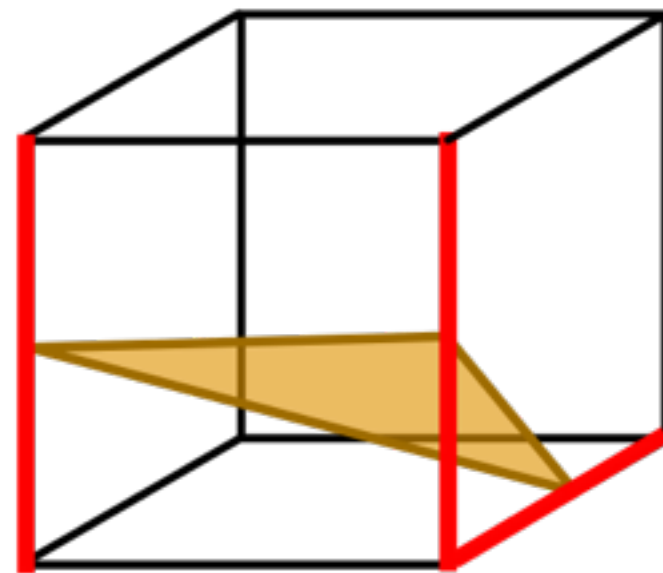


**outside**

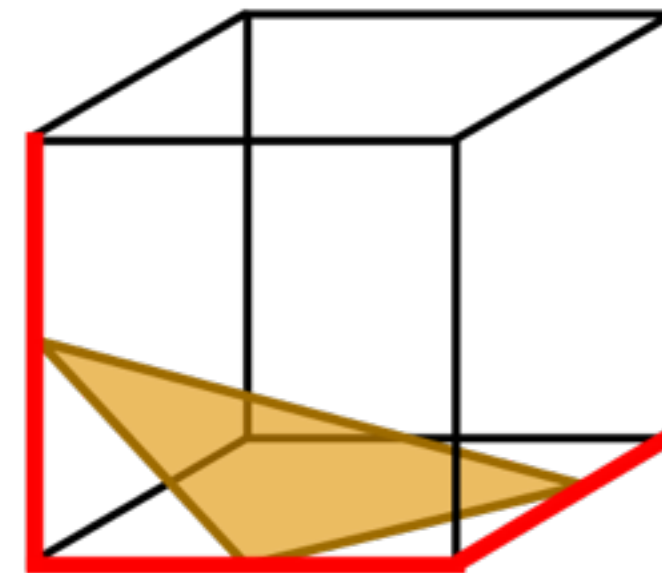
# Triangles Can be Classified by 8 Edge-Groups



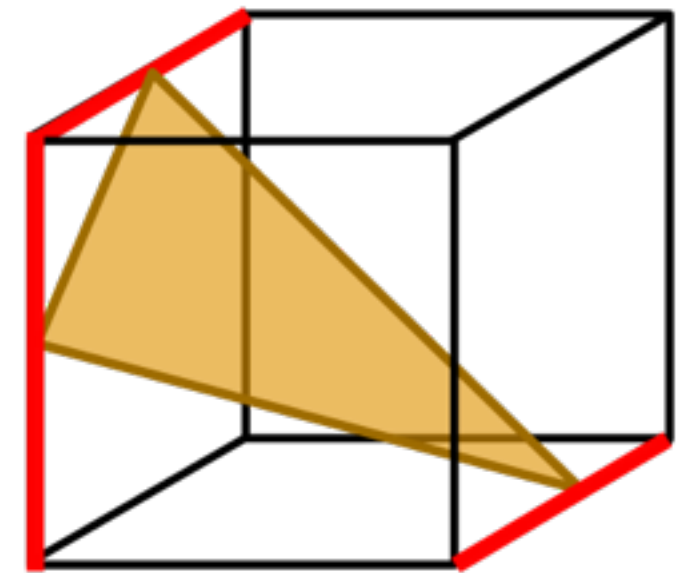
0



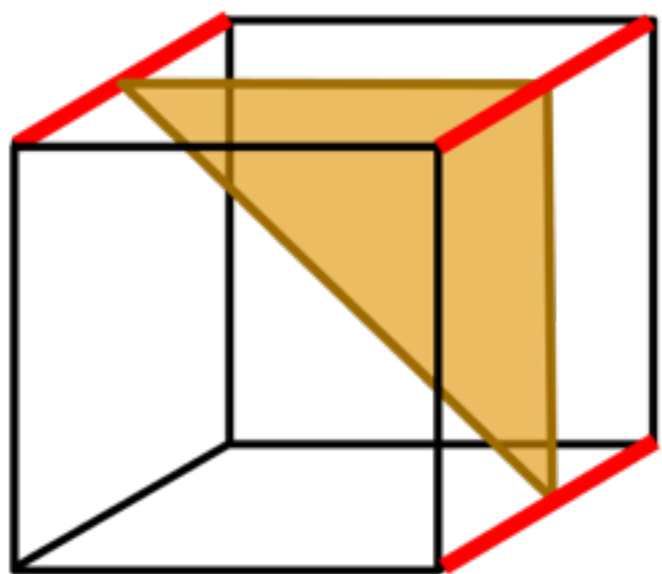
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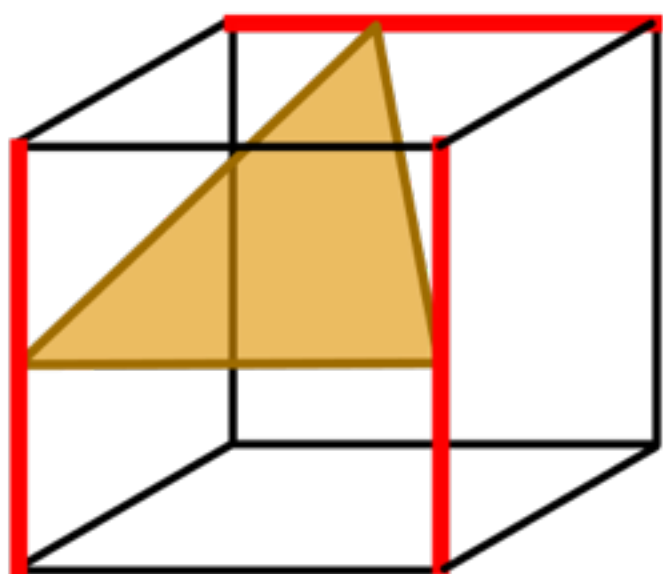
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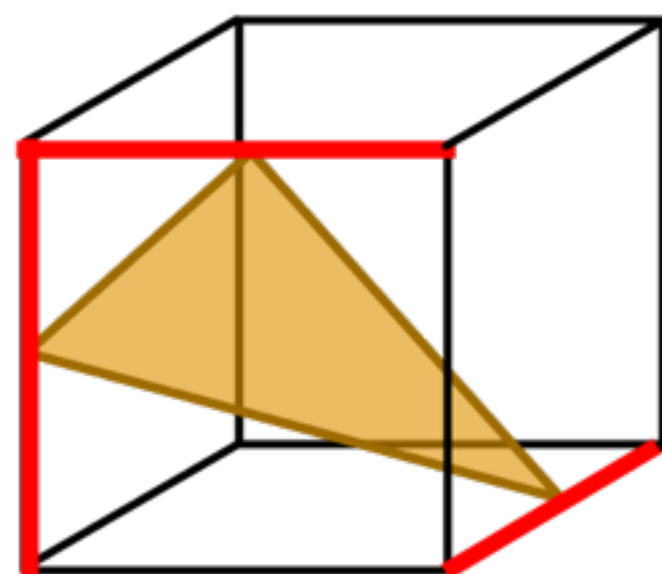
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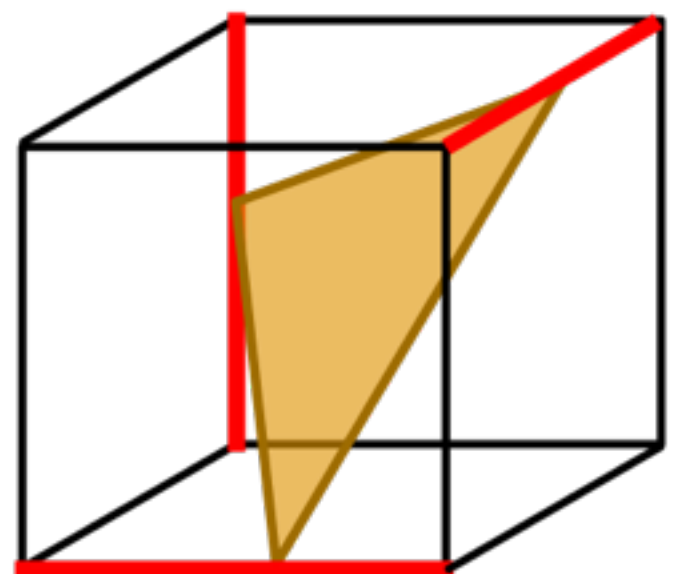
4



5



6



7

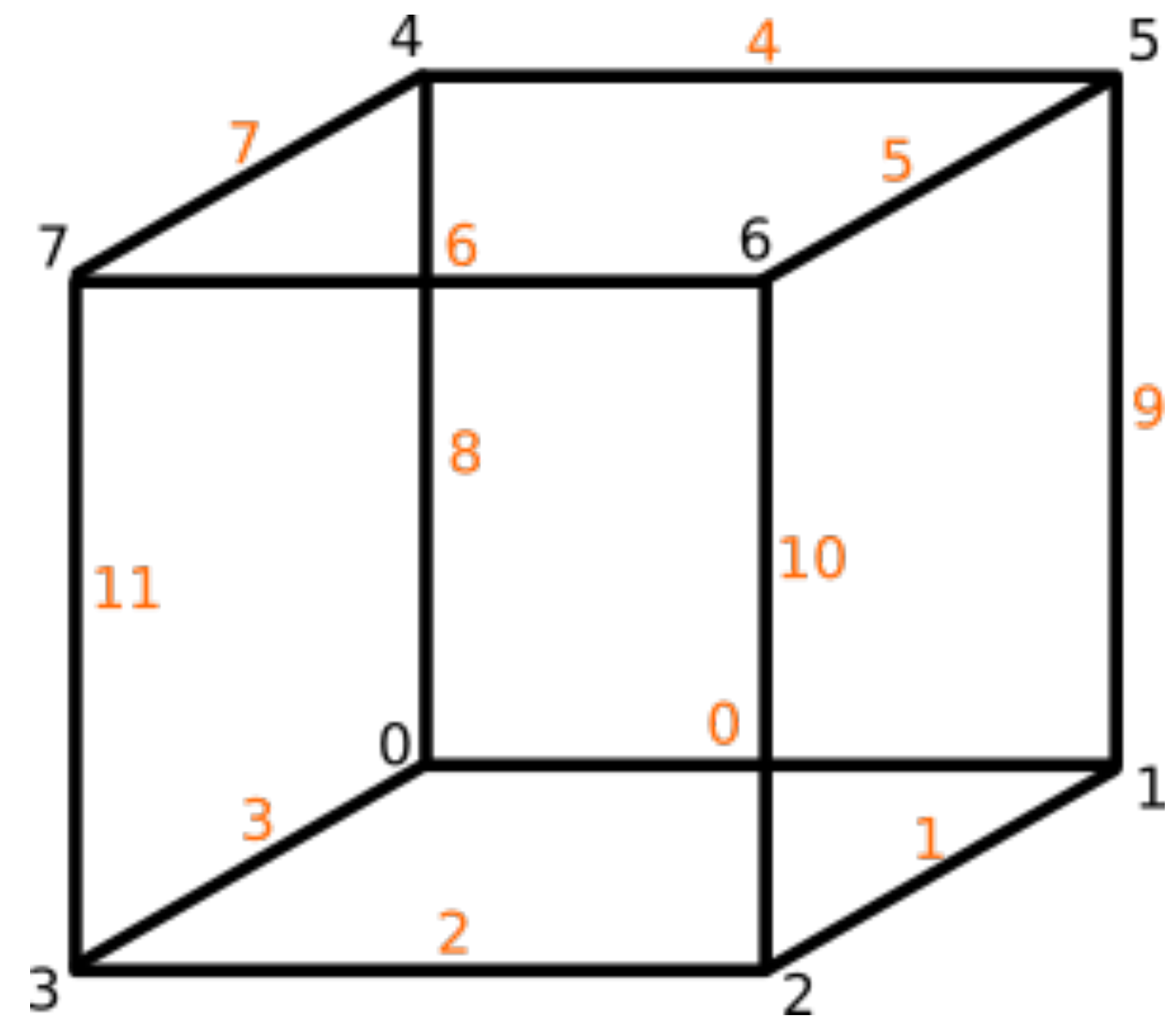


# The Beauty: Pure Speed Through Lookup Tables

A **cube specific 8-bit key** is created from 8 corners (in or out)

```
public static final int[] EDGE_TABLE = {  
    0x000, 0x109, 0x203, 0x30a, 0x406, 0x50f, 0x605, 0x70c,  
    0x80c, 0x905, 0xa0f, 0xb06, 0xc0a, 0xd03, 0xe09, 0xf00,  
    0x190, 0x099, 0x393, 0x29a, 0x596, 0x49f, 0x795, 0x69c,  
    0x99c, 0x895, 0xb9f, 0xa96, 0xd9a, 0xc93, 0xf99, 0xe90, ... }
```

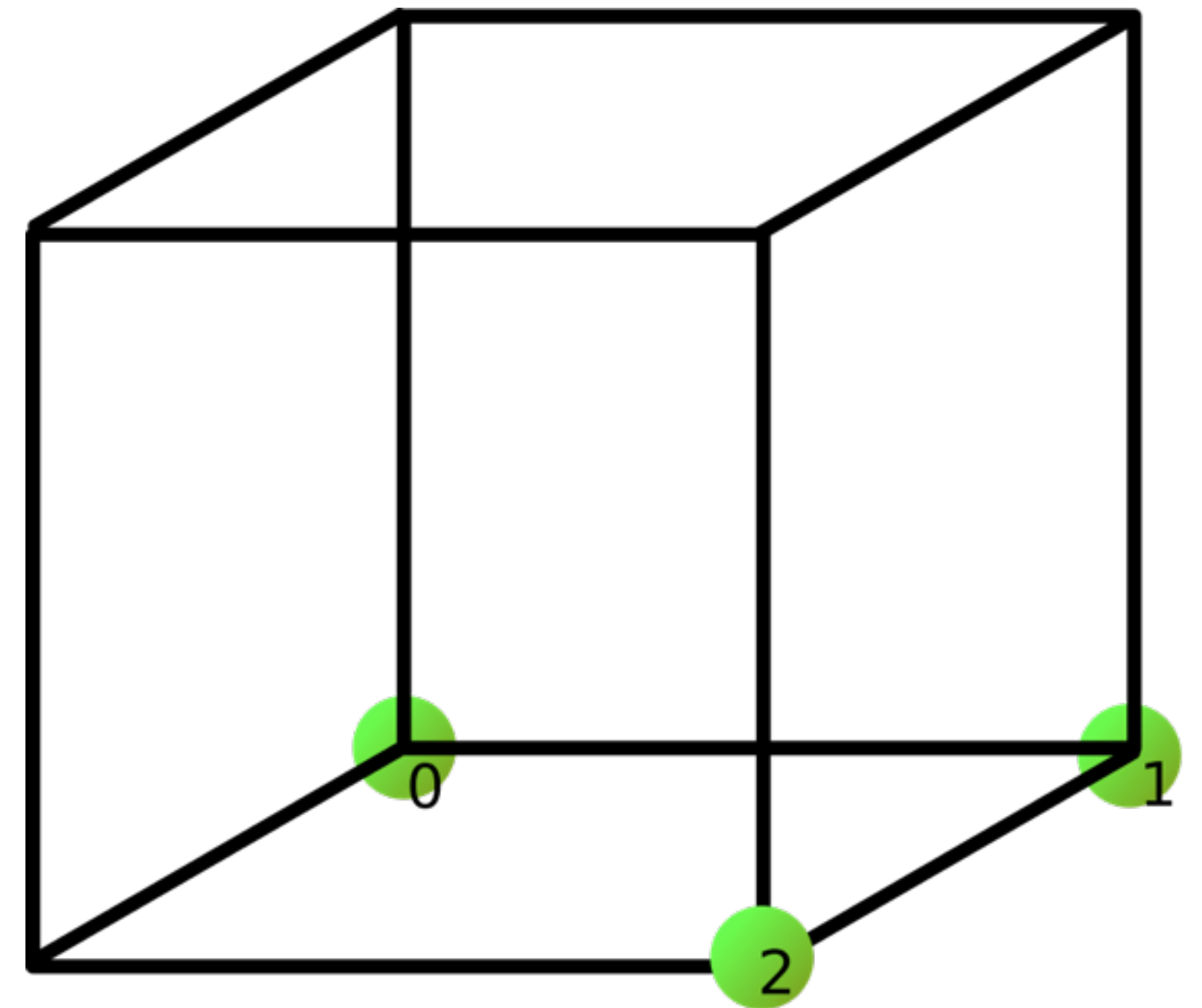
```
public static final int[][] FACE_TABLE =  
    {},  
    {0, 8, 3},  
    {0, 1, 9},  
    {1, 8, 3, 9, 8, 1},  
    {1, 2, 10},  
    {0, 8, 3, 1, 2, 10},  
    {9, 2, 10, 0, 2, 9},  
    {2, 8, 3, 2, 10, 8, 10, 9, 8},  
    {3, 11, 2}, ... }
```



# The Triangulation Process

For all cubes do

- build key from 8 corners (00000111b = 7)
- lookup affected edges: 0x70c
- calculate vertexes on affected edges
- lookup faces: {2, 8, 3, 2, 10, 8, 10, 9, 8}
- create triangles connecting vertexes

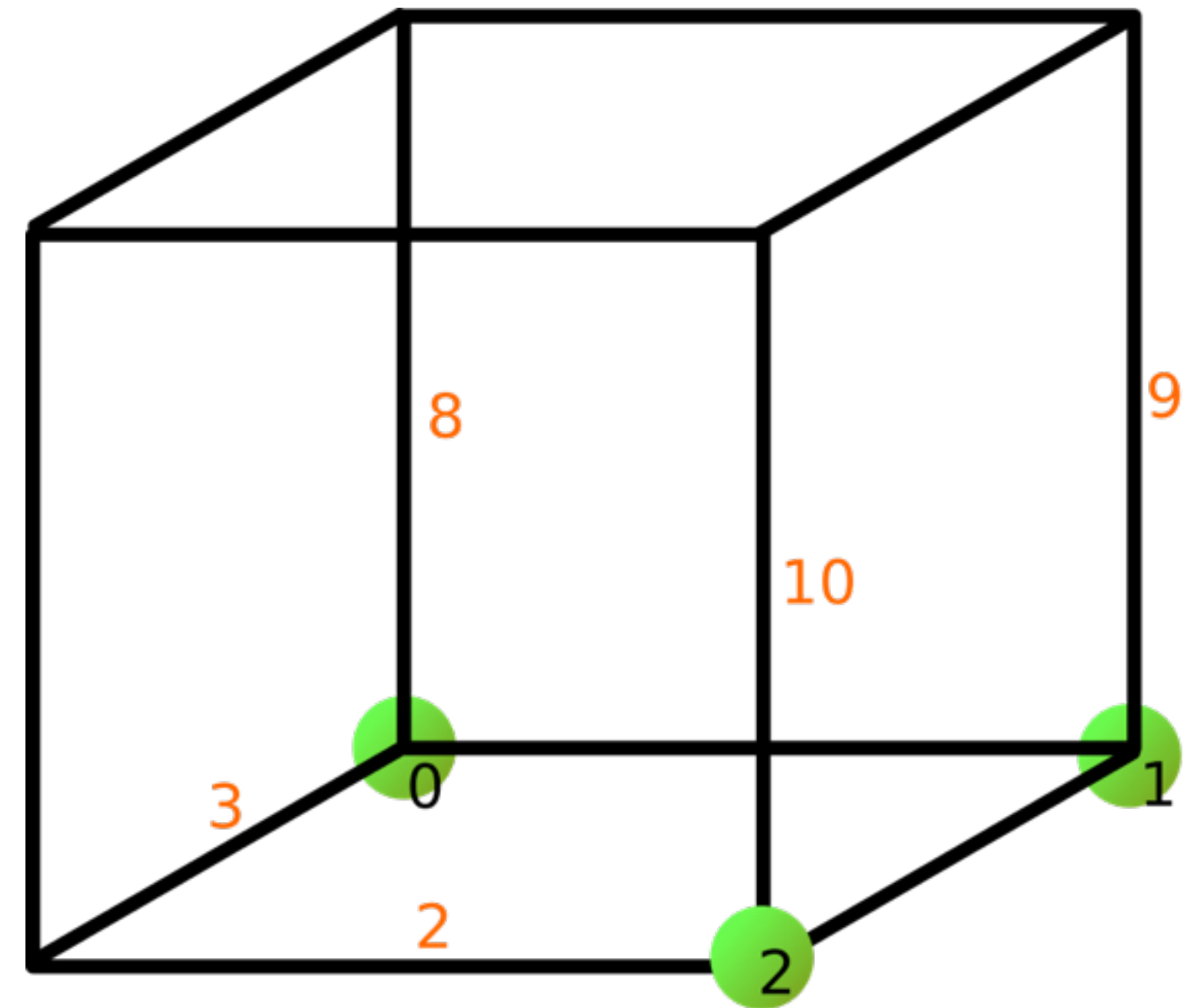




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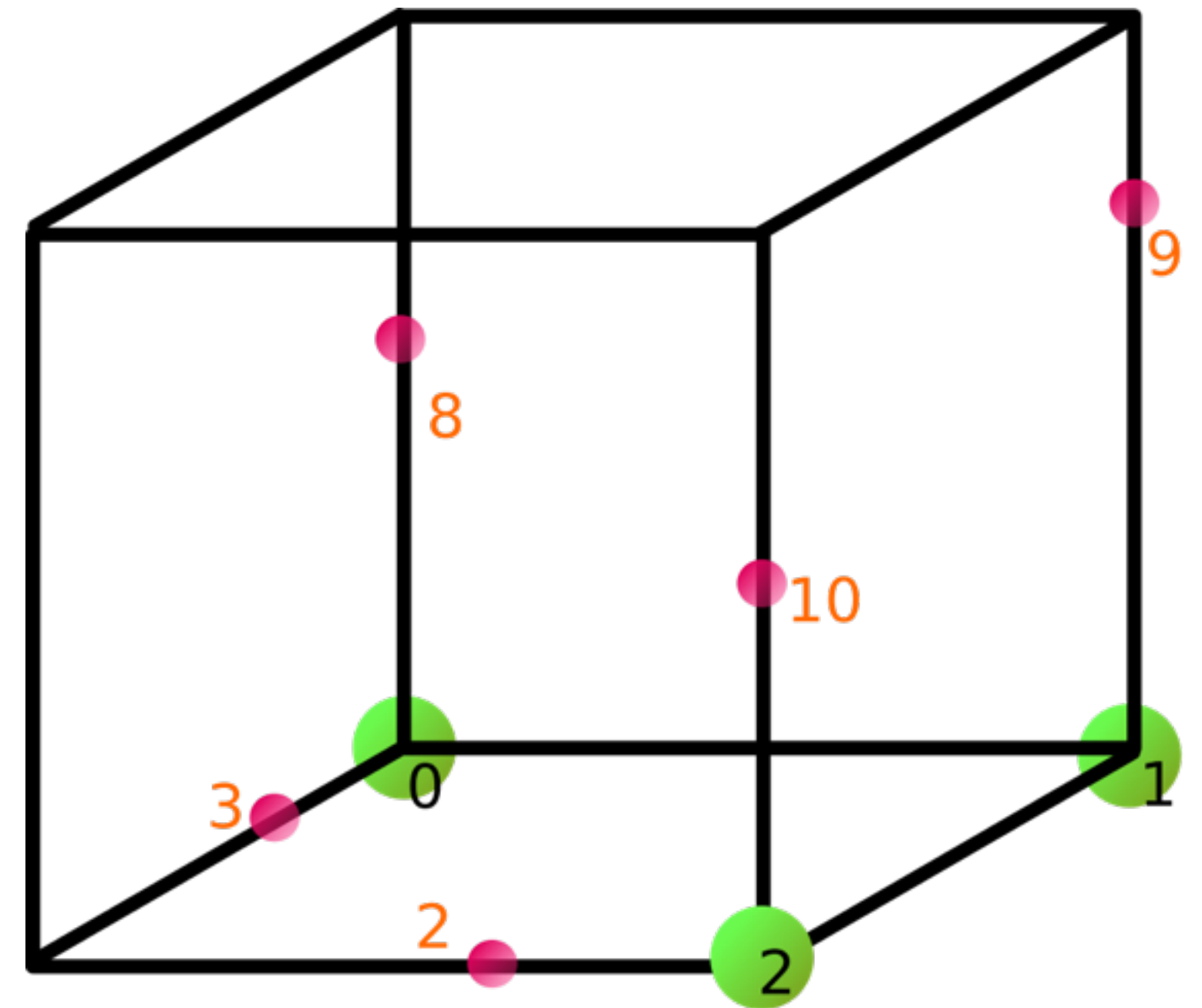
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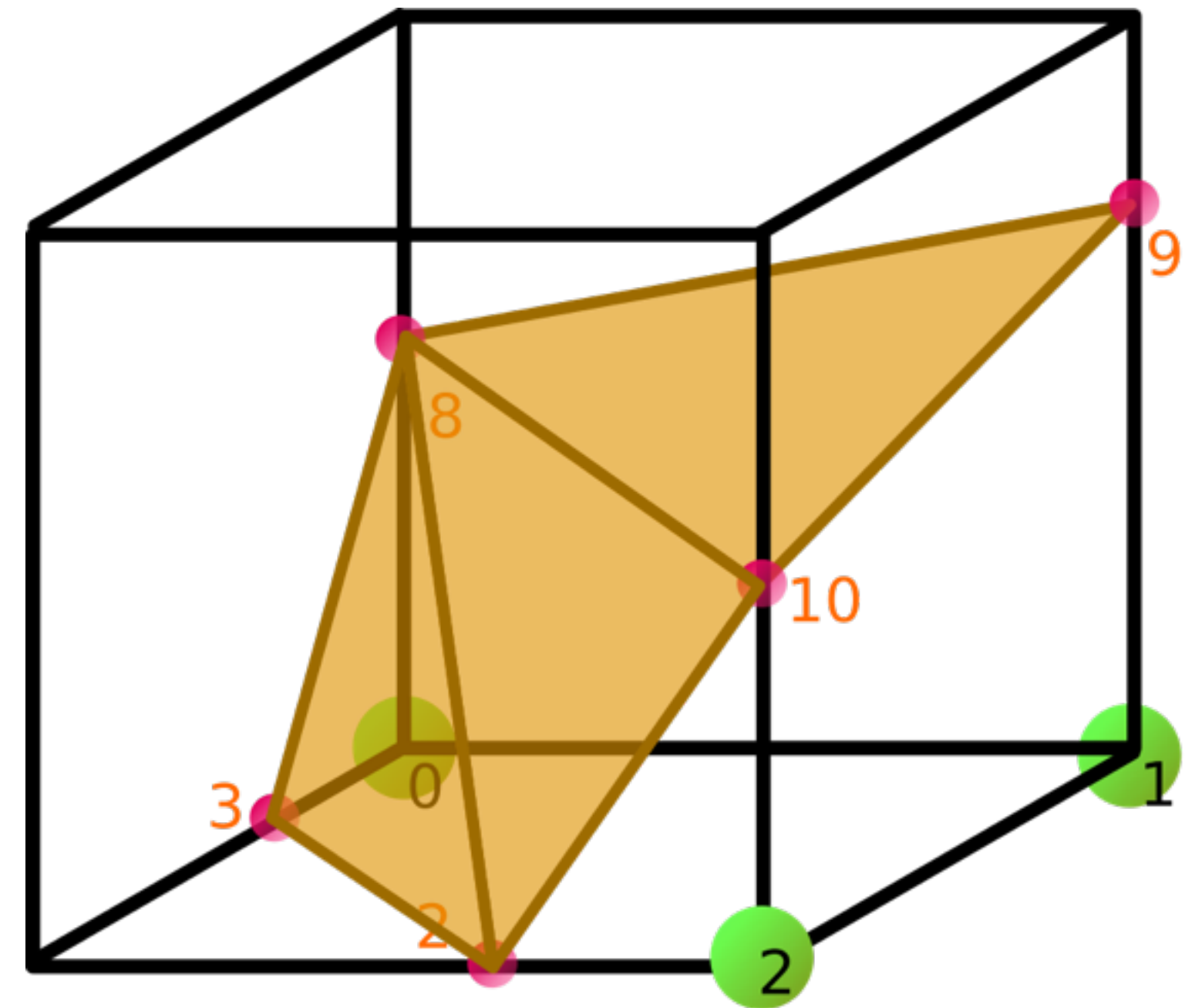
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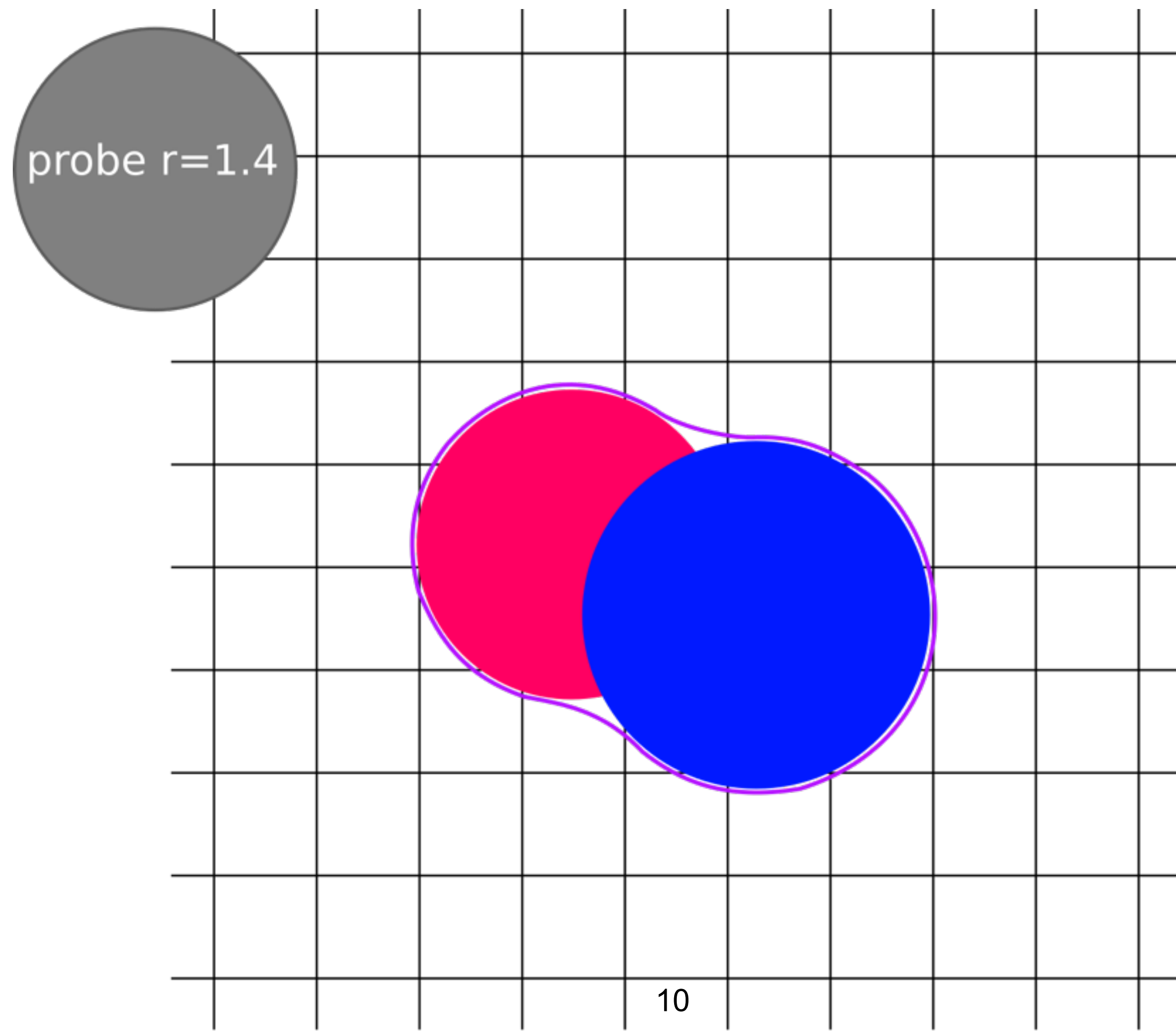




# How Does this help to create a Connolly surface? Create Voxel Grid

For every atom

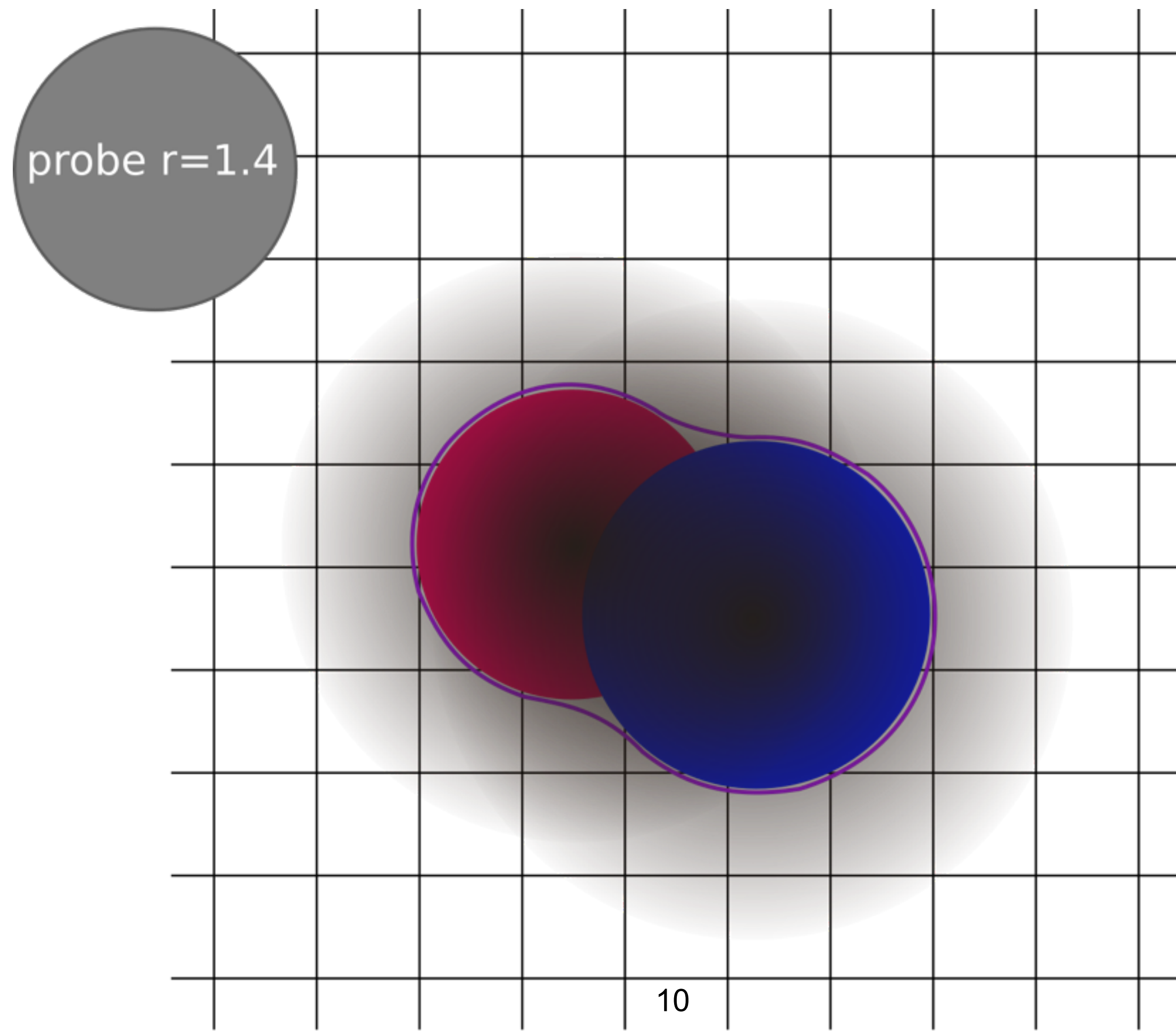
set close grid points to  $\text{VDW-radius} - \text{distance}$



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For every atom

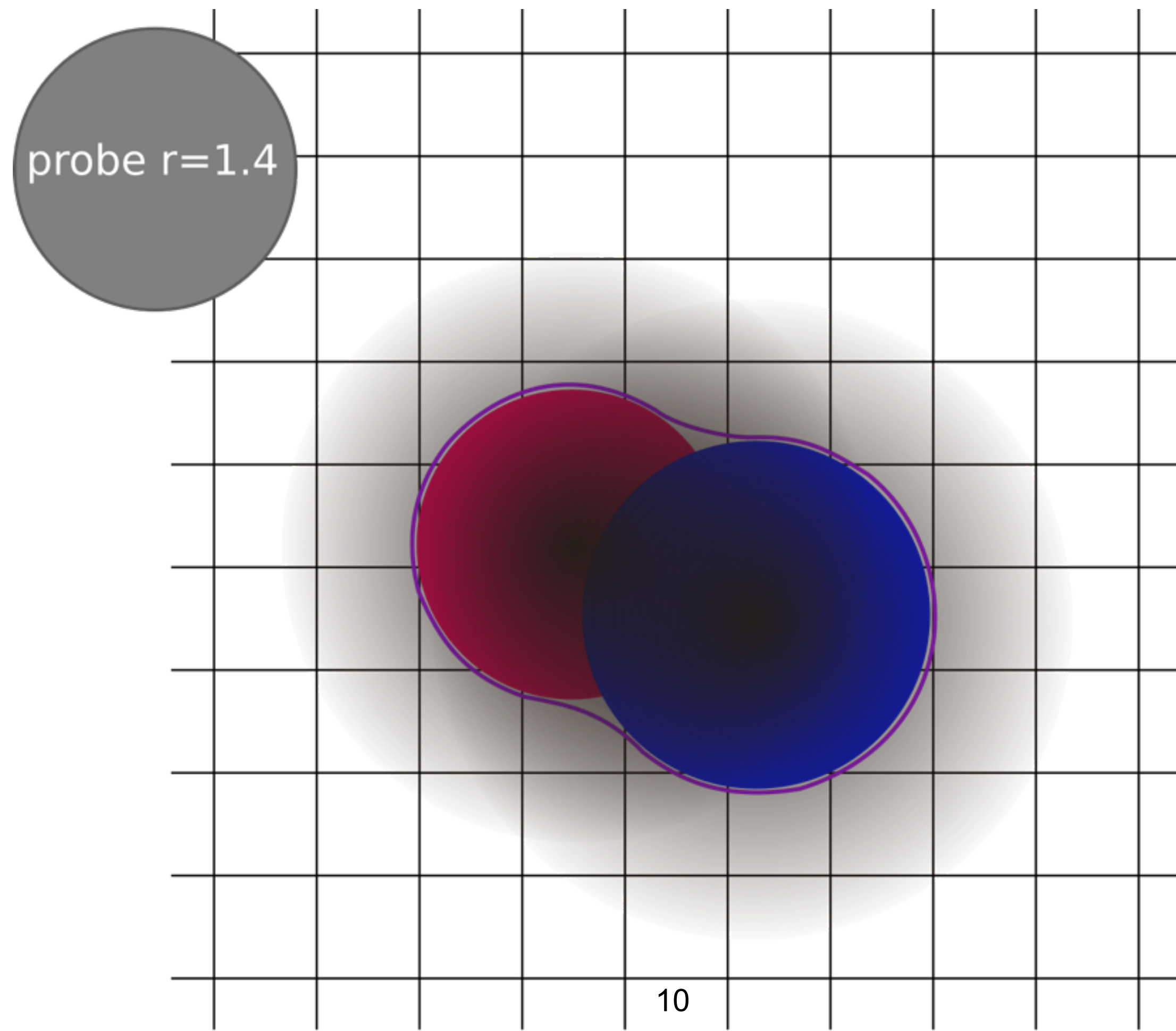
set close grid points to  $\text{VDW-radius} - \text{distance}$



# How Does this help to create a Connolly surface?

## Create Voxel Grid

For every grid edge with separating positive from negative volume  
determine edge position  $p_1$  with  $\text{value}=0$   
for every grid point  $p_2$  within probe radius  
set value to  $\min(\text{value}, \text{distance} - \text{probeRadius})$

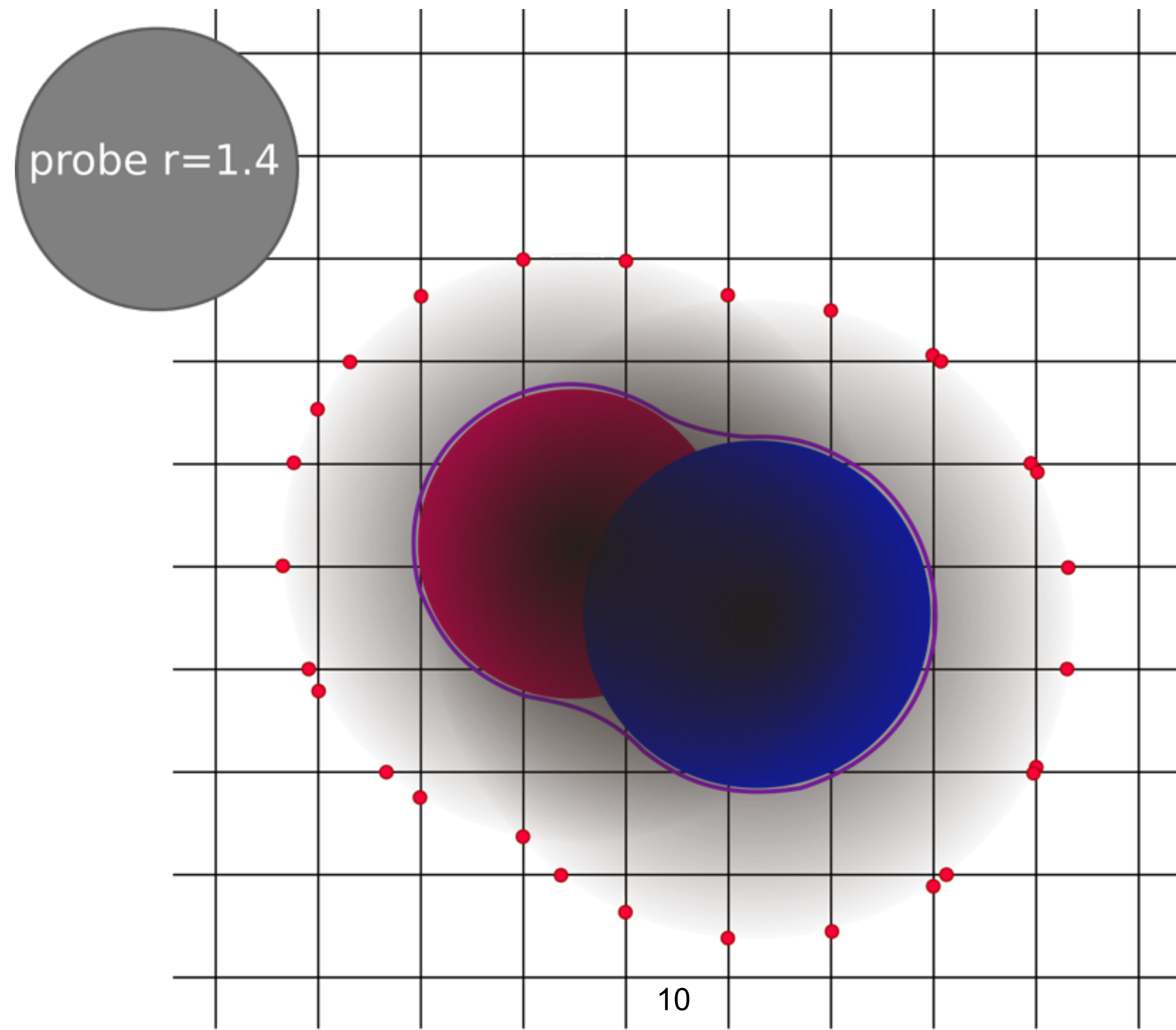




# How Does this help to create a Connolly surface?

## Create Voxel Grid

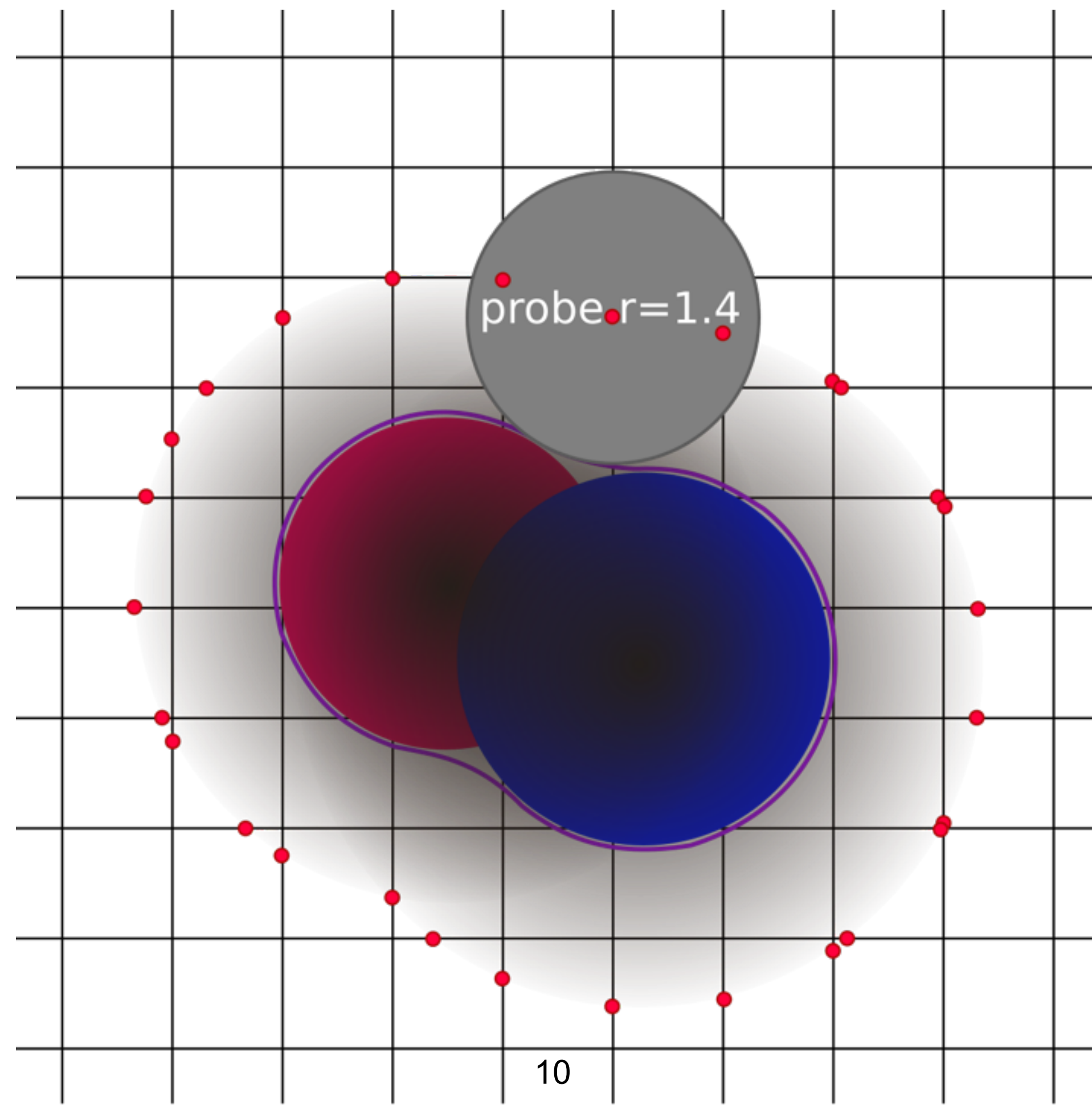
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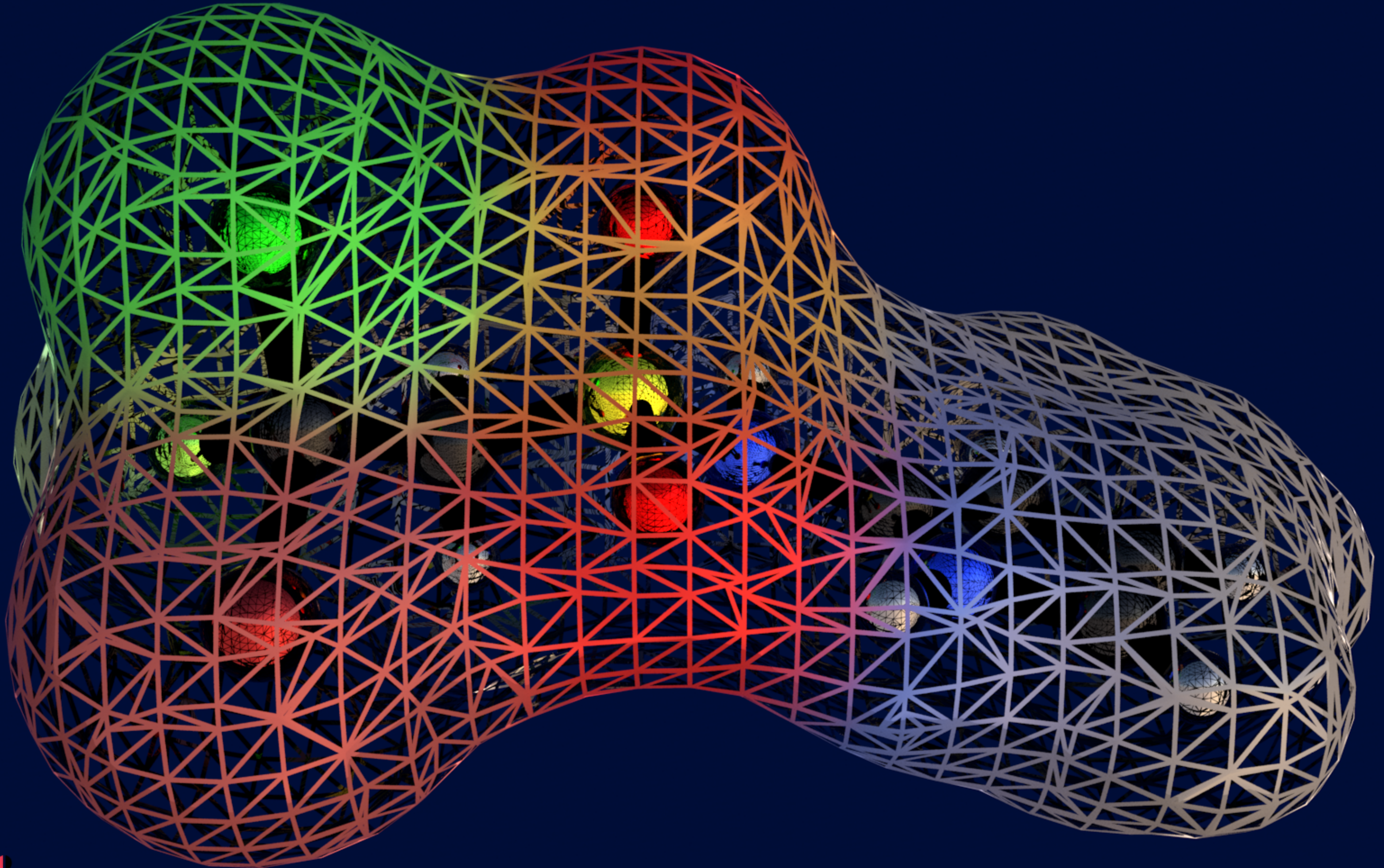
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for every grid point p2 within probe radius  
set value to  $\min(\text{value}, \text{distance} - \text{probeRadius})$

inspired by Tom Goddard,  
“Molecular Surface Algorithm”,  
Oct 14, 2013,  
[www.cgl.ucsf.edu/chimera/data/surface-oct2013/surface.html](http://www.cgl.ucsf.edu/chimera/data/surface-oct2013/surface.html)

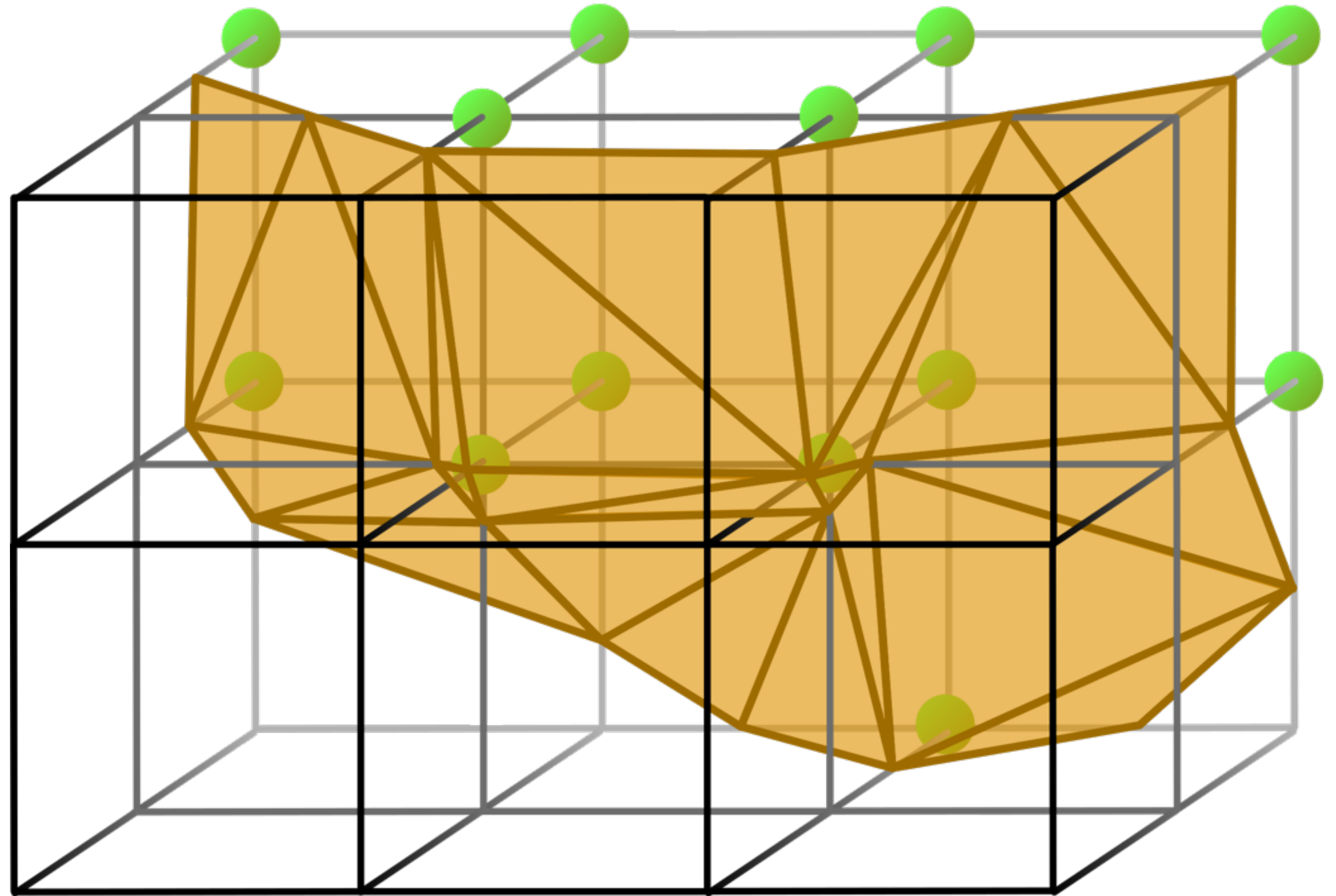


# Problem: Small and skinny triangles





# Avoiding Small-Area-Triangles



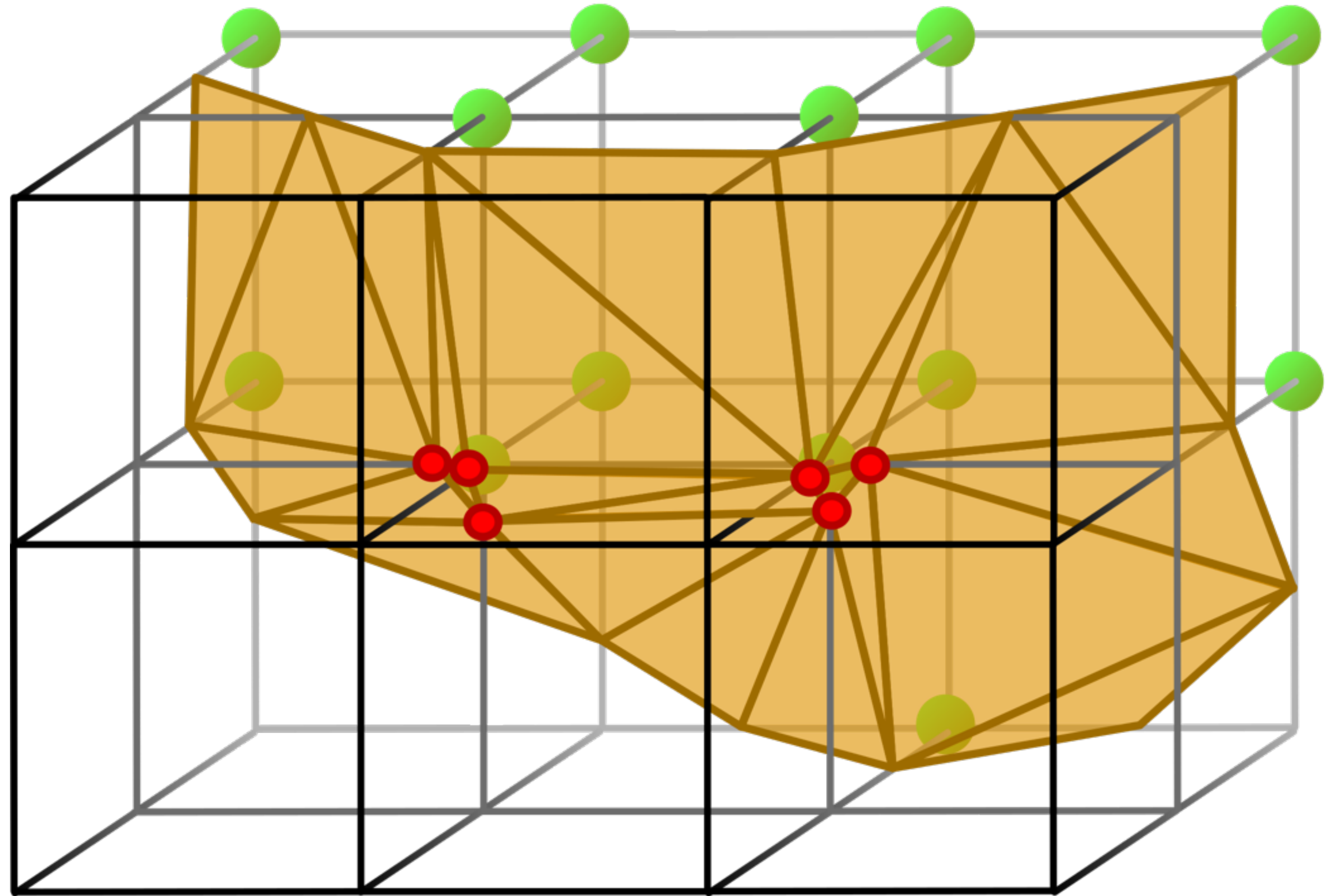
New simple procedure:

For every corner:

if more than 1 close edge cuts

- join these edge cut positions
- remove all triangles with merged corners

# Avoiding Small-Area-Triangles



New simple procedure:

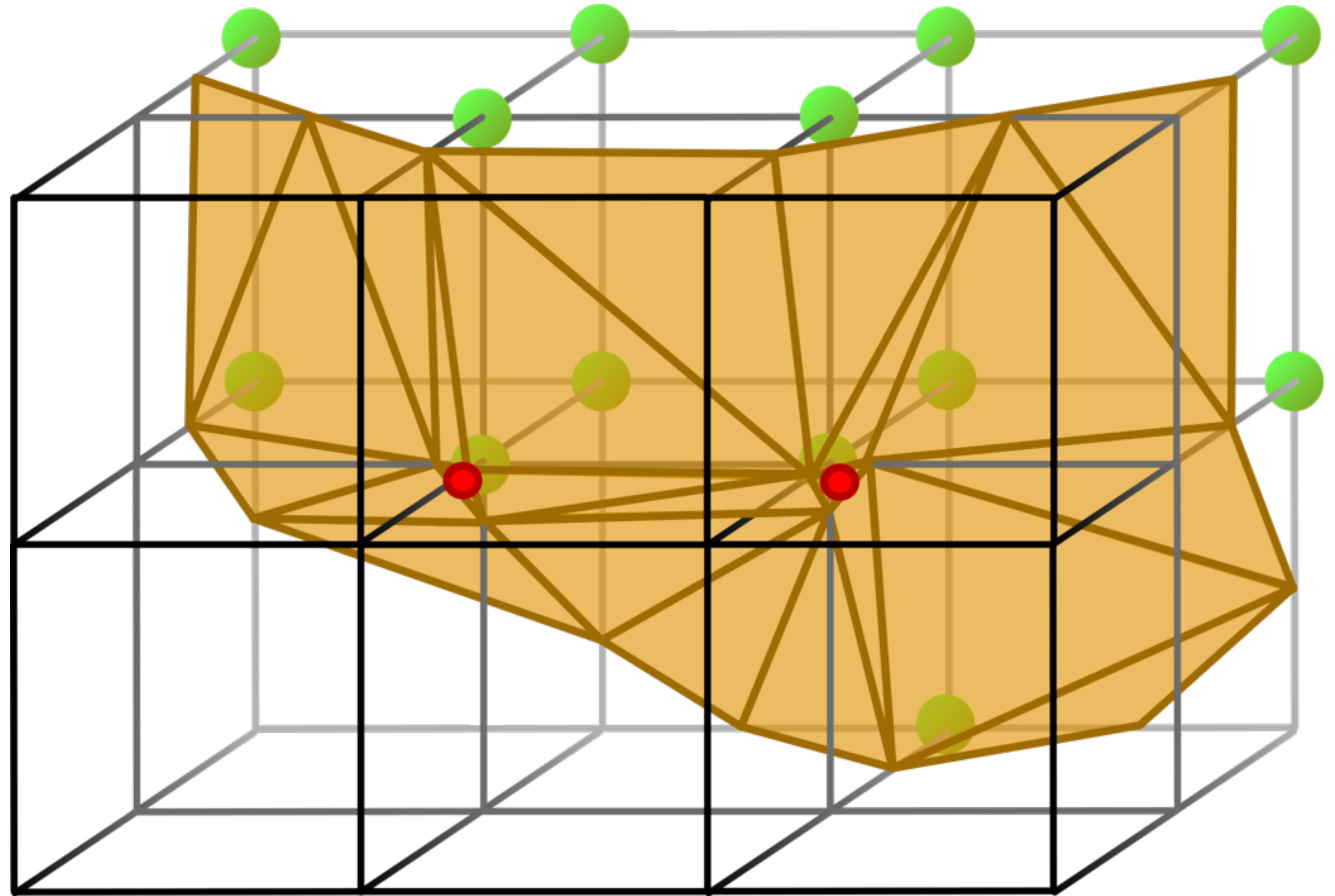
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# Avoiding Small-Area-Triangles



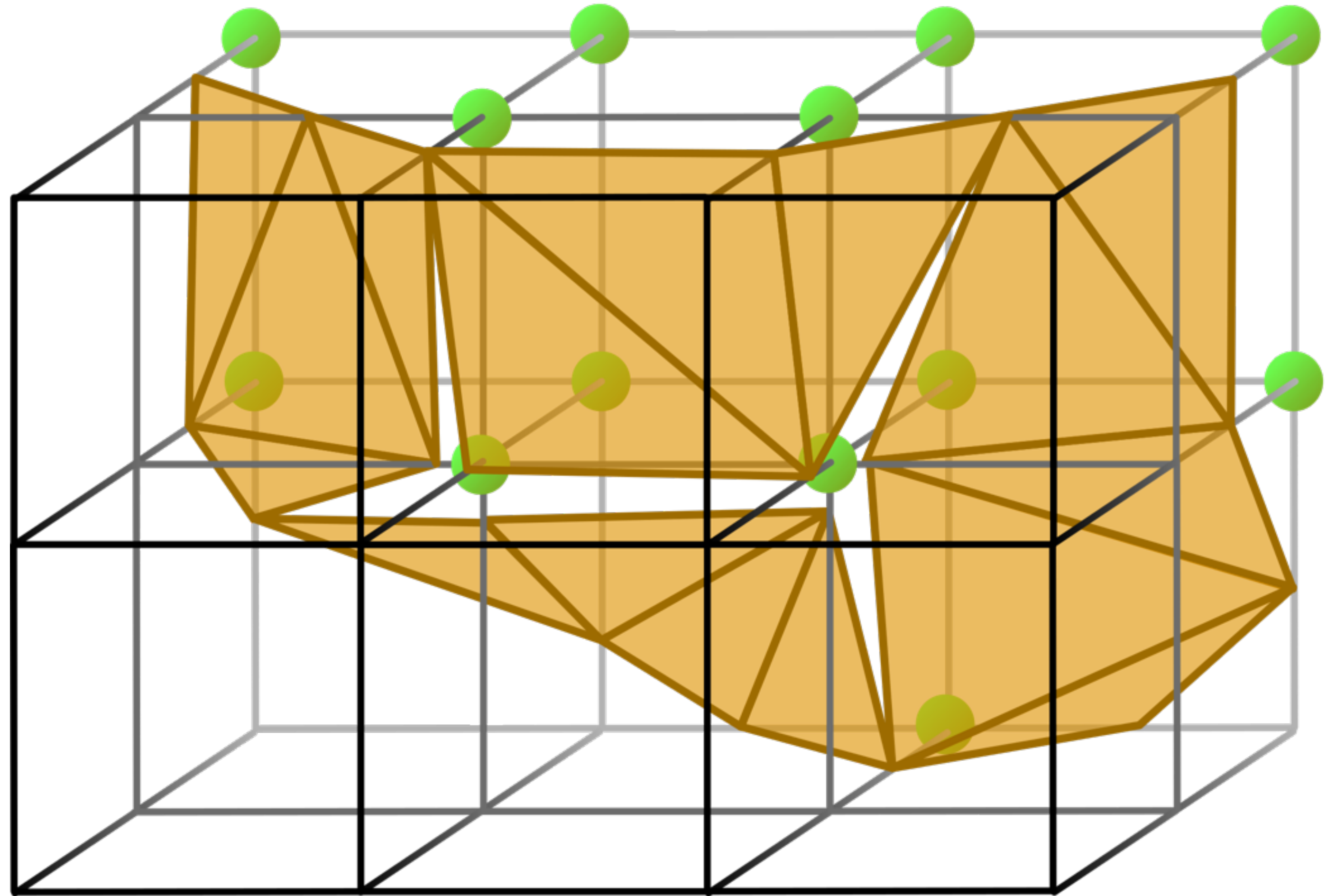
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# Avoiding Small-Area-Triangles



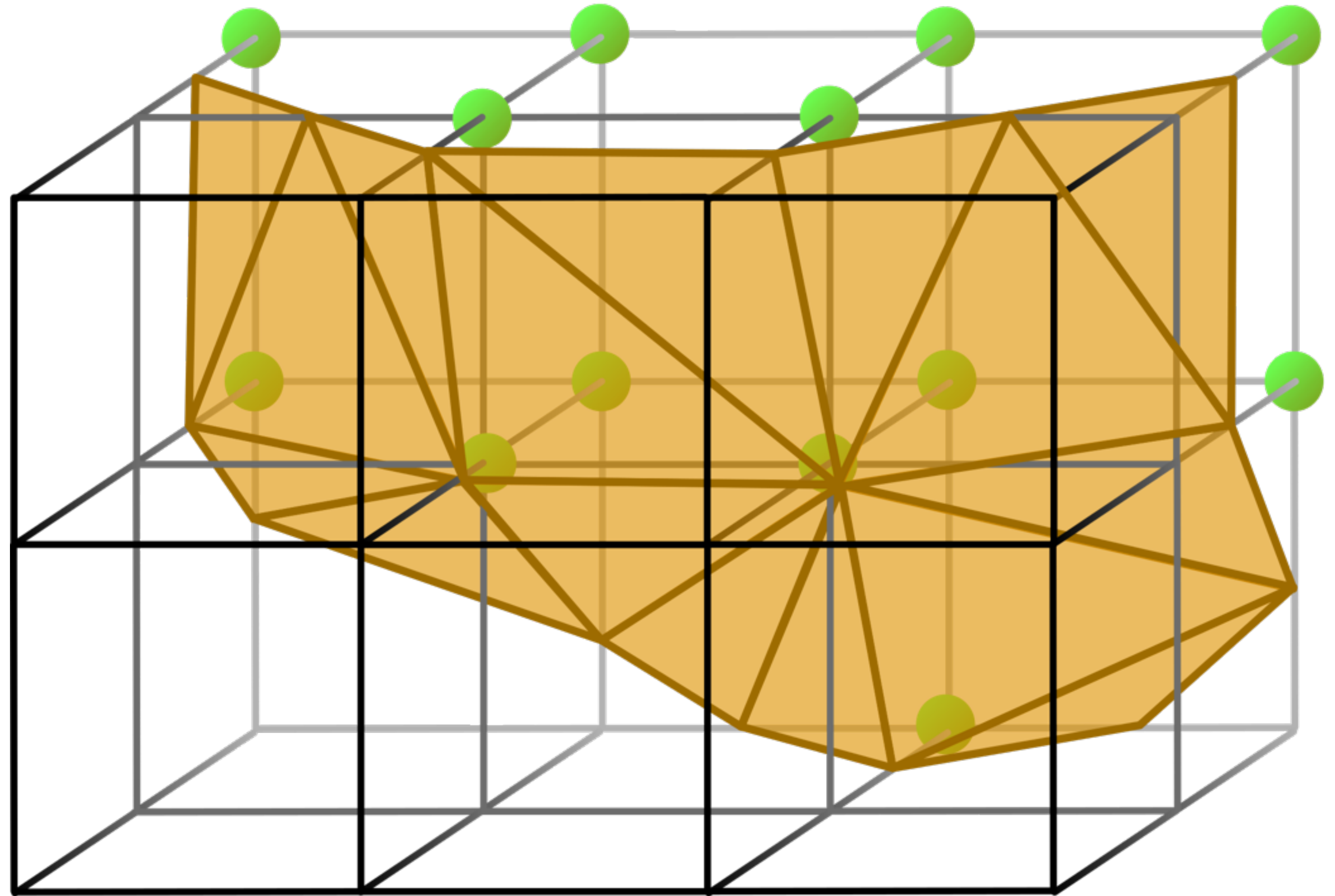
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# Avoiding Small-Area-Triangles



New simple procedure:

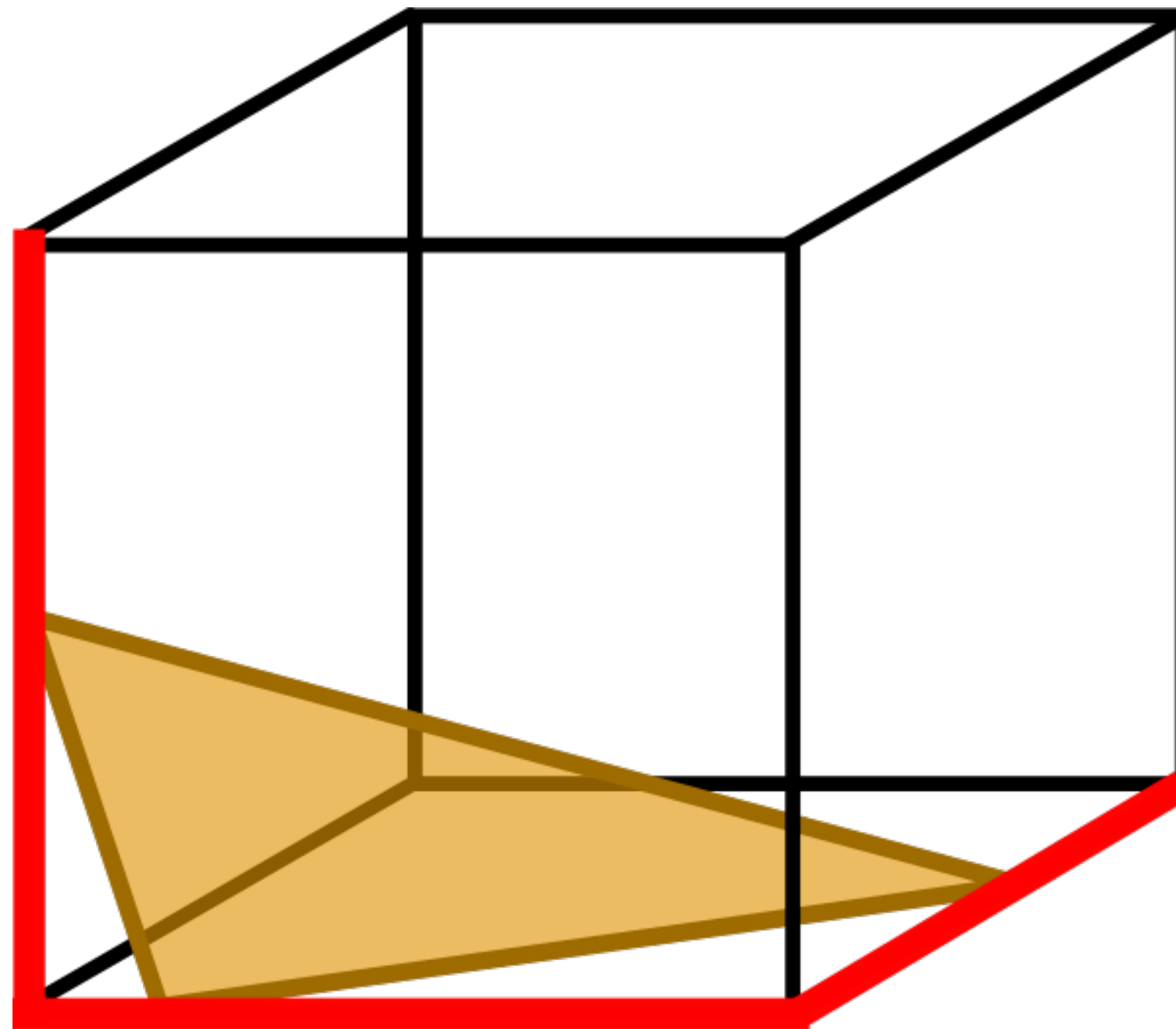
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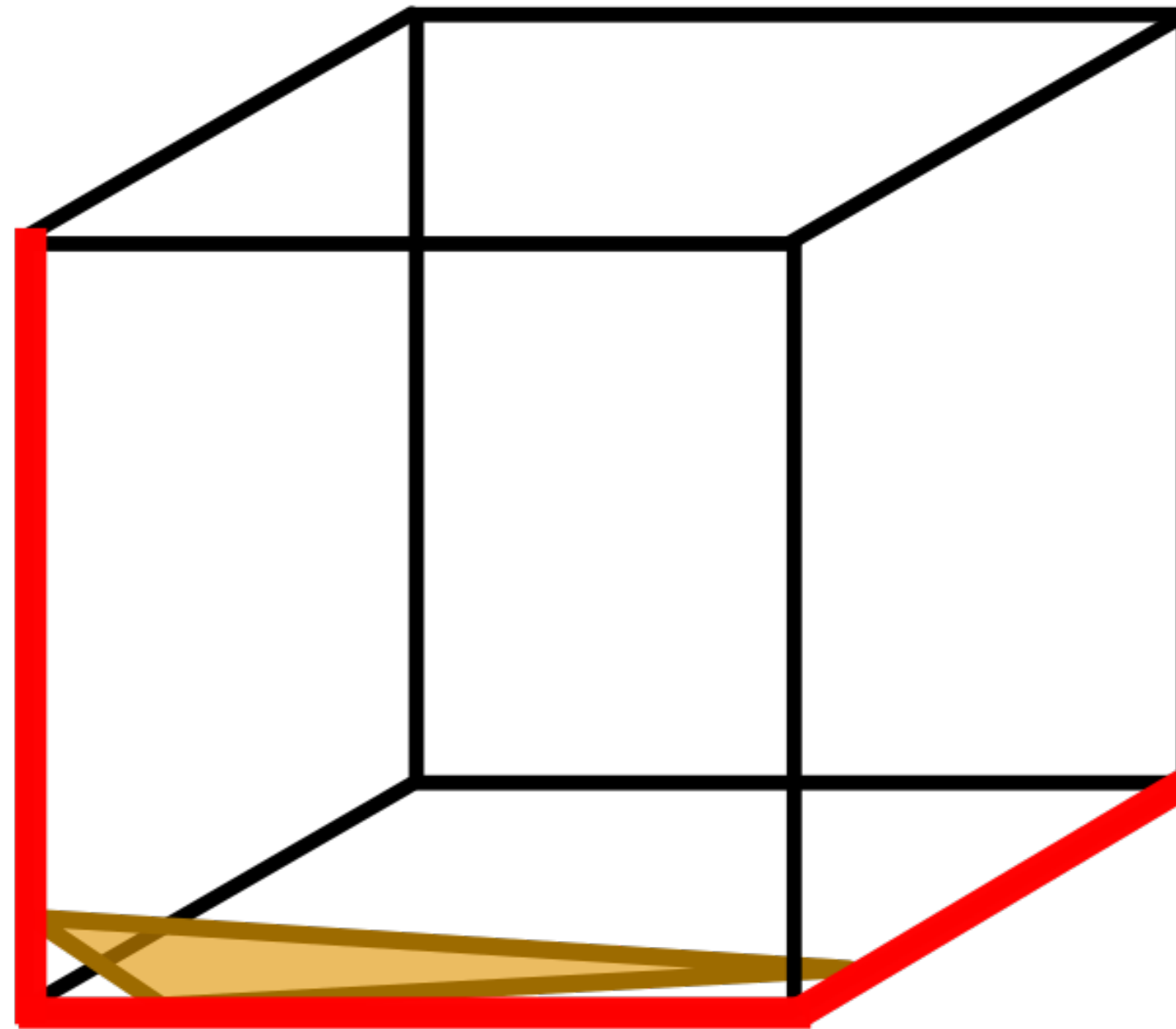


## Second Source for Skinny Triangles: Edge Group 2



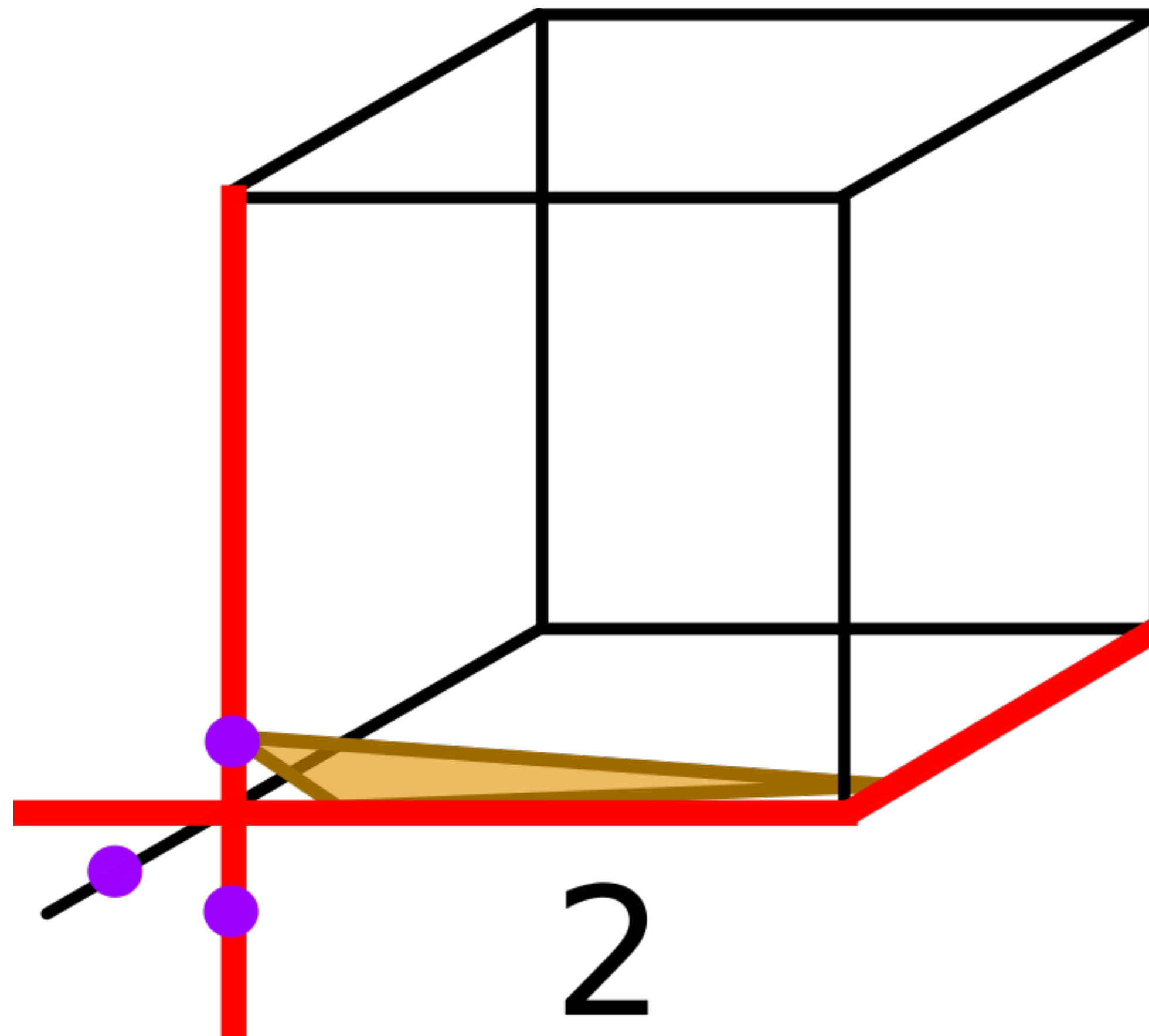
2

# Second Source for Skinny Triangles: Edge Group 2

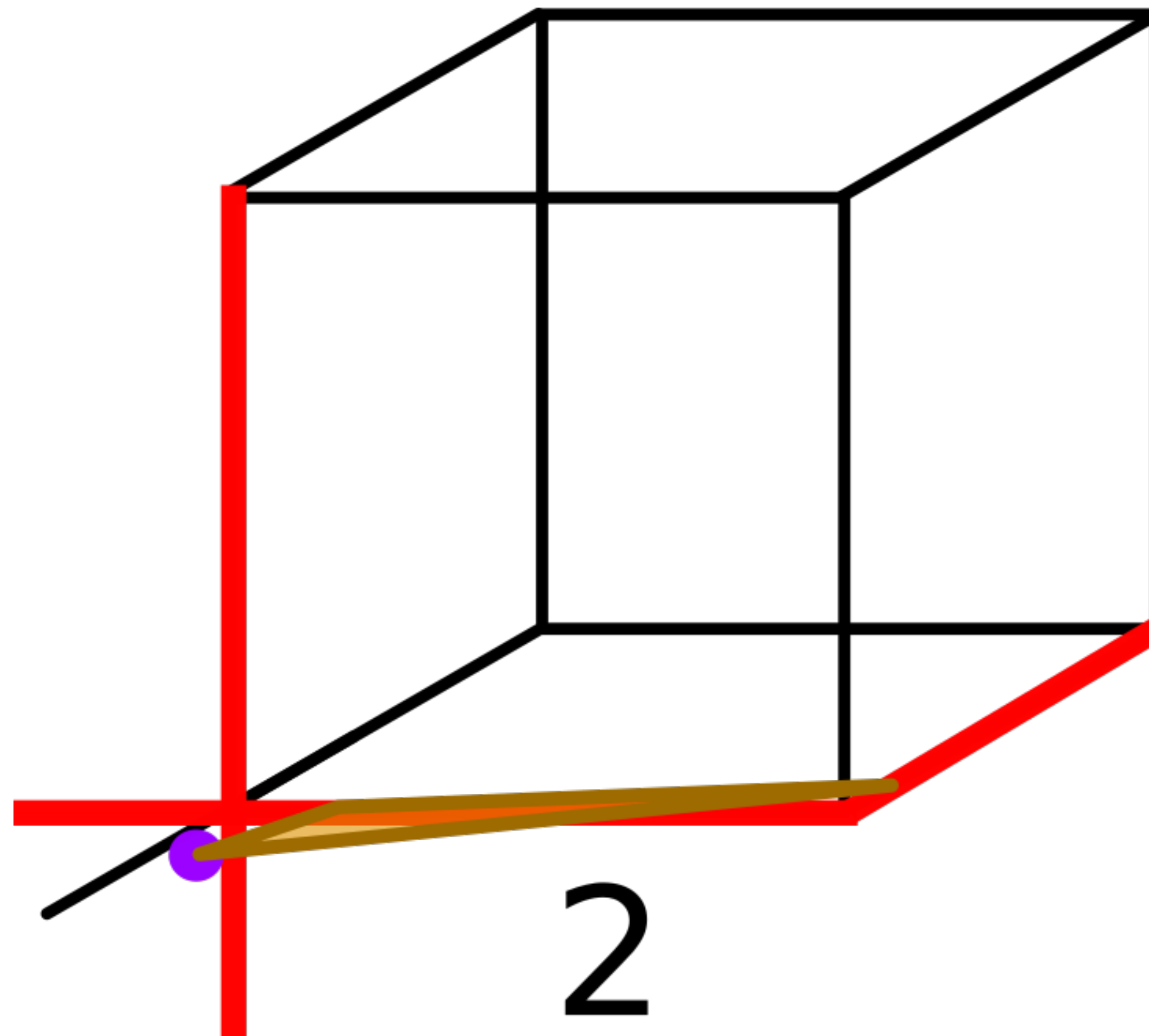


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# Second Source for Skinny Triangles: Edge Group 2

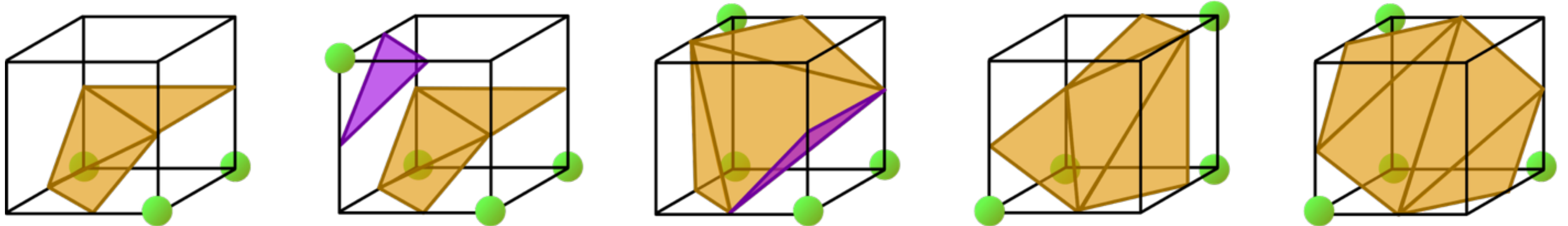


# Second Source for Skinny Triangles: Edge Group 2



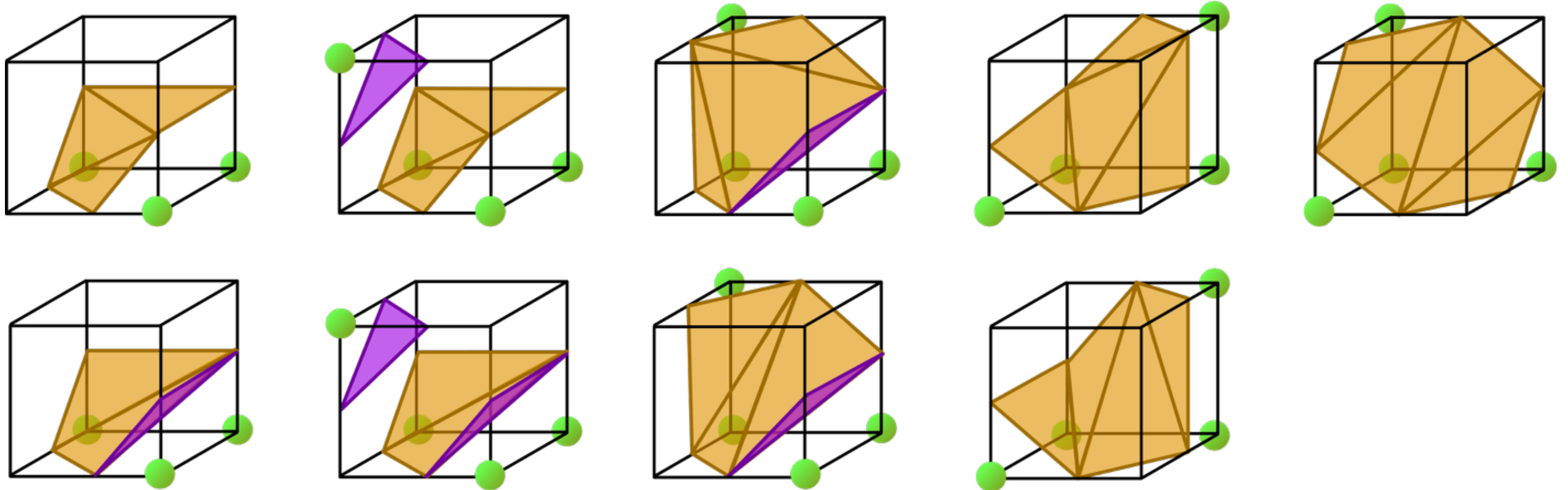


# How To Avoid Edge Group 2



C. A. Dietrich, C. E. Scheidegger, J. L. D. Comba, L. P. Bedel, C. T. Silva, Comput. Sci. Eng. 11, 82 (2009)

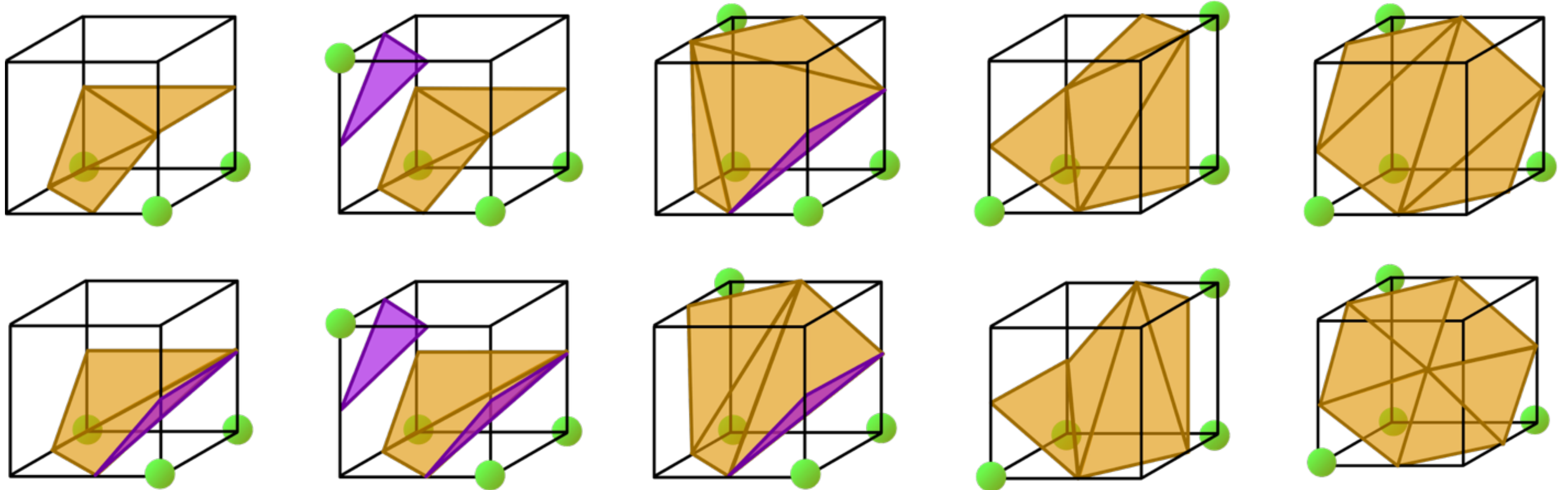
# How To Avoid Edge Group 2



Most cubes can be triangulated in a different way to avoid edge group 2

C. A. Dietrich, C. E. Scheidegger, J. L. D. Comba, L. P. Bedel, C. T. Silva, Comput. Sci. Eng. 11, 82 (2009)

# How To Avoid Edge Group 2



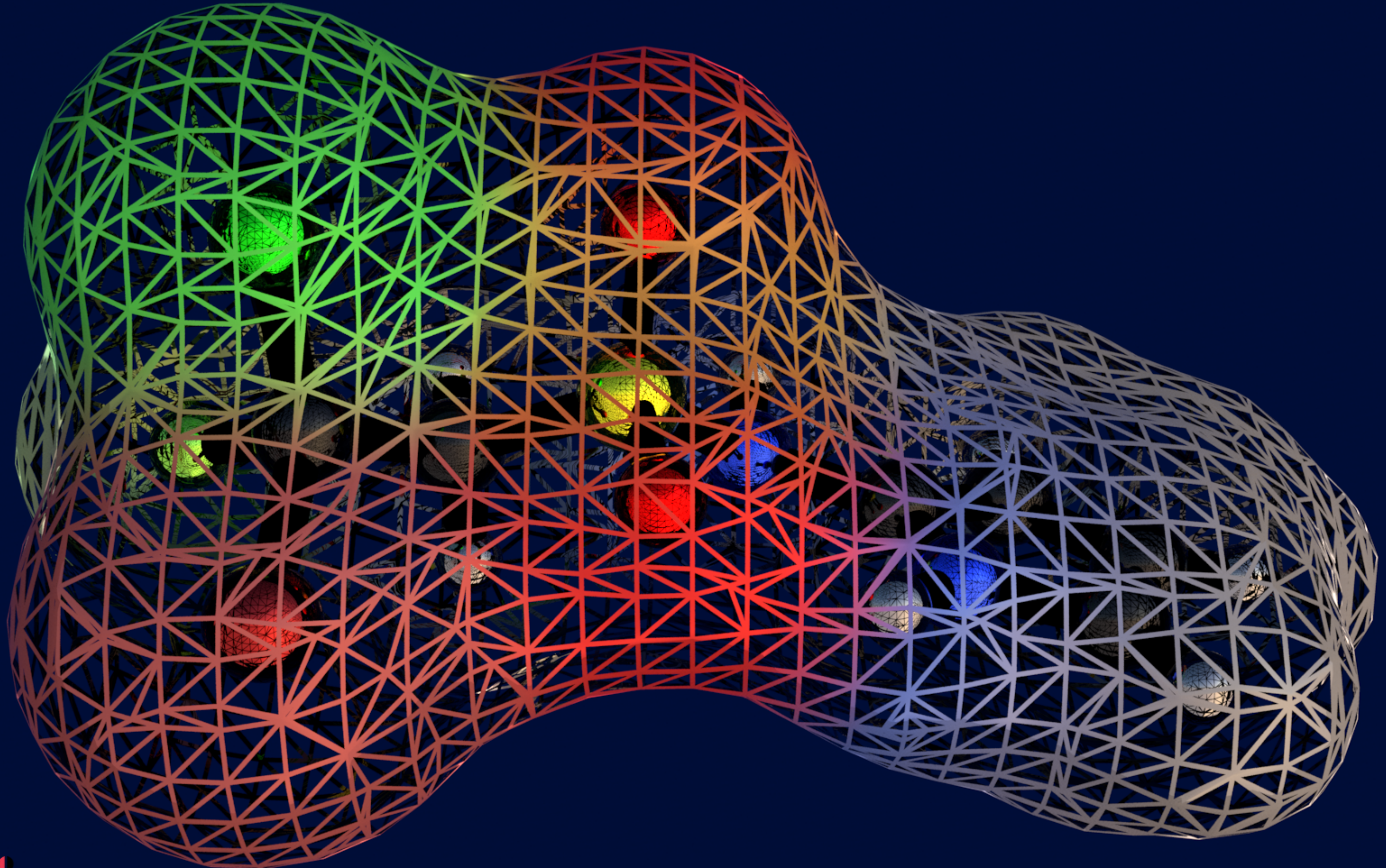
Most cubes can be triangulated in a different way to avoid edge group 2

Where triangulation without edge group 2 is impossible, we add a new vertex

C. A. Dietrich, C. E. Scheidegger, J. L. D. Comba, L. P. Bedel, C. T. Silva, Comput. Sci. Eng. 11, 82 (2009)

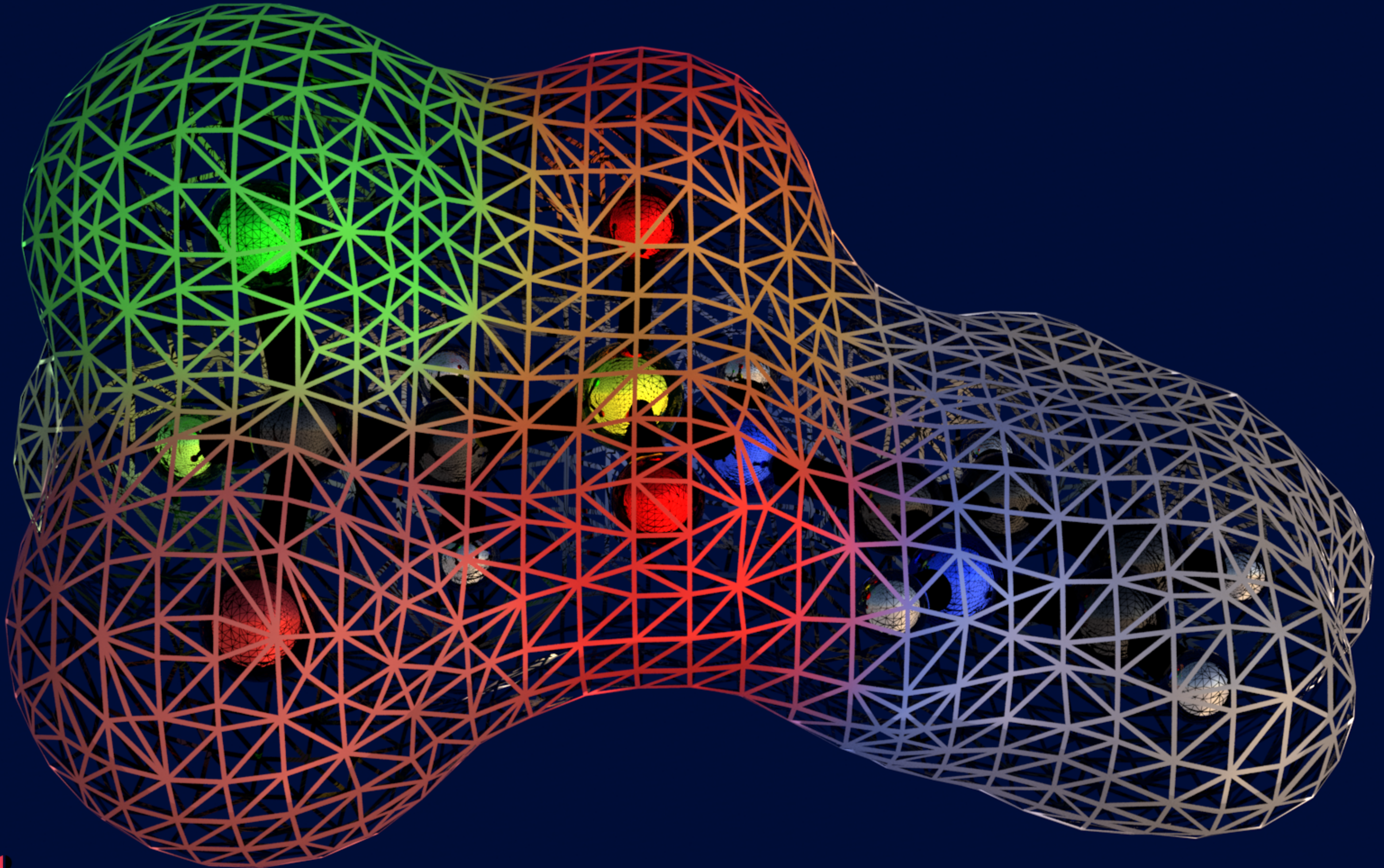


# Original Marching Cubes: 3864 triangles, 1934 vertexes



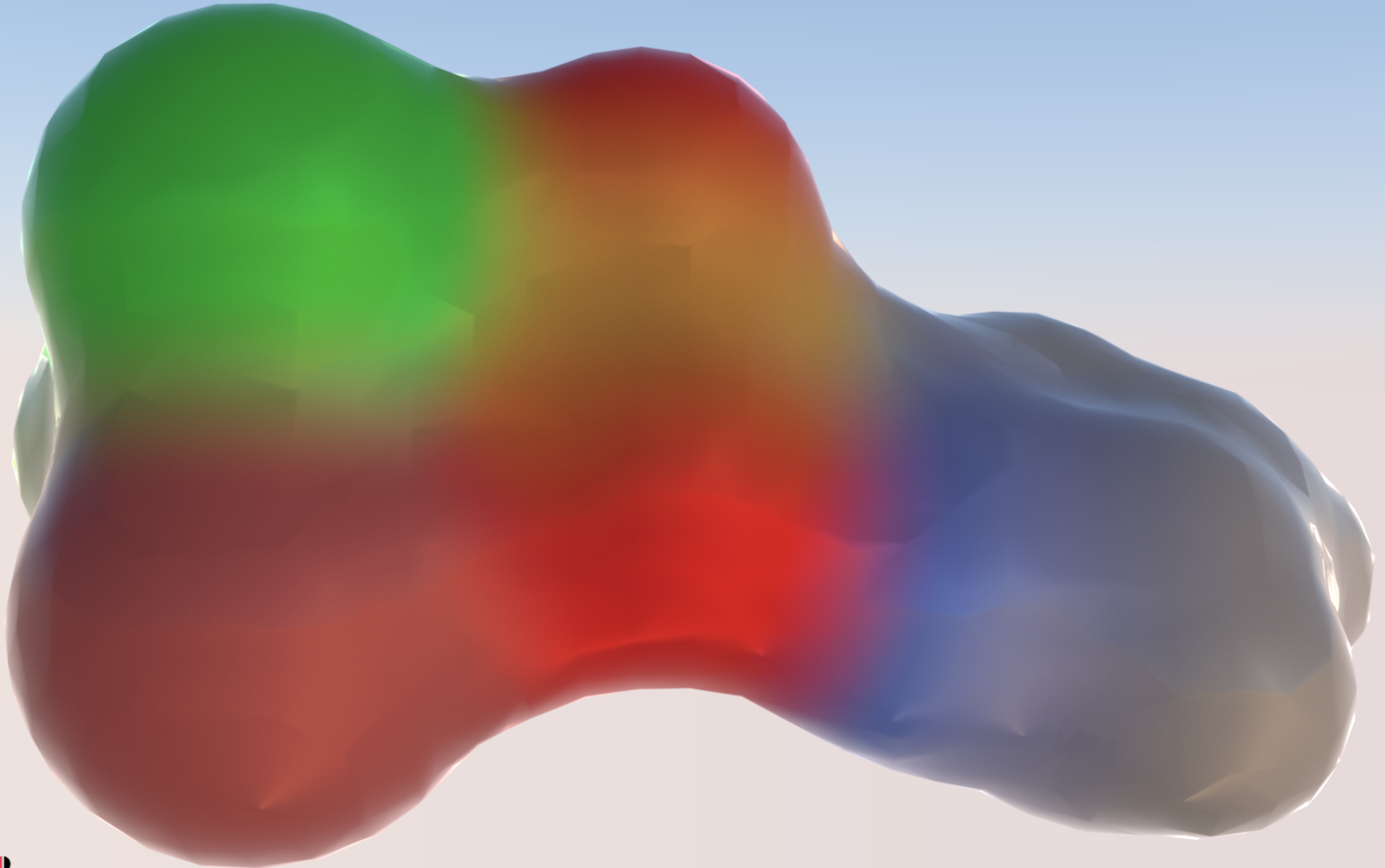


# Smooth Marching Cubes: 2956 triangles, 1480 vertexes



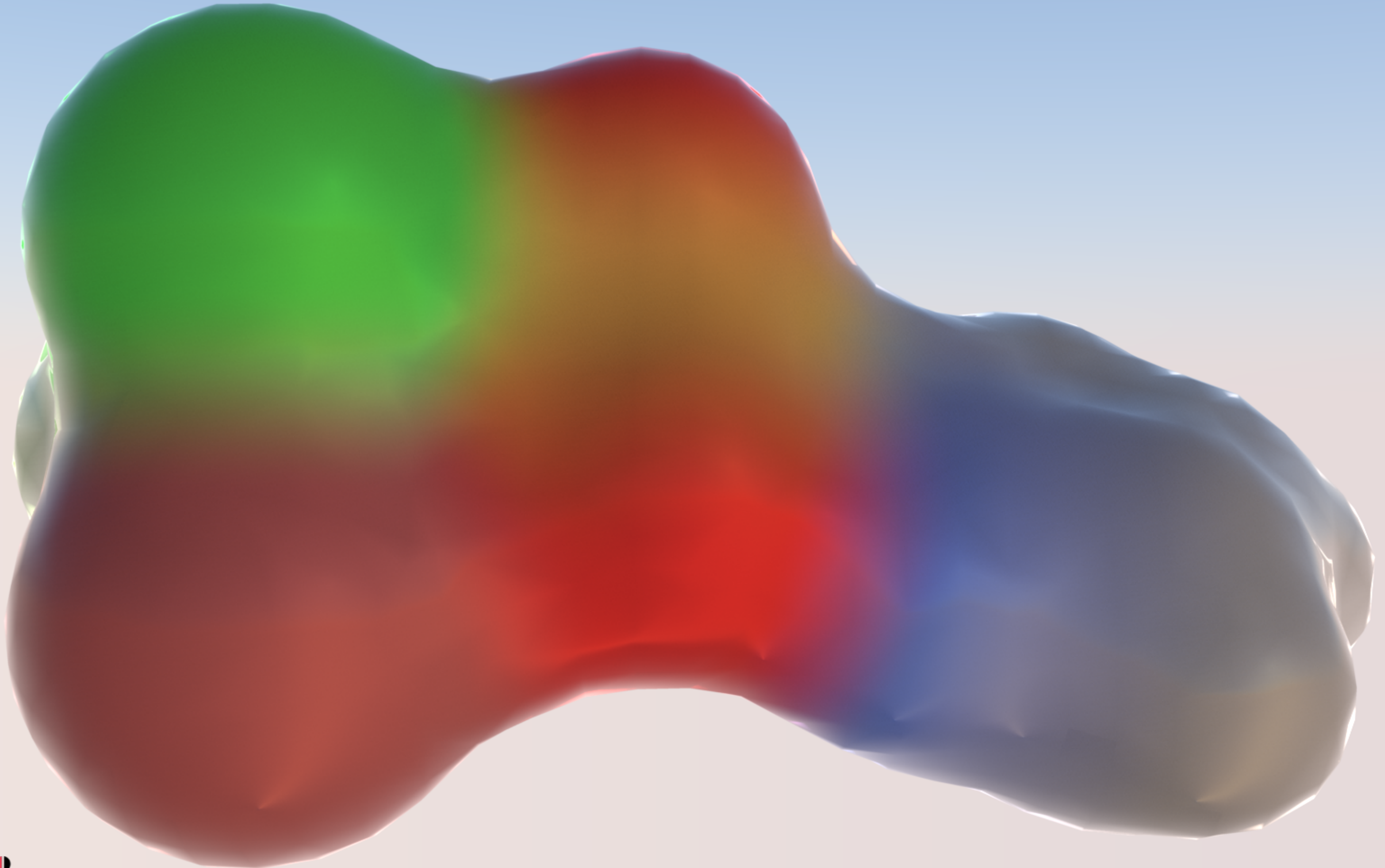


# Original Marching Cubes



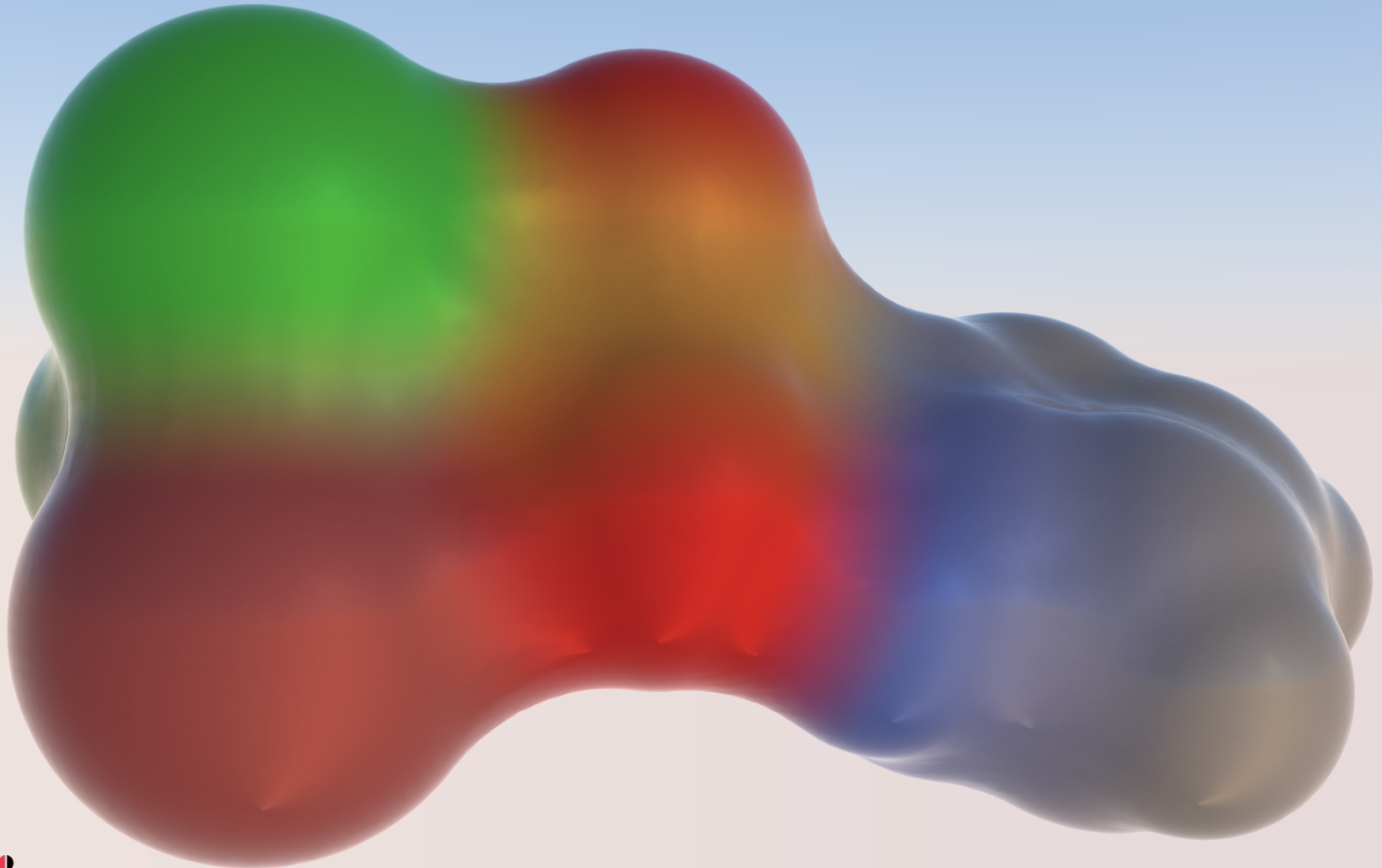


# Smooth Marching Cubes





# Smooth Marching Cubes With 0.1 Angstrom Resolution



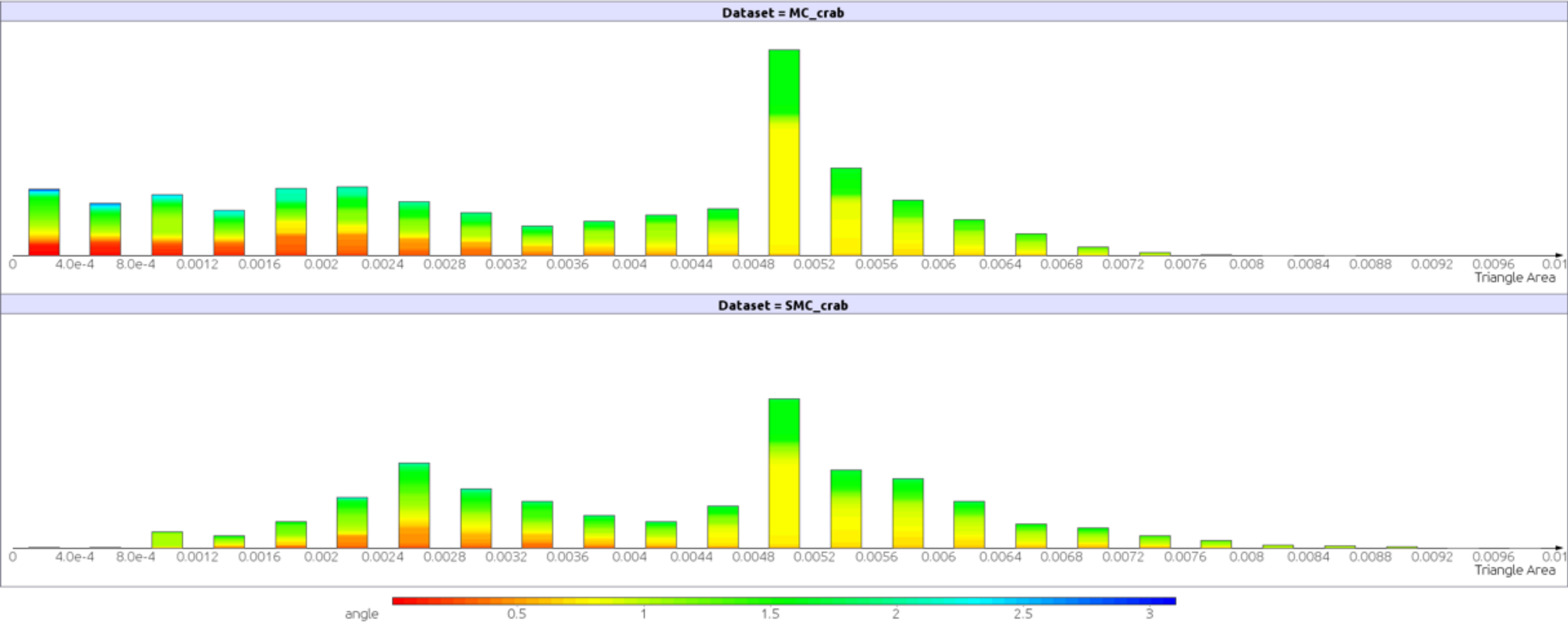
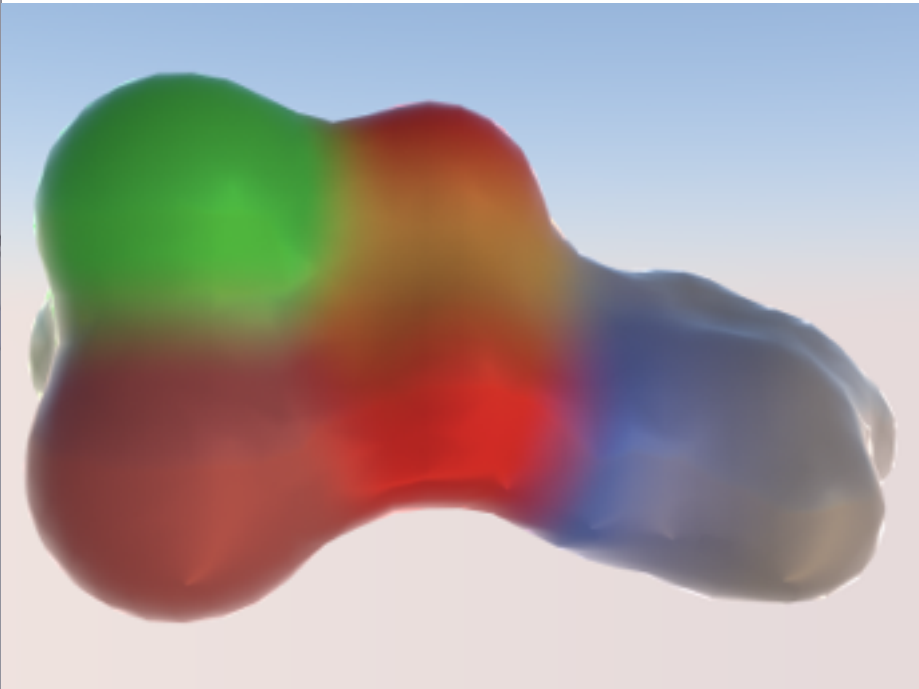
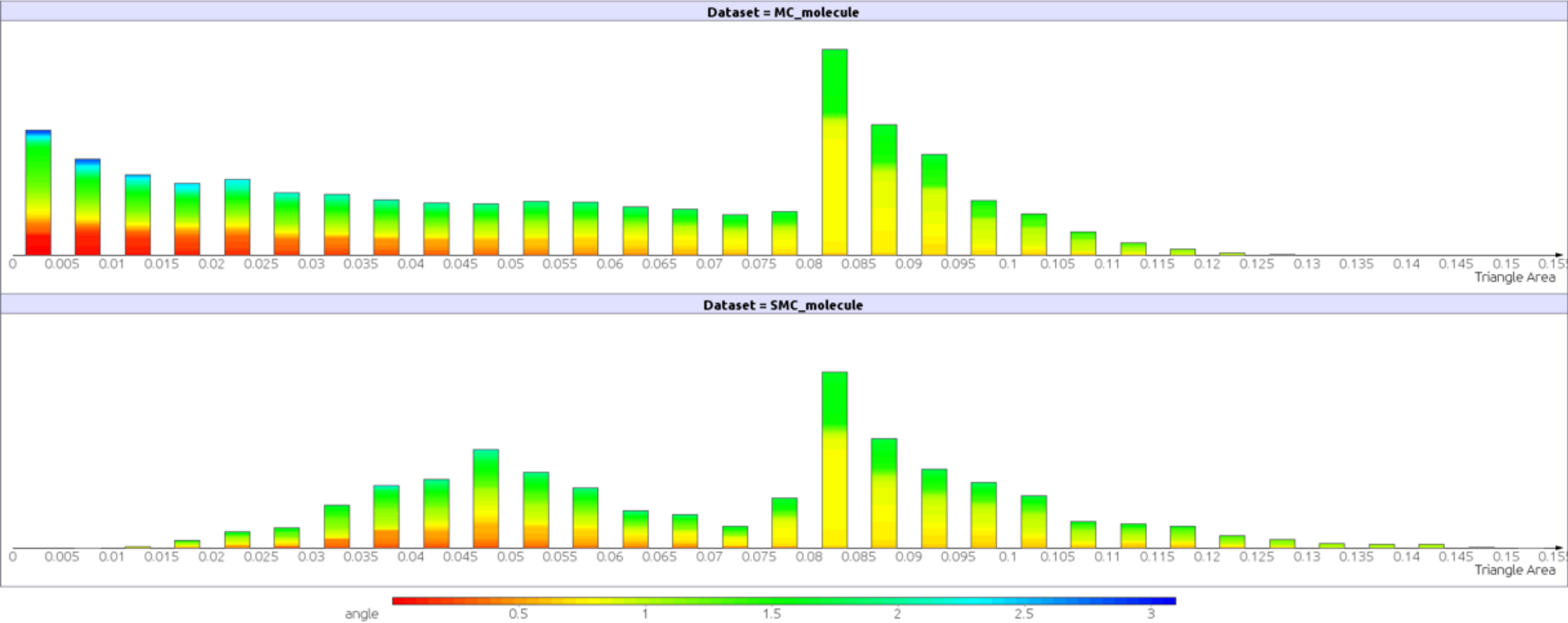


Noisy MRI Dataset: Lobster from [volvis.org](http://volvis.org))  
324\*301\*56 bytes, iso-surface at 40.5  
Gives 360038 triangles, 179174 vertexes



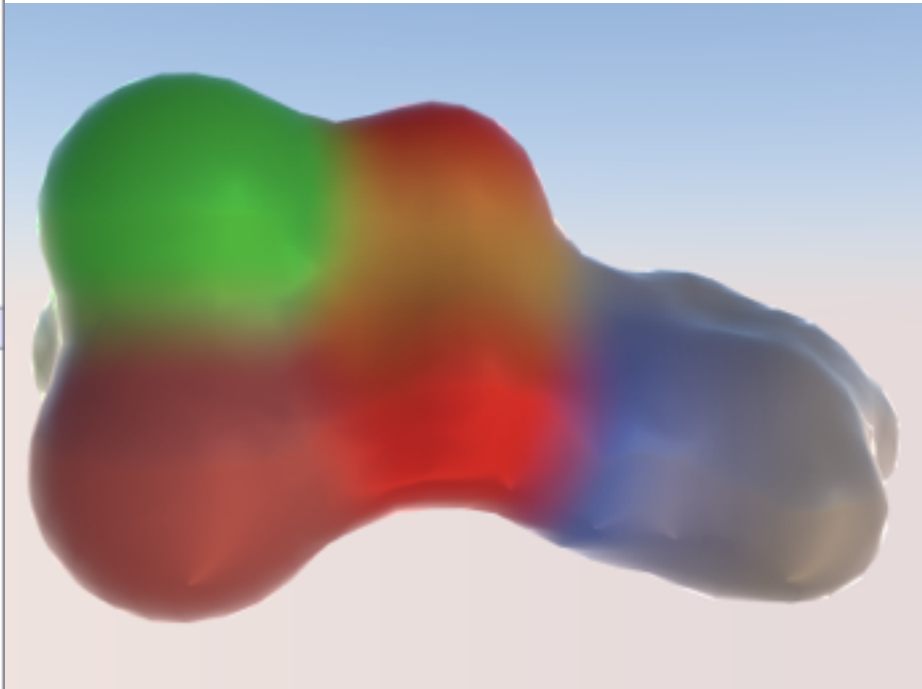
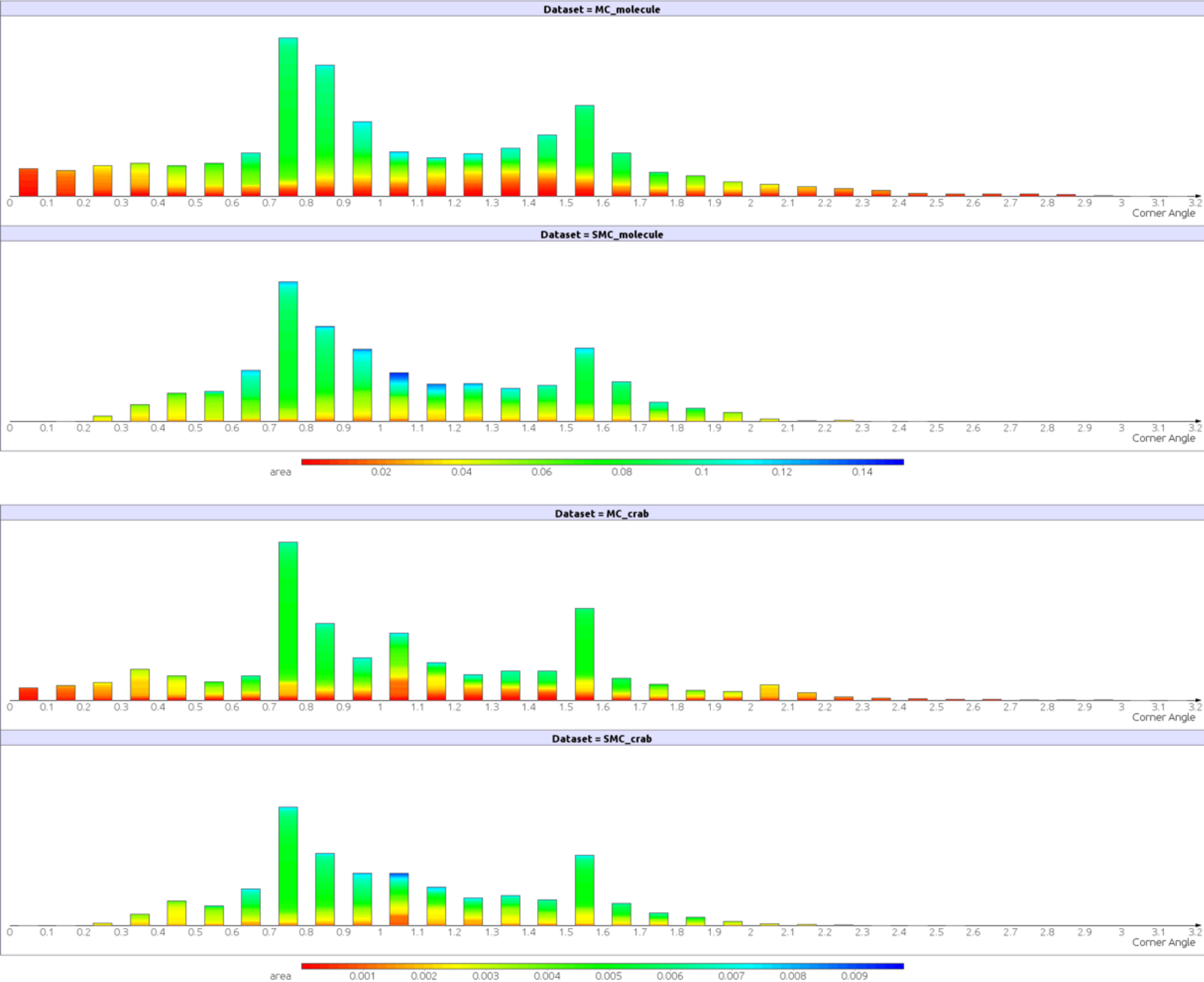


# Triangle Area Distribution: MC vs. SMC





# Corner Angle Distribution: MC vs. SMC



# Result

- ▶ New method improving the Marching Cubes algorithm
- ▶ It entirely avoids small area triangles and, therefore, allows high quality rendering
- ▶ There is a very small performance penalty
- ▶ 3D Environment independent Java source code
- ▶ Robust method applicable for molecular surfaces and noisy data (CT, MRI, etc)



# Thank You

