Derive $(m_*,\ g_*)$ dependence of dim = 6 effective low-energy lagrangian

$$U = e^{iH/f} \qquad\qquad f = \frac{m_*}{g_*}$$

$$\mathcal{L}_{strong} = \frac{m_*^4}{g_*^2}\left[\mathcal{L}^{(0)}(U,\partial/m_*) + \frac{g_*^2}{(4\pi)^2}\mathcal{L}^{(1)}(U,\partial/m_*) + \dots\right] + \mathcal{L}_{break}(U,\Phi_{SM})$$

Basic rules

- each extra $H$ leg costs $\dfrac{g_*}{m_*}$

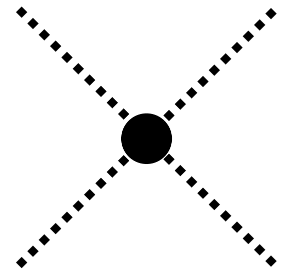
$$\partial_\mu \to \partial_\mu + iA_\mu$$

- each extra $\partial_\mu$ costs $\dfrac{1}{m_*}$

$$\frac{\partial^2}{m_*^2} \to \frac{F_{\mu\nu}}{m_*^2}$$

# ‘Higgs field insertions’

**1)** kinetic term



$$(\partial H)^2 \times \left(\frac{H}{f}\right)^2$$

$$\frac{c_H}{2f^2}\partial^\mu\left(H^\dagger H\right)\partial_\mu\left(H^\dagger H\right) \quad + \quad \frac{c_T}{2f^2}\left(H^\dagger \overleftrightarrow{D^\mu} H\right)\left(H^\dagger \overleftrightarrow{D_\mu} H\right)$$

$$\frac{1}{f^2}H^\dagger H|D_\mu H|^2 \quad \sim \quad \frac{1}{f^2}\partial^\mu\left(H^\dagger H\right)\partial_\mu\left(H^\dagger H\right) +$$ non-derivative terms

by field redefinition $\quad H^\alpha \to H^\alpha + (H^\dagger H)H^\alpha/f^2$

**2)** $\lambda_H (H^\dagger H)^2 \to \mathcal{O}_6 = c_6 \dfrac{\lambda_H}{f^2}(H^\dagger H)^3$

**3)** $y_{ij}\,\bar\psi_i H \psi_j \to \mathcal{O}_y = c_y\, y_{ij}\,\bar\psi_i H \psi_j \dfrac{H^\dagger H}{f^2}$

minimal flavor violation ➡ just one universal operator

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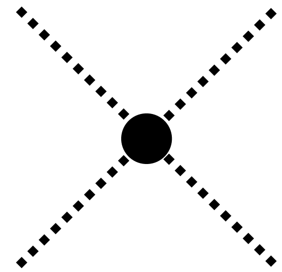

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$$\frac{1}{f^2} H^\dagger H |D_\mu H|^2 \quad \sim \quad \frac{1}{f^2}\partial^\mu \left(H^\dagger H\right) \partial_\mu \left(H^\dagger H\right) + \quad \text{non-derivative terms}$$

by field redefinition $\quad H^\alpha \rightarrow H^\alpha + (H^\dagger H) H^\alpha / f^2$

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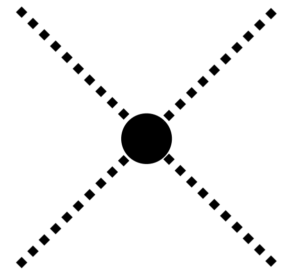

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$$\frac{c_H}{2f^2}\partial^\mu\left(H^\dagger H\right)\partial_\mu\left(H^\dagger H\right) \qquad c_y\, y_{ij}\,\bar{\psi}_i H\psi_j\,\frac{H^\dagger H}{f^2} \qquad c_6\,\frac{\lambda_H}{f^2}(H^\dagger H)^3$$

$O(\frac{v^2}{f^2})$ corrections to on-shell Higgs couplings

By power counting rules, derivative insertions are subleading at low energy

Ex $\quad \frac{1}{m_*^2}(\partial_\mu H)^\dagger \Box (\partial^\mu H)$

$$\frac{\delta\mathcal{O}}{\mathcal{O}} \sim \frac{E^2}{m_*^2} \sim \frac{m_h^2}{m_*^2} \sim \frac{g_{SM}^2}{g_*^2}\frac{v^2}{f^2} \ll \frac{v^2}{f^2}$$

# 2 Higgses & 4 covariant derivatives (CP conserving)

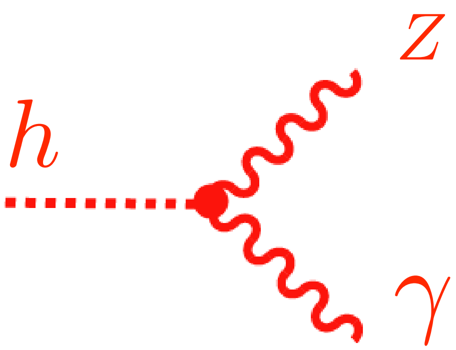
**1)** $O_W = i\frac{c_W}{m_*^2}\left(H^\dagger \sigma^i \overleftrightarrow{D^\mu} H\right)(D^\nu W_{\mu\nu})^i$ $\qquad$ $O_B = i\frac{c_B}{m_*^2}\left(H^\dagger \overleftrightarrow{D^\mu} H\right)(\partial^\nu B_{\mu\nu})$

- generated at tree level by massive vector exchange
- $\widehat{S} = (c_W + c_B)\frac{m_W^2}{m_*^2}$ $\qquad$ $m_* \gtrsim (c_W + c_B)^{1/2}\; 2.5\,\text{TeV}$ at $95\%$ CL

$$\frac{v^2}{f^2} \lesssim \frac{1.5}{c_W + c_B}\left(\frac{g_*}{4\pi}\right)^2$$

$$\widehat{S} \sim \frac{m_W^2}{m_*^2} = \frac{g_{SM}^2}{g_*^2}\frac{v^2}{f^2} = \frac{g_{SM}^2}{16\pi^2}\frac{16\pi^2}{g_*^2}\frac{v^2}{f^2} = \frac{g_{SM}^2}{16\pi^2} N \frac{v^2}{f^2}$$

**2)** $O_{HW} = i\frac{c_{HW}}{m_*^2}\frac{g_*^2}{16\pi^2}(D^\mu H)^\dagger \sigma^i (D^\nu H) W^i_{\mu\nu}$ $\quad$ $O_{HB} = i\frac{c_{HB}}{m_*^2}\frac{g_*^2}{16\pi^2}(D^\mu H)^\dagger (D^\nu H) B_{\mu\nu}$




$\propto c_{HW} - c_{HB}$ $\quad$ $(g-2)_W \propto c_{HW} + c_{HB}$

in a 'minimally' coupled gauge theory these effects arise from loops only
In general one could be more open minded

these affect vector trilinears and $h \to Z\gamma$

**3)** $$\epsilon_H \quad \left\{ \frac{c_\gamma}{m_*^2} H^\dagger H B_{\mu\nu} B^{\mu\nu} + \frac{c_g}{m_*^2} H^\dagger H G_{\mu\nu} G^{\mu\nu} \right\}$$

U(1)$_Q$ and color respect the Goldstone nature of the Higgs:
Higgs potential would stay flat if these where the only SM couplings

$\epsilon_H$ is the same parameter suppressing the Higgs potential $\epsilon_H \sim \dfrac{\lambda_t^2}{16\pi^2}$

$$\frac{\delta\sigma(gg \to h)}{\sigma_{SM}} \sim \frac{\lambda_t^2 v^2}{g_*^2 f^2} < \frac{v^2}{f^2}$$

$$O_{2W} = (D^\mu W_{\mu\nu})^i (D_\rho W^{\rho\nu})^i \quad O_{2B} = (\partial^\mu B_{\mu\nu})(\partial_\rho B^{\rho\nu}) \quad O_{2g} = (D^\mu G_{\mu\nu})^a (D_\rho G^{\rho\nu})^a$$

$$O_{3W} = \epsilon_{ijk} W_\mu^{i\,\nu} W_{\nu\rho}^j W^{k\,\rho\mu} \qquad O_{3g} = f_{abc} G_\mu^{a\,\nu} G_{\nu\rho}^b G^{c\,\rho\mu}$$

suppressed by $\frac{1}{m_*^2}\frac{1}{g_*^2}$

$$\frac{\delta\mathcal{O}}{\mathcal{O}} \sim \frac{E^2}{m_*^2}\frac{g_{SM}^2}{g_*^2}$$

# Effective lagrangian from composite Higgs sector

$$\begin{aligned}
\mathcal{L}_{\rm SILH} = &\frac{c_H}{2f^2}\partial^\mu \left(H^\dagger H\right) \partial_\mu \left(H^\dagger H\right) + \frac{c_T}{2f^2}\left(H^\dagger \overleftrightarrow{D^\mu} H\right)\left(H^\dagger \overleftrightarrow{D}_\mu H\right) \\
&- \frac{c_6 \lambda}{f^2}\left(H^\dagger H\right)^3 + \left(\frac{c_y y_f}{f^2} H^\dagger H \bar{f}_L H f_R + \text{h.c.}\right) \\
&+ \frac{i c_W g}{2m_\rho^2}\left(H^\dagger \sigma^i \overleftrightarrow{D^\mu} H\right)\left(D^\nu W_{\mu\nu}\right)^i + \frac{i c_B g'}{2m_\rho^2}\left(H^\dagger \overleftrightarrow{D^\mu} H\right)\left(\partial^\nu B_{\mu\nu}\right) \\
&+ \frac{i c_{HW} g}{16\pi^2 f^2}(D^\mu H)^\dagger \sigma^i (D^\nu H) W^i_{\mu\nu} + \frac{i c_{HB} g'}{16\pi^2 f^2}(D^\mu H)^\dagger (D^\nu H) B_{\mu\nu} \\
&+ \frac{c_\gamma g'^2}{16\pi^2 f^2}\frac{g^2}{g_\rho^2} H^\dagger H B_{\mu\nu} B^{\mu\nu} + \frac{c_g g_S^2}{16\pi^2 f^2}\frac{y_t^2}{g_\rho^2} H^\dagger H G^a_{\mu\nu} G^{a\mu\nu}.
\end{aligned}$$