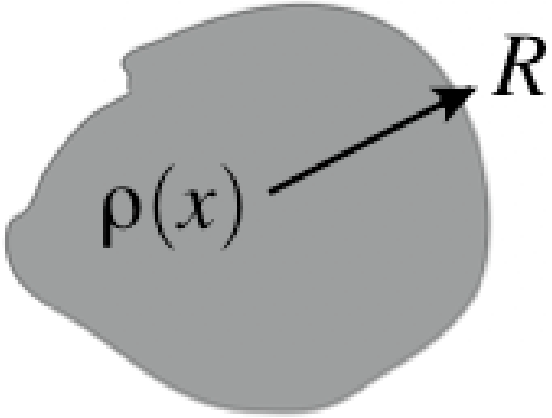
Natural and un-natural mass hierarchies

Particle mass versus size

- Classical computation



$$e \equiv \int \rho(x)\, d^3x$$

$$\Delta m \sim \frac{e^2}{4\pi}\frac{1}{R} \sim \frac{e^2}{4\pi}\Lambda$$

- Quantum result

scalar $$m^2 = m_0^2 + \frac{3e^2}{16\pi^2}\Lambda^2 + O(e^4)$$

fermion $$m = m_0\left(1 + \frac{3e^2}{8\pi^2}\ln\frac{\Lambda}{m_0} + O(e^4)\right)$$

For a fermion only a mild logarithmic divergence remains !!

concrete example: $$\frac{e^2}{16\pi^2}\ln\frac{M_{\text{Planck}}}{m_{\text{electron}}} \sim 0.37 = O(1)$$

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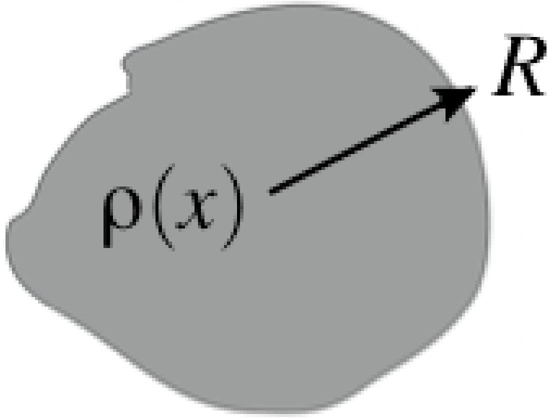

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$$\Lambda \sim \frac{1}{R}$$

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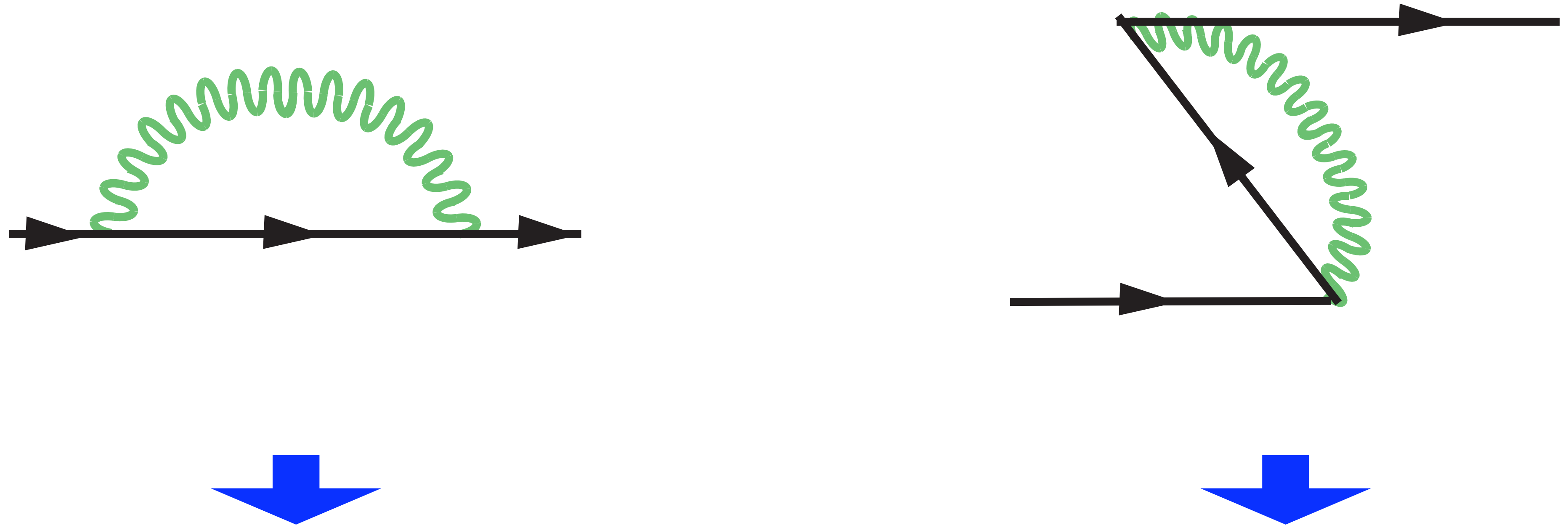
☑ cancellation is due to virtual positron contribution to mass



$$\Delta m_e = +\frac{e^2}{4\pi}\Lambda \qquad -\frac{e^2}{4\pi}\Lambda \qquad = 0$$

This result is more directly understood in terms of symmetries

Naturally small masses ⟷ Symmetry

1) Fermion: $$\mathcal{L}_{\text{electron}} = i\bar{\mathbf{e}}_{\mathbf{L}}\gamma^\mu D_\mu \mathbf{e}_{\mathbf{L}} + i\bar{\mathbf{e}}_{\mathbf{R}}\gamma^\mu D_\mu \mathbf{e}_{\mathbf{R}} + m_e\bar{\mathbf{e}}_{\mathbf{L}}\mathbf{e}_{\mathbf{R}} + m_e\bar{\mathbf{e}}_{\mathbf{R}}\mathbf{e}_{\mathbf{L}}$$

electron
+
positron

→

limit $m_e = 0$ respects
chiral symmetry

$$\mathbf{e}_{\mathbf{L}} \to \mathbf{e}_{\mathbf{L}}$$

$$\mathbf{e}_{\mathbf{R}} \to e^{i\theta}\mathbf{e}_{\mathbf{R}}$$

↓

$$\delta m_e \propto \frac{\alpha}{4\pi} m_e$$

$$\xcancel{\propto \frac{\alpha}{4\pi}\Lambda}$$

2) vector → gauge symmetry: $$A_\mu \to A_\mu + \partial_\mu a$$

mass term: $m_\gamma^2 A_\mu A^\mu$ not invariant

$$m_\gamma = 0$$

3) interacting massless scalar $\mathcal{L}_\varphi = \partial_\mu \varphi \partial^\mu \varphi + \lambda \varphi^4$

$\int d^4x \, \mathcal{L}_\varphi$ is classically invariant under dilatations $\varphi(x) \to k\varphi(kx)$

however the very existence of any UV scale explictly breaks dilatations

$$\delta m_\varphi^2 \sim \frac{\lambda}{16\pi^2} \Lambda^2$$

3a) Nambu-Goldstone boson $\varphi \to \varphi + c$

$$\mathcal{L} = \mathcal{L}(\partial\varphi) = (\partial_\mu\varphi)^2 + \frac{1}{\Lambda^4}(\partial_\mu\varphi)^2(\partial_\nu\varphi)^2 + \ldots$$

$E \ll \Lambda$ the scalar becomes a free particle

The Higgs looks only mildly like a NG boson!

$$\mathcal{L}_{top} = \lambda_t \bar{Q}_L H t_R + \text{h.c.}$$

$$\Big\downarrow$$

$$\delta m_H^2 \sim \frac{3\lambda_t^2}{8\pi^2} \Lambda^2$$

OK as long as

$$\Lambda \lesssim 1\,\text{TeV}$$

still interesting to build models at weak scale (see Pomarol)

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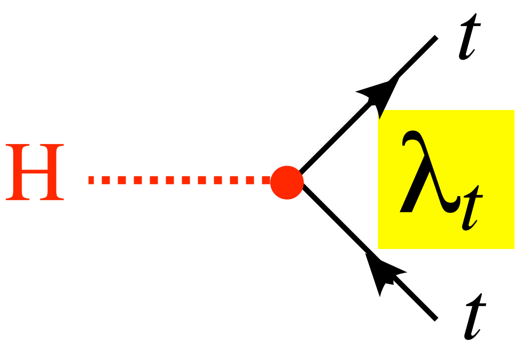
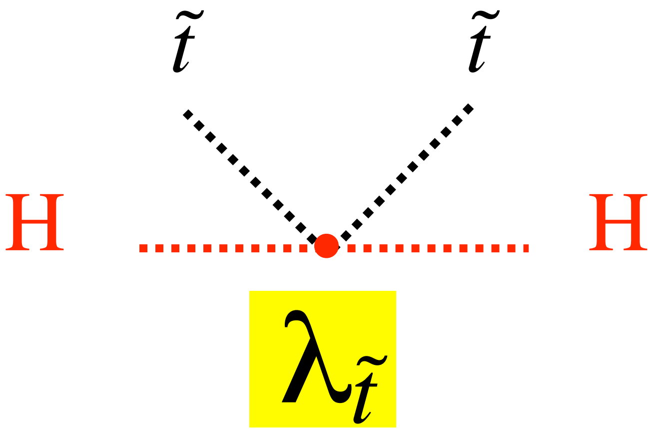
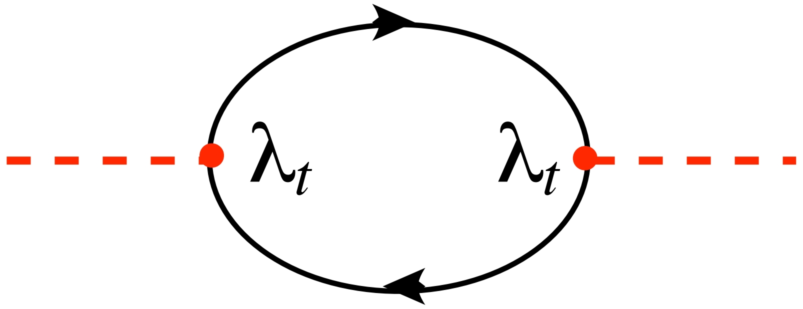
... we must speculate

Try to make Higgs scalar naturally light by following positron example: add new particles

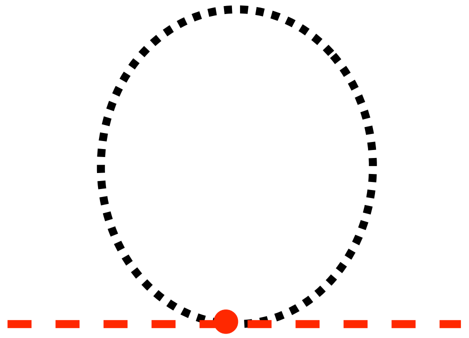
Ex: top quark contribution







+



$$\delta\mu^2 = \boxed{-} \frac{3\lambda_t^2}{8\pi^2}\Lambda^2 \qquad \boxed{+} \frac{3\lambda_{\tilde{t}}}{8\pi^2}\Lambda^2$$

fermion

boson

- Fermion and boson loops cancel each other for $\lambda_t^2 = \lambda_{\tilde{t}}$

$\lambda_t^2 = \lambda_{\tilde{t}}$ needs a symmetry relating bosons to fermions

Does such a symmetry exist ?

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YES !

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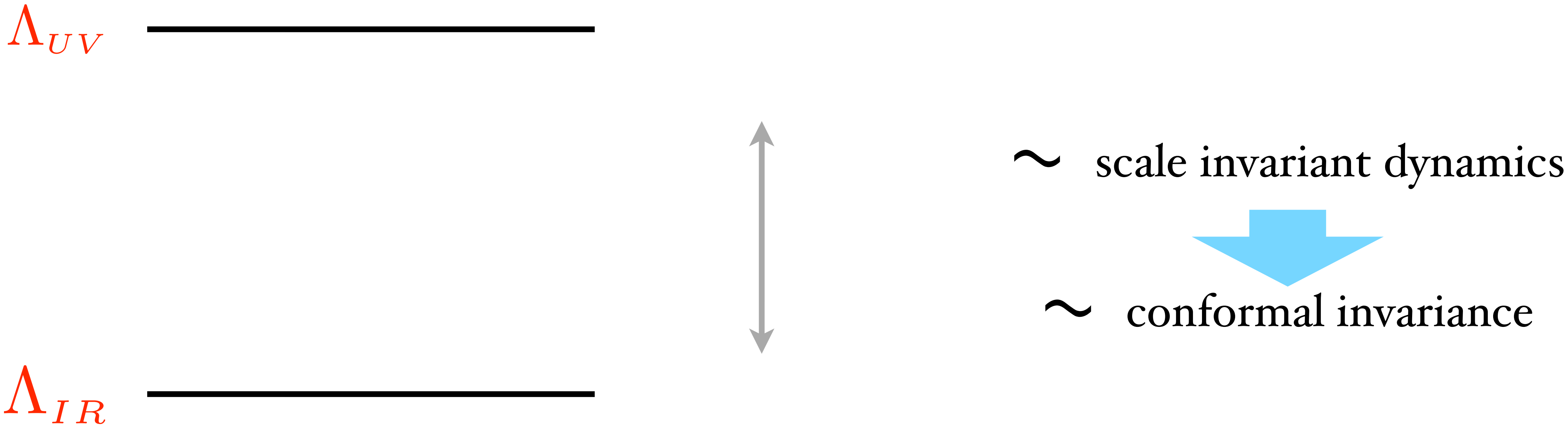
Does such a symmetry exist ?

YES !

SuperSymmetry

Volkov, Akulov 1973
Wess, Zumino 1974

A more `mature' and general viewpoint on mass hierarchies (based on Renormalization Group)

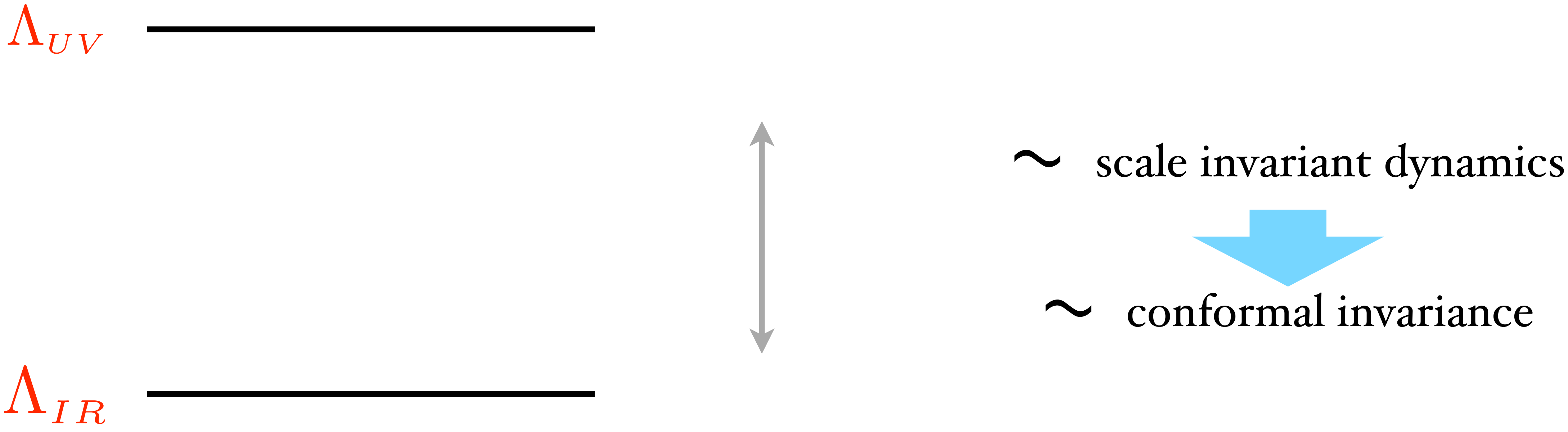


Λ_{UV}

$\sim$ scale invariant dynamics

$\sim$ conformal invariance

Λ_{IR}



Λ_{UV}

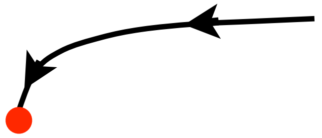
Λ_{IR}

$\sim$ scale invariant dynamics

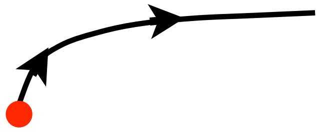
$\sim$ conformal invariance

❖ *stability* of $\Lambda_{IR} \ll \Lambda_{UV}$ characterized by dimensionality of *perturbations* at fixed point

$$\Delta\mathcal{L} = \lambda\mathcal{O} \qquad \lambda(E) = \lambda(\Lambda_{UV})\left(\frac{E}{\Lambda_{UV}}\right)^{d_{\mathcal{O}}-4}$$

$d_{\mathcal{O}} - 4 > 0$ irrelevant 

$d_{\mathcal{O}} - 4 = 0$ marginal 

$d_{\mathcal{O}} - 4 < 0$ relevant 

Ex. scalar mass $\lambda(E) = \left(\frac{m}{E}\right)^2$

Natural Hierarchy

Natural Hierarchy

A. There exists no strongly relevant operator

most relevant $4 - d_{\mathcal{O}} = \epsilon \ll 1$ $\lambda(E) = \lambda_0 \left(\frac{\Lambda_{UV}}{E}\right)^{\epsilon}$

$\Lambda_{IR} \longleftrightarrow \lambda(\Lambda_{IR}) \sim 1$ $\Rightarrow$ $\Lambda_{IR} \sim \Lambda_{UV} \lambda_0^{1/\epsilon}$ exponential hierarchy

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B. Strongly relevant operators exist, but can be controlled by a symmetry

Ex.

- ✦ quark mass in QCD $d_{\mathcal{O}} = 3$ controlled by chiral symmetry
- ✦ scalar masses in MSSM $d_{\mathcal{O}} = 2$ SUSY + chiral symm

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The Standard Model belongs to neither cathegory