Topological matter and
its exploration with quantum gases

The Kosterlitz-Thouless transition explored with atomic and photonic fluids

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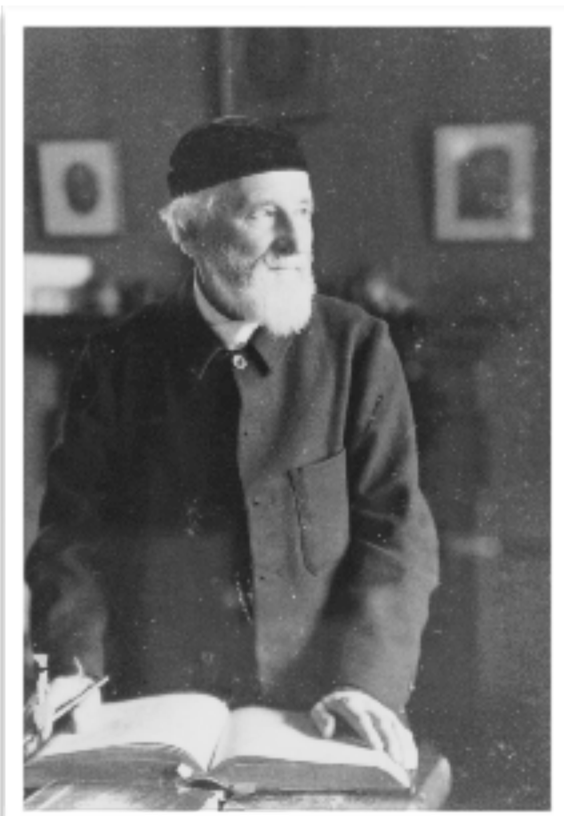
Lectures at EPFL
November 2019

COLLÈGE DE FRANCE
1530

A two-dimensional world

From sociology



E.A. Abbott 1838-1926

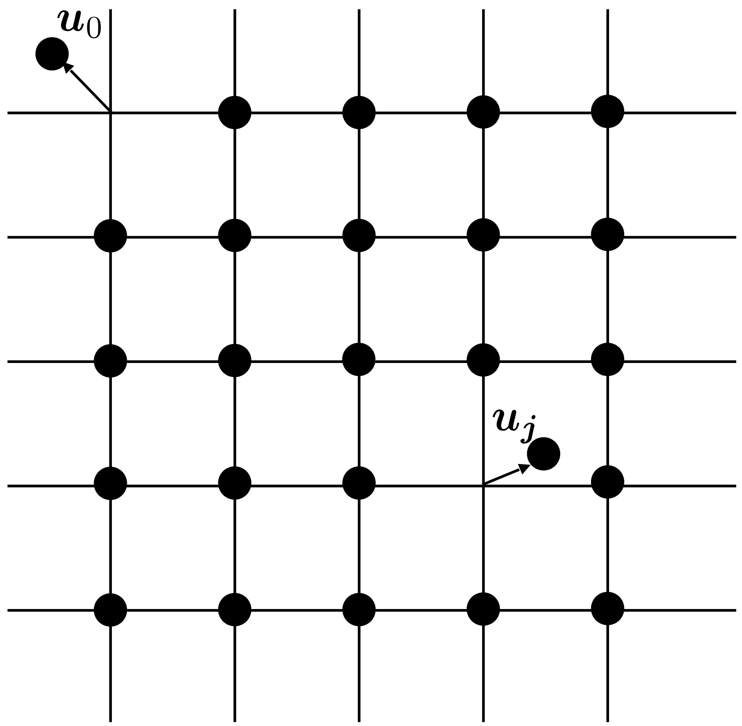
… to physics

R. Peierls 1907-95

Would the usual physical objects like crystals or magnets exist if we were living in a 2D world?

Peierls's argument (1935)

At non-zero temperature there is no true crystalline order in dimension 1 or 2



$$\langle (\boldsymbol{u_j} - \boldsymbol{u}_0)^2 \rangle \propto T \ \ln(R_j)$$

diverges at long distance R_j

Generalization by Mermin-Wagner and Hohenberg (1966) for any system with short-range interactions: no breaking of a continuous symmetry leading to a long-range order in the system ($T \neq 0$)

This result also applies to the case of Bose-Einstein condensation (U(1) symmetry)

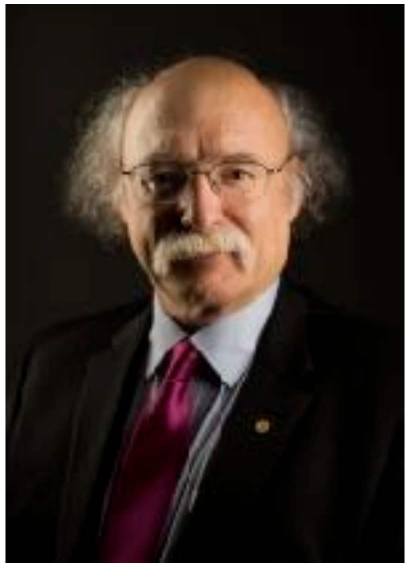
Topological order

1973, Kosterlitz & Thouless :

Ordering, metastability and phase transitions in two-dimensional systems

J.M. Kosterlitz

D.J. Thouless



F.D. Haldane

In spite of Mermin-Wagner-Hohenberg theorem, “unconventional” phase transitions can still take place in 2D systems

Transition between two different kinds of disordered phases, that are topologically distinct

Outline of the first two hours

1. The Peierls argument

The role of thermal phonons in 1D, 2D, 3D

2. The 2D ideal Bose gas

- Uniform vs. trapped systems
- The case of a 2D gas made with photons

3. The Gross-Pitaevskii approach

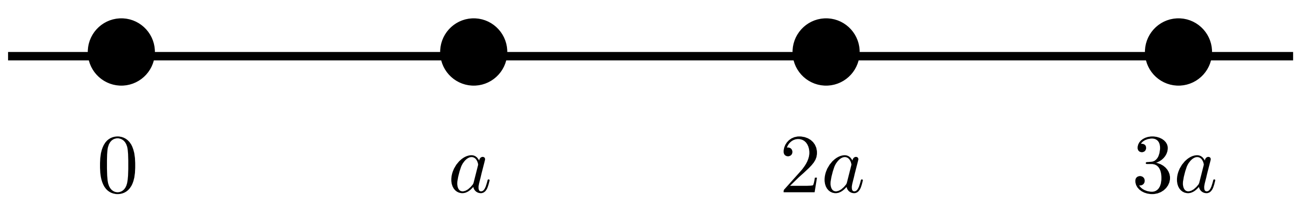
- Phase and density fluctuations
- Quasi-long range order
- Bogoliubov spectrum and sound propagation

Second part of the lecture:
The role of vortices and the Kosterlitz-Thouless mechanism

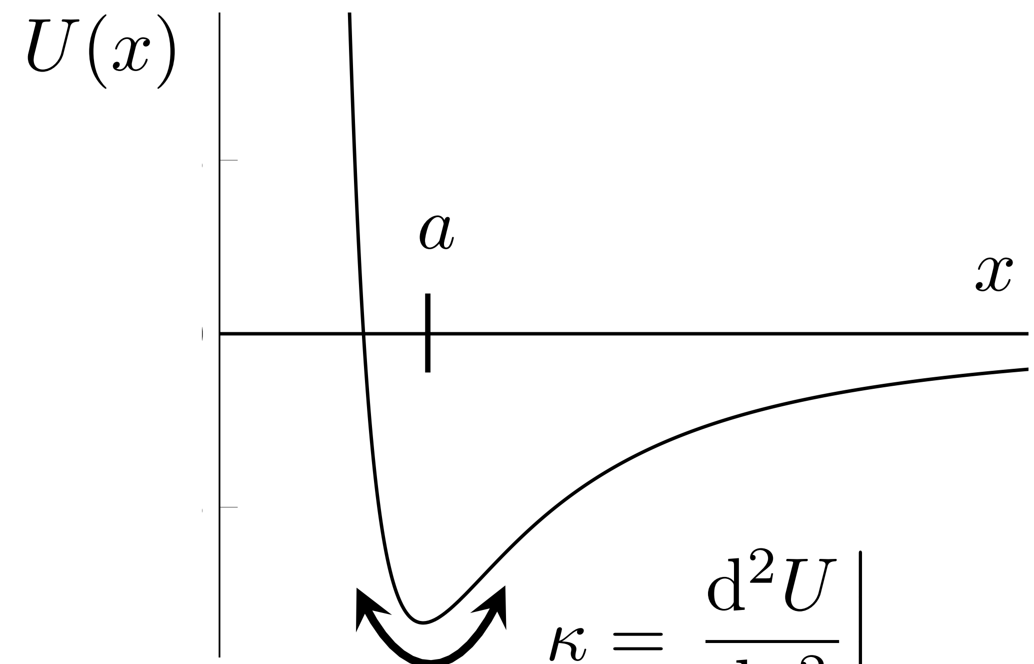
Hadzibabic & Dalibard, Riv. Nuo. Cim. 34, 389 (2011) + College de France lectures 2016-17
For superconductors: Benfatto, Castellani & Giamarchi, arXiv 1201.2307

Peierls argument in one dimension

A simple qualitative version: Piling up defects



Zero temperature: **ordered chain**



$$\kappa = \left.\frac{\mathrm{d}^2 U}{\mathrm{d}x^2}\right|_{x=a}$$

Non-zero temperature:

We fix the position x_0 of the atom $j = 0$. The position of atom $j = 1$ can fluctuate:

$$x_1 = x_0 + a + \delta_1 \qquad \langle \delta_1 \rangle = 0 \qquad \langle \delta_1^2 \rangle \sim \frac{k_\mathrm{B} T}{\kappa}$$

Piling up defects (2)

$$\left\{\begin{array}{l} x_1 = x_0 + a + \delta_1 \\ x_2 = x_1 + a + \delta_2 \\ . \\ . \\ . \\ x_j = x_{j-1} + a + \delta_j \end{array}\right. \qquad \begin{array}{l} \langle \delta_1^2 \rangle \sim \dfrac{k_\mathrm{B} T}{\kappa} \\ \\ \\ \\ \\ \langle \delta_j^2 \rangle \sim \dfrac{k_\mathrm{B} T}{\kappa} \end{array}$$

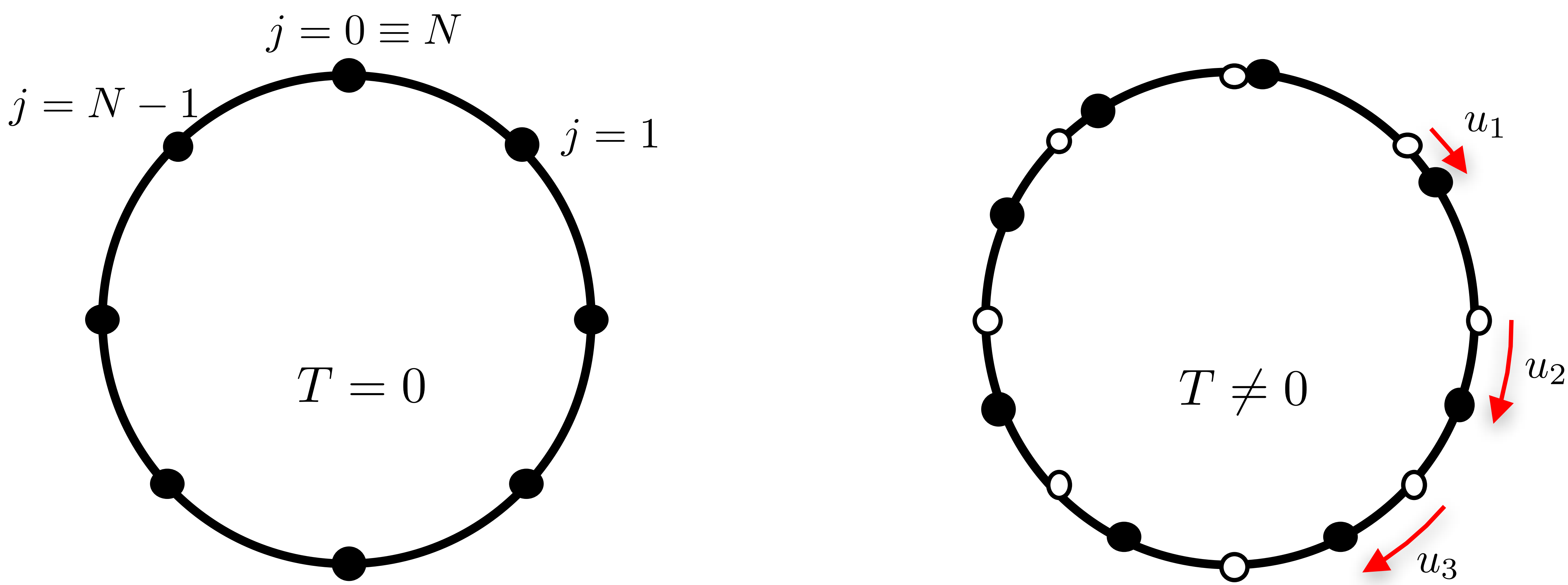
$$\longrightarrow \quad x_j = x_0 + ja + \Delta_j \qquad \Delta_j = \delta_1 + \delta_2 + \ldots + \delta_j$$

Sum of independent variables: $\langle \Delta_j^2 \rangle \sim \dfrac{k_\mathrm{B} T}{\kappa}\, j.$

If $\langle \Delta_j^2 \rangle \gtrsim a^2$, i.e. if $j \gtrsim \dfrac{\kappa a^2}{k_\mathrm{B} T}$, we have lost all information regarding the position of atom *j* with respect to the crystal period: no long-range order

A more quantitative analysis: The 1D harmonic crystal

Periodic boundary conditions



Energy : $$E = \sum_{j=1}^{N} \frac{1}{2} m \dot{u}_j^2 \; + \; \frac{\kappa}{2}(u_{j+1} - u_j)^2$$

Hypothesis: $|u_{j+1} - u_j| \ll a$ but not necessarily $|u_j| \ll a$

Equations of motion: $m\, \ddot{u}_j = \kappa(u_{j+1} - 2u_j + u_{j-1})$

Solutions of the equations of motion

Analysis in Fourier space: $\hat{u}_q = \frac{1}{\sqrt{N}} \sum_j \mathrm{e}^{-\mathrm{i}\, q X_j}\, u_j$ $\qquad X_j = ja$

wave number q : $\quad q = -\frac{\pi}{a}, \ldots, -\frac{2\pi}{Na},\ 0,\ \frac{2\pi}{Na}, \ldots, +\frac{\pi}{a}$

N independent equations for each value of q :

$$\ddot{\hat{u}}_q + \omega_q^2 \hat{u}_q = 0 \qquad \text{with} \qquad \omega_q = 2\sqrt{\frac{\kappa}{m}}\, |\sin \frac{qa}{2}|$$

For small wave numbers, $q \ll \pi/a$, we find a linear dispersion relations:

$$\omega_q = c\, |q| \qquad \text{with} \qquad c = a\sqrt{\frac{\kappa}{m}}$$

Sound waves (phonons)

Thermodynamic equilibrium for this 1D system

System described by a collection of independent harmonic oscillators ω_q

Average at thermal equilibrium: $\langle \hat{u}_q \hat{u}^*_{q'} \rangle = 0$ if $q \neq q'$

$$\frac{1}{2} m \omega_q^2 \langle |\hat{u}_q|^2 \rangle = \frac{1}{2} k_\mathrm{B} T \longrightarrow \langle |u_q|^2 \rangle = \frac{k_\mathrm{B} T}{m \omega_q^2}$$

Back to position space: what is the correlation between the displacements of two atoms separated by *j* sites?

$$u_j - u_0 = \frac{1}{\sqrt{N}} \sum_q \left(\mathrm{e}^{\mathrm{i} q X_j} - 1 \right) \hat{u}_q \qquad X_j = ja$$

$$\longrightarrow \langle u_j - u_0 \rangle = 0$$

$$\longrightarrow \langle (u_j - u_0)^2 \rangle = \frac{k_\mathrm{B} T}{m} \frac{4}{N} \sum_q \frac{\sin^2 (q X_j / 2)}{\omega_q^2}$$

to be calculated

Estimation of $\langle (u_j - u_0)^2 \rangle$

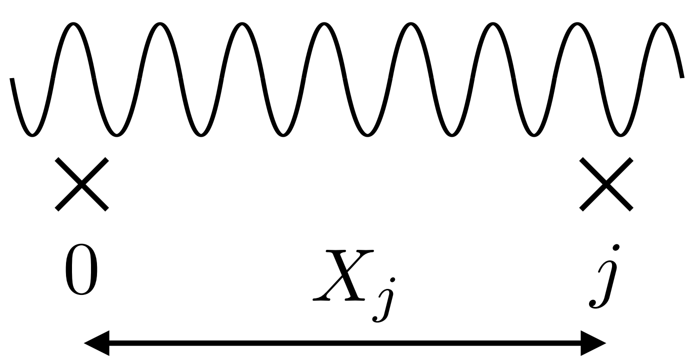
Passage from a discrete sum over q to an integral, and use of $\omega_q \approx c\,|q|$

$$\langle (u_j - u_0)^2 \rangle \approx \frac{4}{\pi}\,\frac{k_\mathrm{B}T}{\kappa a} \int_0^{\pi/a} \frac{\sin^2(qX_j/2)}{q^2}\,\mathrm{d}q \quad < \quad \frac{4}{\pi}\,\frac{k_\mathrm{B}T}{\kappa a} \int_0^{\pi/a} \frac{1}{q^2}\mathrm{d}q \quad ?$$

useless: divergent integral in $q = 0$!

Cut the integral in two pieces, which are estimated using different approximations:

- Larger wave number (i.e., shorter wavelength) : $q > \pi/X_j$



$$\sin^2(qX_j/2) \approx \frac{1}{2}$$

- Smaller wave number (i.e., larger wavelength) : $q < \pi/X_j$

$$\sin^2(qX_j/2) \approx (qX_j/2)^2$$

Estimation of $\langle (u_j - u_0)^2 \rangle$

Contribution of the two pieces of the integral

Large q:

$$\langle (u_j - u_0)^2 \rangle \approx \frac{4}{\pi} \frac{k_\mathrm{B}T}{\kappa a} \int_{\pi/X_j}^{\pi/a} \frac{1}{2q^2} \, \mathrm{d}q$$
$$\sim \frac{k_\mathrm{B}T}{\kappa a} X_j$$

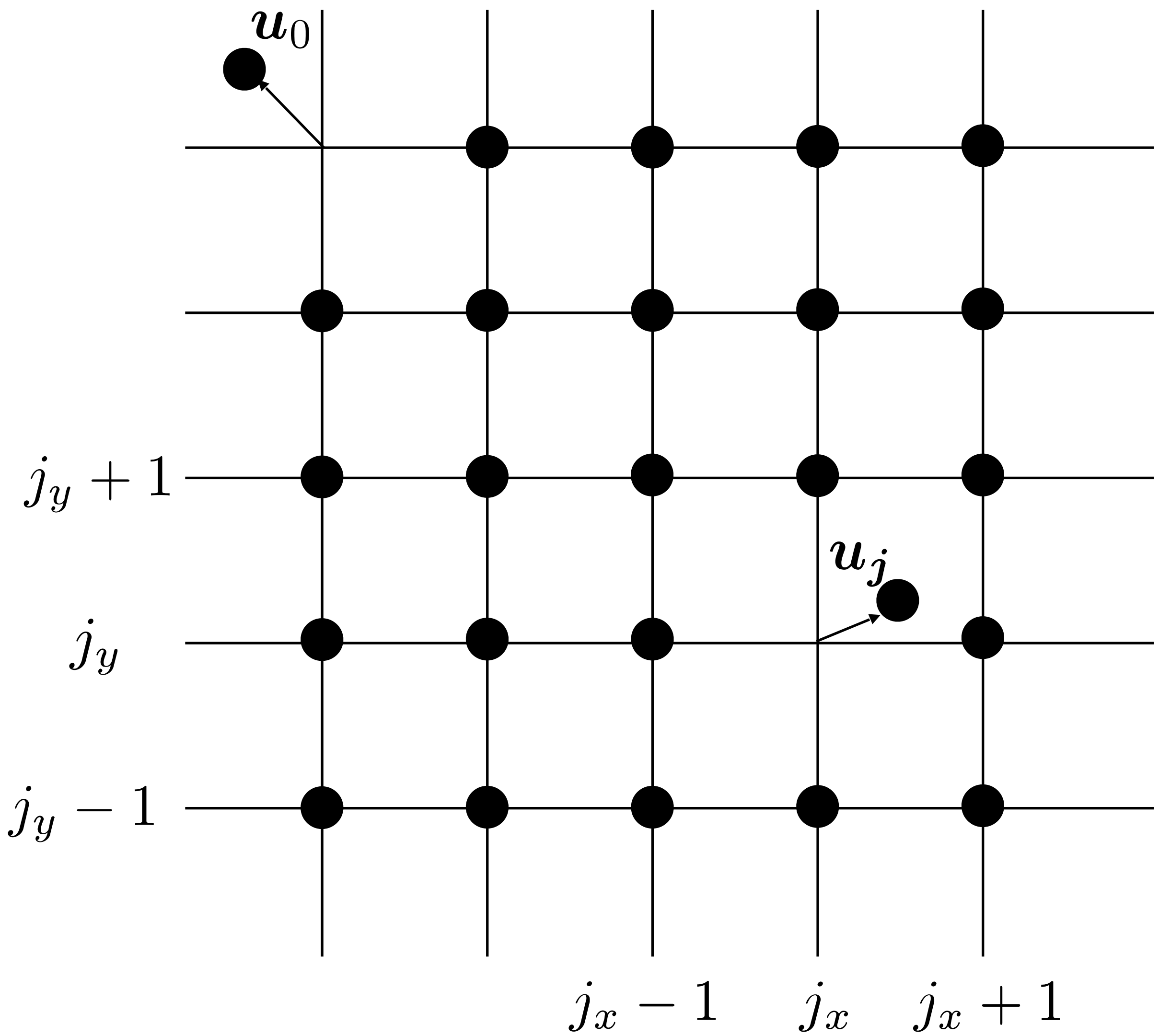
Small q:

$$\langle (u_j - u_0)^2 \rangle \approx \frac{4}{\pi} \frac{k_\mathrm{B}T}{\kappa a} \int_{0}^{\pi/X_j} \frac{(qX_j/2)^2}{q^2} \, \mathrm{d}q$$
$$\sim \frac{k_\mathrm{B}T}{\kappa a} X_j$$

The two parts have a similar magnitude and we recover the result obtained by “piling up” the defects. The main contribution comes from the modes:

$$q \sim \frac{\pi}{X_j}$$ *responsible for the loss of long-range order*

The two-dimensional case



$$\boldsymbol{j} \equiv (j_x, j_y)$$

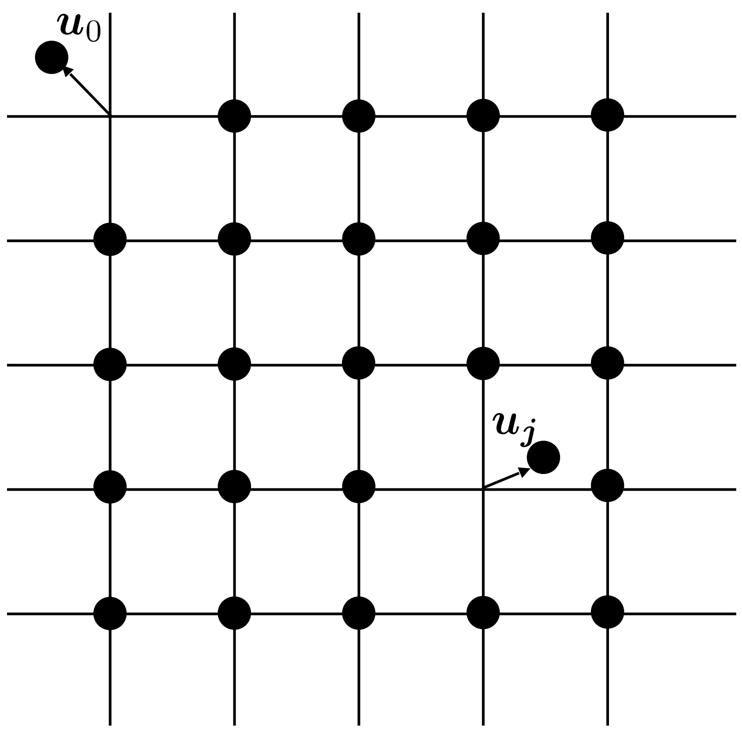
Similar analysis to the 1D case, looking for modes of wave vector $\boldsymbol{q}$

$$\boldsymbol{q} \equiv (q_x, q_y)$$

The detailed analysis is more complicated because of the two possible polarizations of the modes: parallel or perpendicular to $\boldsymbol{q}$

Here we look only for scaling laws

Deviation with respect to equilibrium in 2D



A treatment similar to the 1D case leads to

$$\langle (\boldsymbol{u}_{\boldsymbol{j}} - \boldsymbol{u}_0)^2 \rangle \sim \frac{k_{\mathrm{B}}T}{\kappa}\,\frac{2}{\pi^2}\int_{\mathrm{ZB}} \frac{\sin^2(\boldsymbol{q}\cdot\boldsymbol{R}_{\boldsymbol{j}}/2)}{q^2}\,\mathrm{d}^2q.$$

where the vector $\boldsymbol{q} = (q_x, q_y)$ is in the Brillouin zone

$$-\frac{\pi}{a} < q_{x,y} < +\frac{\pi}{a}$$

Again cut the integral in two pieces:

- Wave vectors such that: $q > \pi/R_j$

$$\sim \frac{1}{\pi^2}\,\frac{k_{\mathrm{B}}T}{\kappa}\int_{\pi/R_{\boldsymbol{j}}}^{\pi/a}\frac{1}{q^2}\,2\pi q\,\mathrm{d}q \quad \sim \frac{2}{\pi}\,\frac{k_{\mathrm{B}}T}{\kappa}\log(R_{\boldsymbol{j}}/a)$$

dominant

- Wave vectors such that: $q < \pi/R_j$

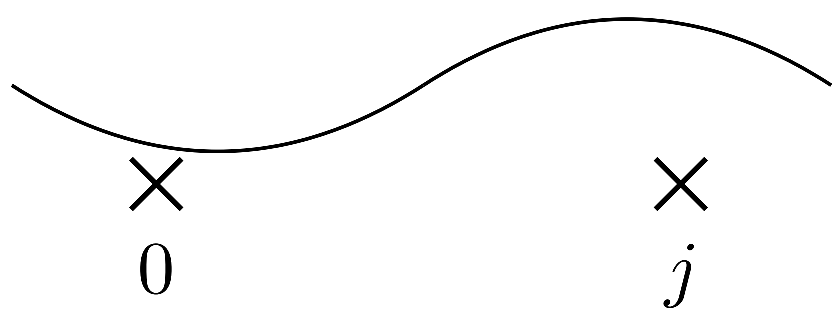
$$\sim \frac{1}{2\pi^2}\,\frac{k_{\mathrm{B}}T}{\kappa}\,R_{\boldsymbol{j}}^2\int_0^{\pi/R_{\boldsymbol{j}}}\pi q\,\mathrm{d}q \quad \sim \frac{\pi}{4}\,\frac{k_{\mathrm{B}}T}{\kappa}$$

No long range order in 2D

R. Peierls

Main result: $\langle (\boldsymbol{u_j} - \boldsymbol{u}_0)^2 \rangle \sim \dfrac{k_\mathrm{B}T}{\kappa} \log(R_{\boldsymbol{j}}/a)$

with a dominant contribution of wave vectors $q \sim \pi/R_j$ as in 1D.



No long-range order

$$\langle (\boldsymbol{u_j} - \boldsymbol{u}_0)^2 \rangle \to \infty \quad \text{quand} \quad R_j \to \infty$$

but only a logarithmic divergence, to be compared with a linear divergence in 1D

"quasi-long range order"

What about 3D?

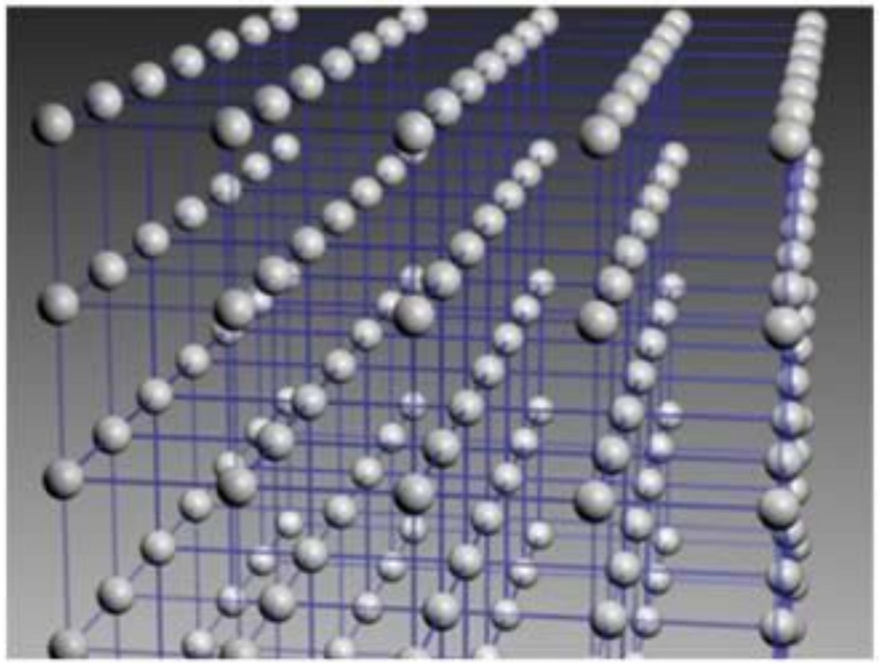


image I. Bloch

Same type of analysis:

$$\langle (\boldsymbol{u_j} - \boldsymbol{u}_0)^2 \rangle \sim \frac{a k_\mathrm{B} T}{\kappa} \int_\mathrm{ZB} \frac{\sin^2(\boldsymbol{q} \cdot \boldsymbol{R_j}/2)}{q^2} \, \mathrm{d}^3 q$$

$\mathrm{d}^3 q = q^2 \, \mathrm{d}q \, \mathrm{d}^2\Omega \longrightarrow$ no divergence anymore in $\boldsymbol{q} = 0$

One then finds $\langle (\boldsymbol{u_j} - \boldsymbol{u}_0)^2 \rangle \sim \dfrac{k_\mathrm{B} T}{\kappa}$: independent of R_j

If $\dfrac{k_\mathrm{B} T}{\kappa} \ll a^2$, cristalline order can exist over an infinite range

Mermin - Wagner - Hohenberg theorem

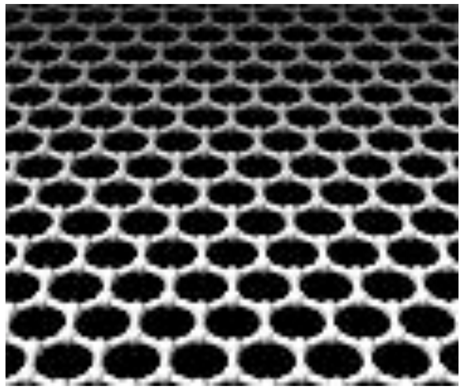
For a system with a dimension lower or equal to 2 and short-range interactions, there is no spontaneous breaking of a continuous symmetry at a non-zero temperature

- If the range is infinite, mean-field theory is valid and standard phase transitions predicted in this case can occur.

- Continuous symmetry: translation, Heisenberg magnetism, Bose-Einstein condensation. The theorem does not apply as such to discrete symmetries (Ising).

- At zero temperature the interacting Bose gas is condensed.

What about graphene?

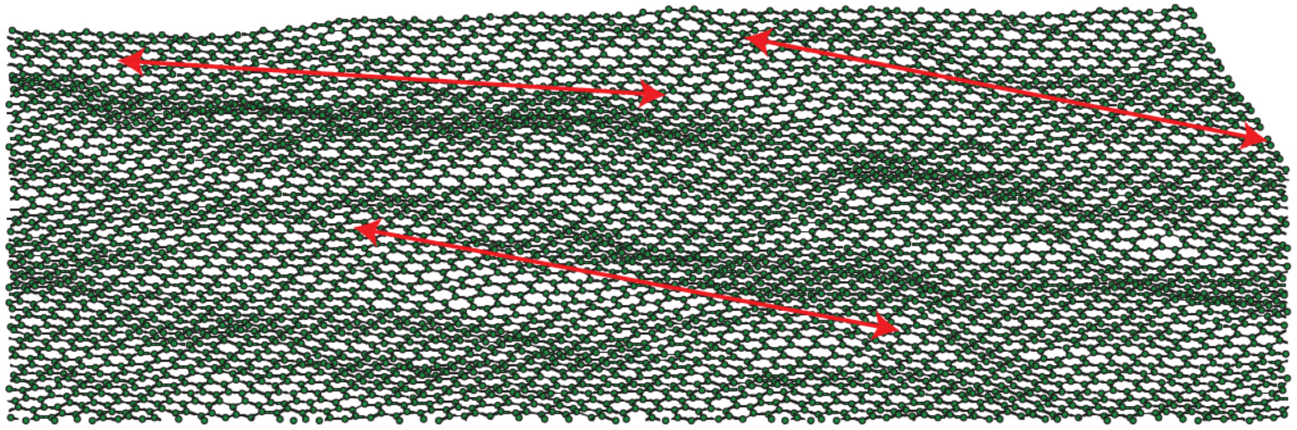


Compatibility with Mermin - Wagner - Hohenberg theorem?

- Because of the large value of the bounds, the loss of crystal order appears only on very long length scales

  A finite size sample may exhibit a crystalline order

- The surface is rippled and the non-linear coupling between the fluctuations of the height and the displacements parallel to the surface induce an effective lon-range component



Fasolino et al, 2007
Nature materials 6.11, p. 858–861.

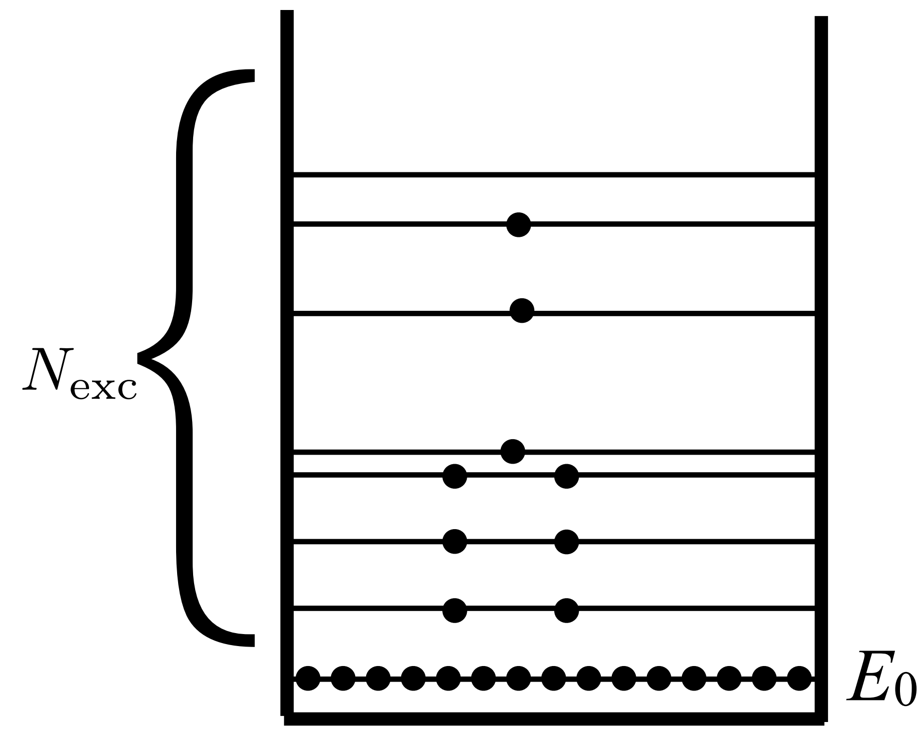
2.

The 2D ideal Bose gas

- Uniform vs. trapped systems
- The case of a 2D gas made with photons

Einstein's saturated ideal gas

Bose particles confined in a box at fixed temperature



The number of particles that can be placed in the excited sates is bounded. Indeed the Bose law

$$N_{\boldsymbol{p}} = \frac{1}{\mathrm{e}^{(E_{\boldsymbol{p}}-\mu)/k_{\mathrm{B}}T} - 1}$$

is meaningful only if $\mu < E_0 = 0$

$$N_{\mathrm{exc}}(T,\mu) = \sum_{\boldsymbol{p}\neq 0} \frac{1}{\mathrm{e}^{(E_{\boldsymbol{p}}-\mu)/k_{\mathrm{B}}T} - 1}$$

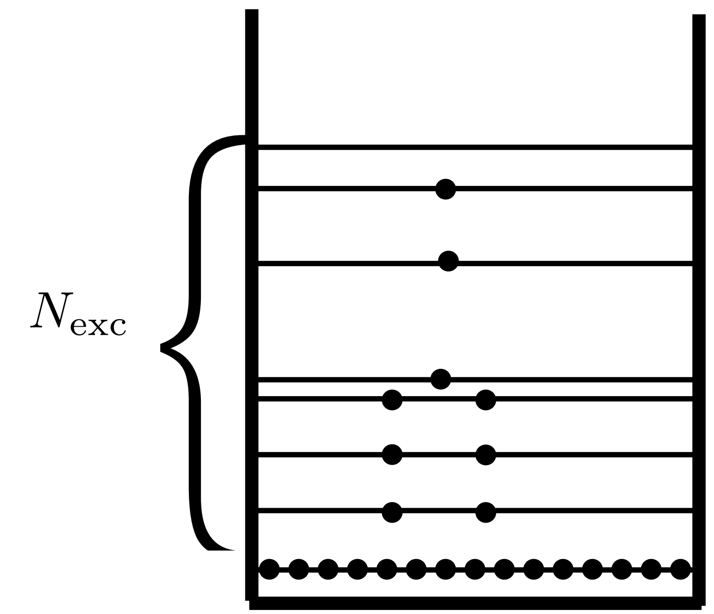
$$< \sum_{\boldsymbol{p}\neq 0} \frac{1}{\mathrm{e}^{E_{\boldsymbol{p}}/k_{\mathrm{B}}T} - 1}$$

obtained for $\mu \to 0$

Continuum limit using the density of states: $N_{\mathrm{exc}}(T,\mu) < \int_0^{+\infty} \frac{D(E)}{\mathrm{e}^{E/k_{\mathrm{B}}T} - 1}\,\mathrm{d}E$

Does this integral converge in $E=0$?

Einstein's saturated ideal gas (2)



$$N_{\mathrm{exc}}(T,\mu) < \int_0^{+\infty} \frac{D(E)}{\mathrm{e}^{E/k_{\mathrm{B}}T}-1}\,\mathrm{d}E$$

The convergence in $E = 0$ depends on $D(E)$

- In 3D: $D(E) \propto \sqrt{E}$ and $\dfrac{1}{\mathrm{e}^{E/k_{\mathrm{B}}T}-1} \approx \dfrac{1}{\left(1+\frac{E}{k_{\mathrm{B}}T}\right)-1} = \dfrac{k_{\mathrm{B}}T}{E}$

$$\longrightarrow \quad k_{\mathrm{B}}T \int_0 \frac{1}{\sqrt{E}}\,\mathrm{d}E$$ converges in E=0 : BEC !

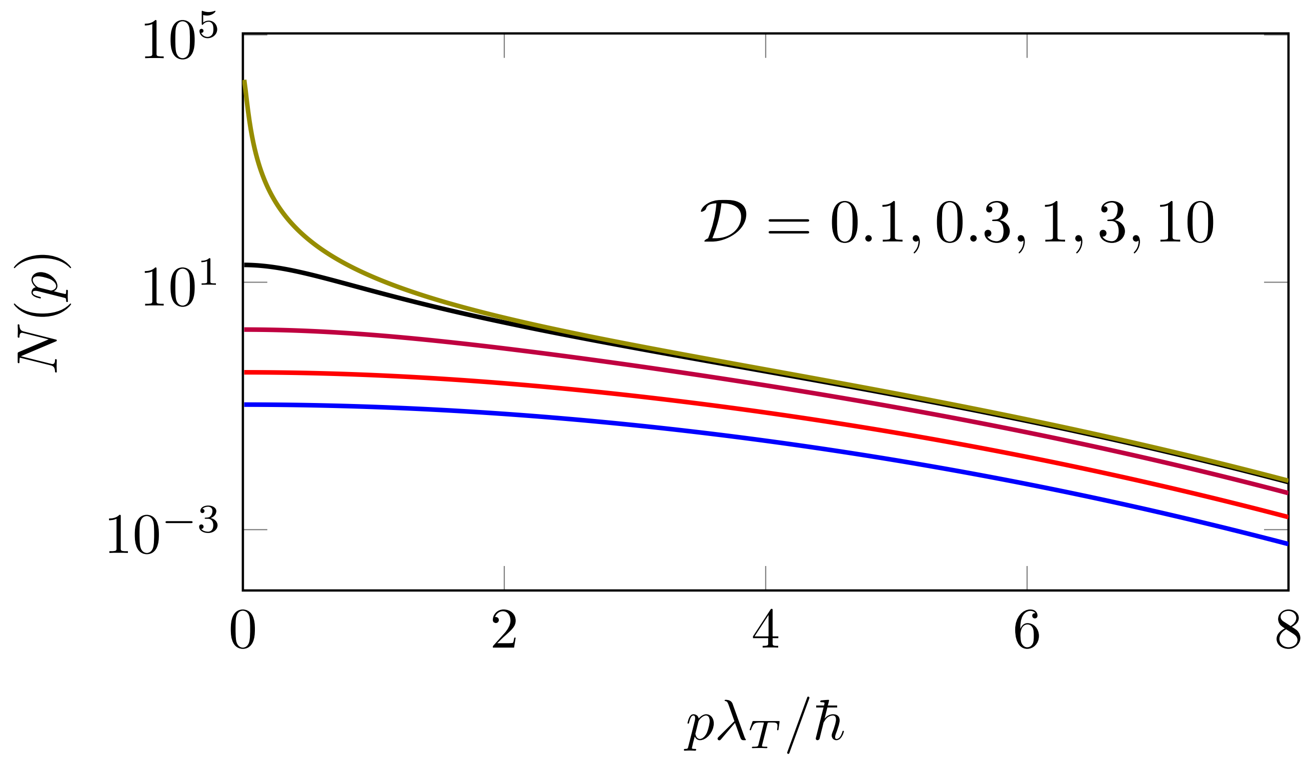
- In 2D: $D(E)$ is constant and the integral $k_{\mathrm{B}}T \int_0 \frac{1}{E}\,\mathrm{d}E$ diverges

For a given T, one can put an arbitrarily large number of particles in the excited states by letting $\mu \to 0$: no BEC

Momentum distribution of the ideal 2D Bose gas

Quantum statistics still play a role, even in the absence of condensation:
Particles accumulate in the region of small momenta

Varying phase space density from << 1 to >>1



$\lambda_T = \dfrac{\hbar\sqrt{2\pi}}{\sqrt{mk_\mathrm{B}T}}$: thermal wavelength

$\mathcal{D} = \rho\lambda_T^2$: phase space density

$$\frac{p\lambda_T}{\hbar} = 1 \;\Leftrightarrow\; \frac{p^2}{2m} = \frac{1}{4\pi}k_\mathrm{B}T$$

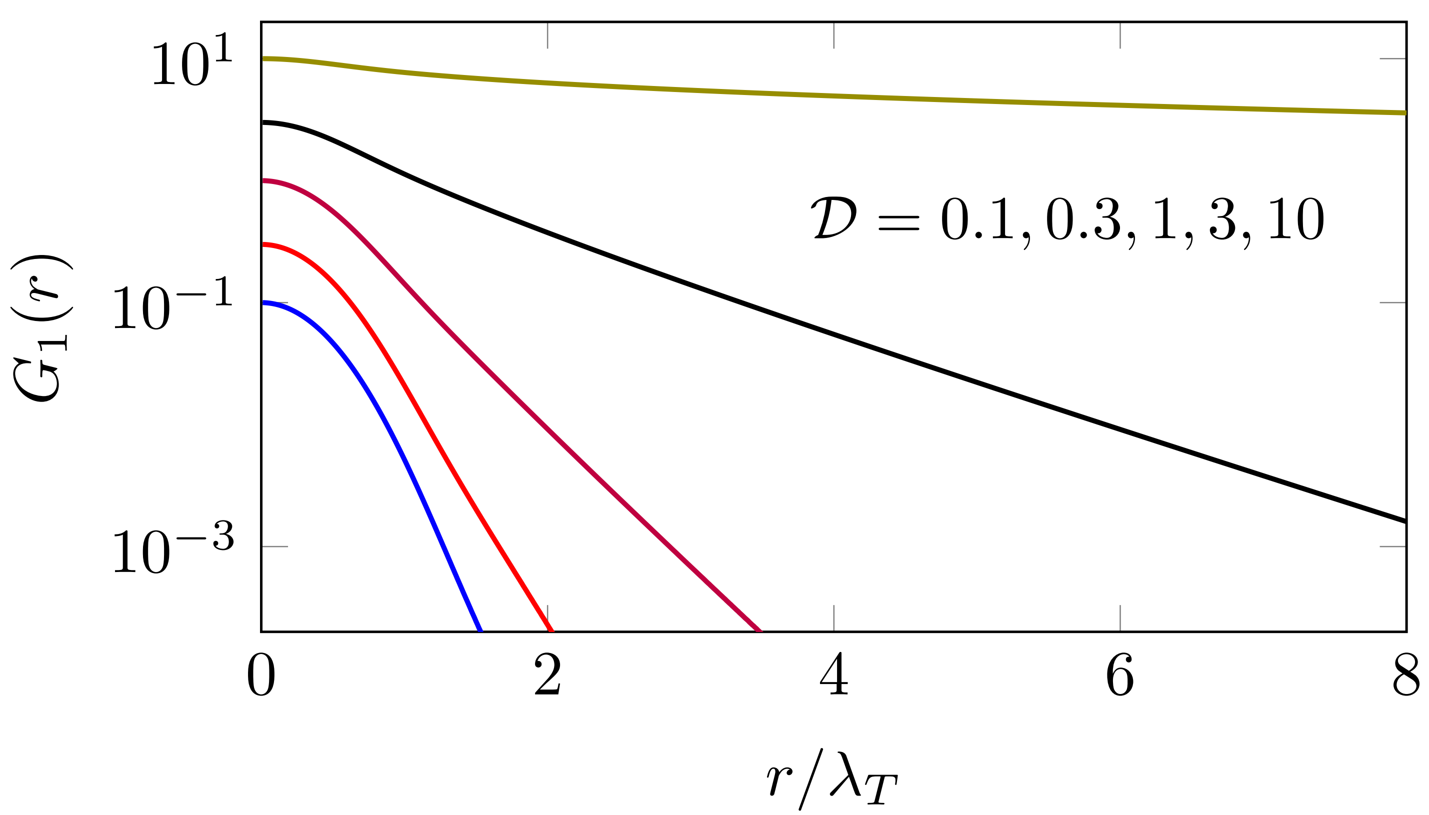
$$N(\boldsymbol{p}) = \frac{1}{\mathrm{e}^{(p^2/2m-\mu)/k_\mathrm{B}T} - 1} \approx \frac{k_\mathrm{B}T}{\frac{p^2}{2m} + |\mu|}$$

Lorentz distribution

Spatial coherence of the 2D Bose gas

Characterized by the one-body correlation function

$G_1(\boldsymbol{r}, \boldsymbol{r}') = \langle \boldsymbol{r} | \hat{\rho}_1 | \boldsymbol{r}' \rangle$ *: Fourier transform of the momentum distribution*



Lorentz momentum distribution

$\longrightarrow$ Exponential G_1

$$G_1(r, 0) \propto \mathrm{e}^{-r/\ell}$$

The coherence length ℓ increases with $\mathcal{D}$

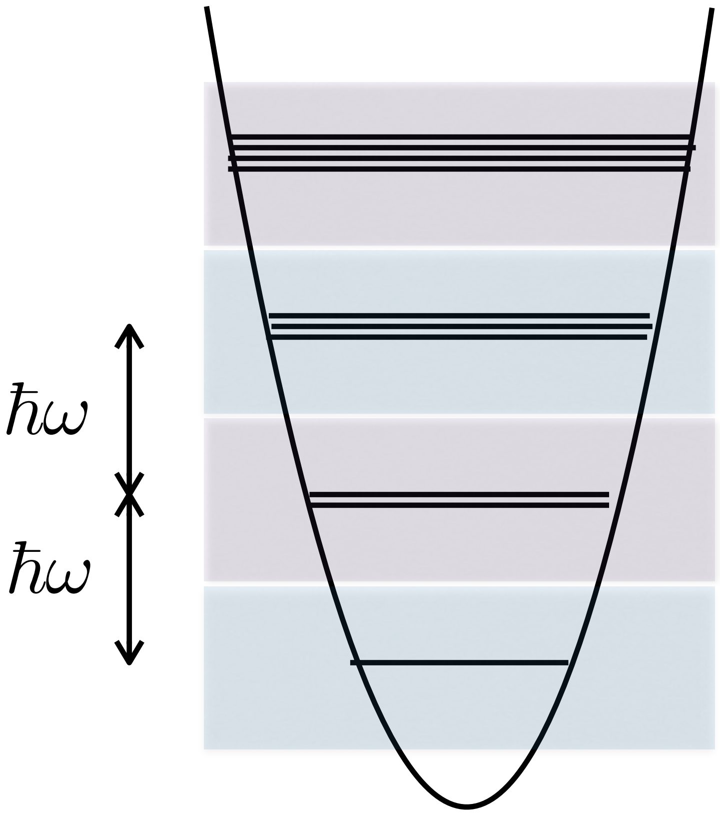
An exponential decay is by essence "fast" (even if ℓ can be large): no emergence of quasi-long range order at this stage…

2.

The 2D ideal Bose gas (continued)

- The case of a trapped gas
- The case of a 2D gas made with photons

Density of states in a 2D harmonic potential



$$E_{\boldsymbol{n}} = (n_x + n_y + 1)\hbar\omega \qquad\qquad D(E) \propto E$$

This ensures the convergence of

$$\int_0^{+\infty} \frac{D(E)}{\mathrm{e}^{E/k_{\mathrm{B}}T} - 1}\,\mathrm{d}E$$

Saturation of excited state populations in a harmonic potential

$$N_{\mathrm{exc}} < 1.64 \left(\frac{k_{\mathrm{B}}T}{\hbar\omega}\right)^2 \qquad\qquad 1.64... = \frac{\pi^2}{6}$$

Condensate like in 3D ?

This condensate is singular

Description within semi-classical approximation $(k_\mathrm{B}T \gg \hbar\omega)$ with

$$N(E) = \frac{1}{\mathrm{e}^{(E-\mu)/k_\mathrm{B}T} - 1} \longrightarrow W(\boldsymbol{r},\boldsymbol{p}) \propto \frac{1}{\mathrm{e}^{\left(\frac{\boldsymbol{p}^2}{2m}+\frac{1}{2}m\omega^2 r^2-\mu\right)/k_\mathrm{B}T} - 1}$$

Spatial density in the limit $\mu \to 0$, i.e., $Z \to 1$

$$\rho(\boldsymbol{r}) = \int W(\boldsymbol{r},\boldsymbol{p})\ \mathrm{d}^2p \quad = -\frac{1}{\lambda_T^2}\ \ln\left(1 - \mathrm{e}^{-m\omega^2 r^2/(2k_\mathrm{B}T)}\right)$$

which gives in the trap center $r \to 0$:

$$\rho(\boldsymbol{r}) \approx -\frac{1}{\lambda_T^2}\ \ln(r^2) + \mathrm{constant}$$

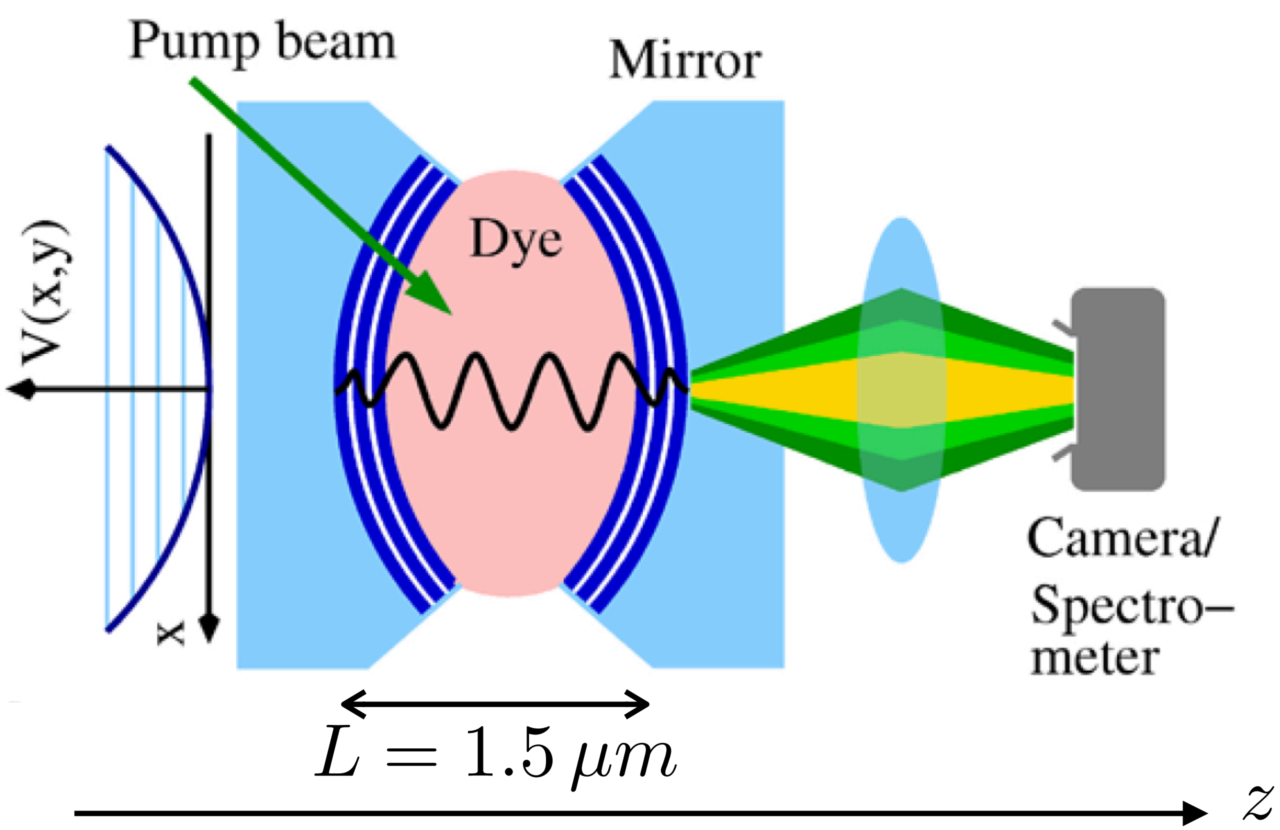
diverges in $r = 0$

Problem for interacting particles…

A nearly ideal 2D gas in a harmonic potential: Photons in a small cavity

How to give a mass to photons?
How to harmonically trap photons in 2D?
How to give non-zero chemical potential to photons?

Bonn experiment (M. Weitz group)



Optical cavity with finesse 10^5

Methanol + dye (Rhodamine 6G)

Optical index $n_0 = 1.33$

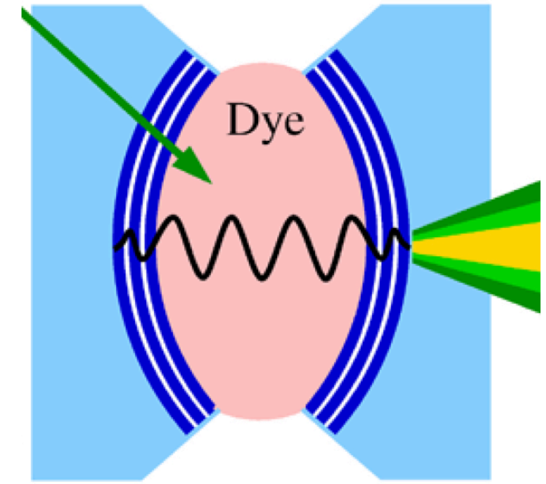
Photon wavelength in the range 500-580 nm

$$L \approx N \frac{\lambda}{2n_0} \quad \text{with} \quad N = 7$$

Longitudinal modes $N = 6$ or $N = 8$ correspond to wavelengths that cannot be reached by fluorescence of the dye molecules

The degree of freedom along z is frozen for the photons in the cavity

A mass for photons?



Along the cavity axis z: $k_z = N\pi/L$ with $N = 7$

Transverse (xy) degrees of freedom: $E = \hbar\omega = \dfrac{\hbar c|\boldsymbol{k}|}{n_0}$ $\quad |\boldsymbol{k}| = \sqrt{k_z^2 + \boldsymbol{k}_\perp^2}$

Paraxial approximation $\quad |\boldsymbol{k}_\perp| \ll k_z$

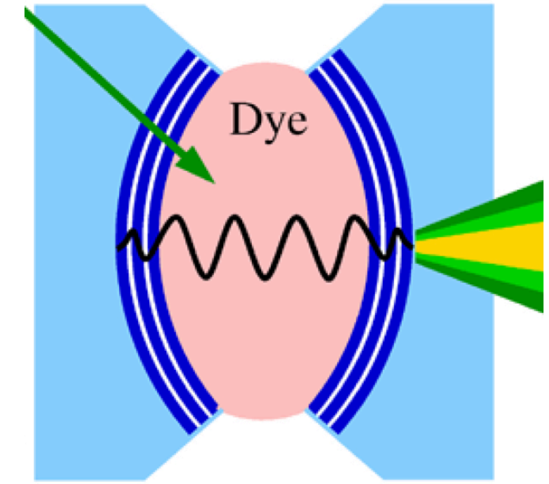
$$E \approx \frac{\hbar c}{n_0} k_z \left(1 + \frac{\boldsymbol{k}_\perp^2}{2k_z^2}\right) \approx \hbar\omega_0 + \frac{\hbar^2 \boldsymbol{k}_\perp^2}{2m_{\mathrm{ph}}}$$

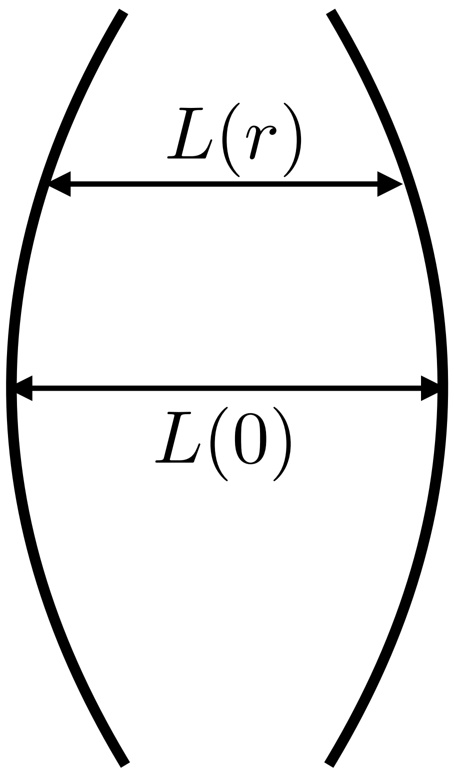
where : $\quad \hbar\omega_0 = \dfrac{N\pi\hbar c}{n_0 L} \qquad m_{\mathrm{ph}} = \dfrac{\hbar n_0 k_z}{c}$

$$m_{\mathrm{ph}} \approx 0.7 \times 10^{-35}\,\mathrm{kg}$$

100 000 fois lighter than an electron

A 2D harmonic trap for photons





$$L(r) = L_0 - 2\left(R - \sqrt{R^2 - r^2}\right)$$
$$\approx L_0 - \frac{r^2}{R}$$

to be injected into : $\hbar\omega_0 = \dfrac{N\pi\hbar c}{n_0 L}$

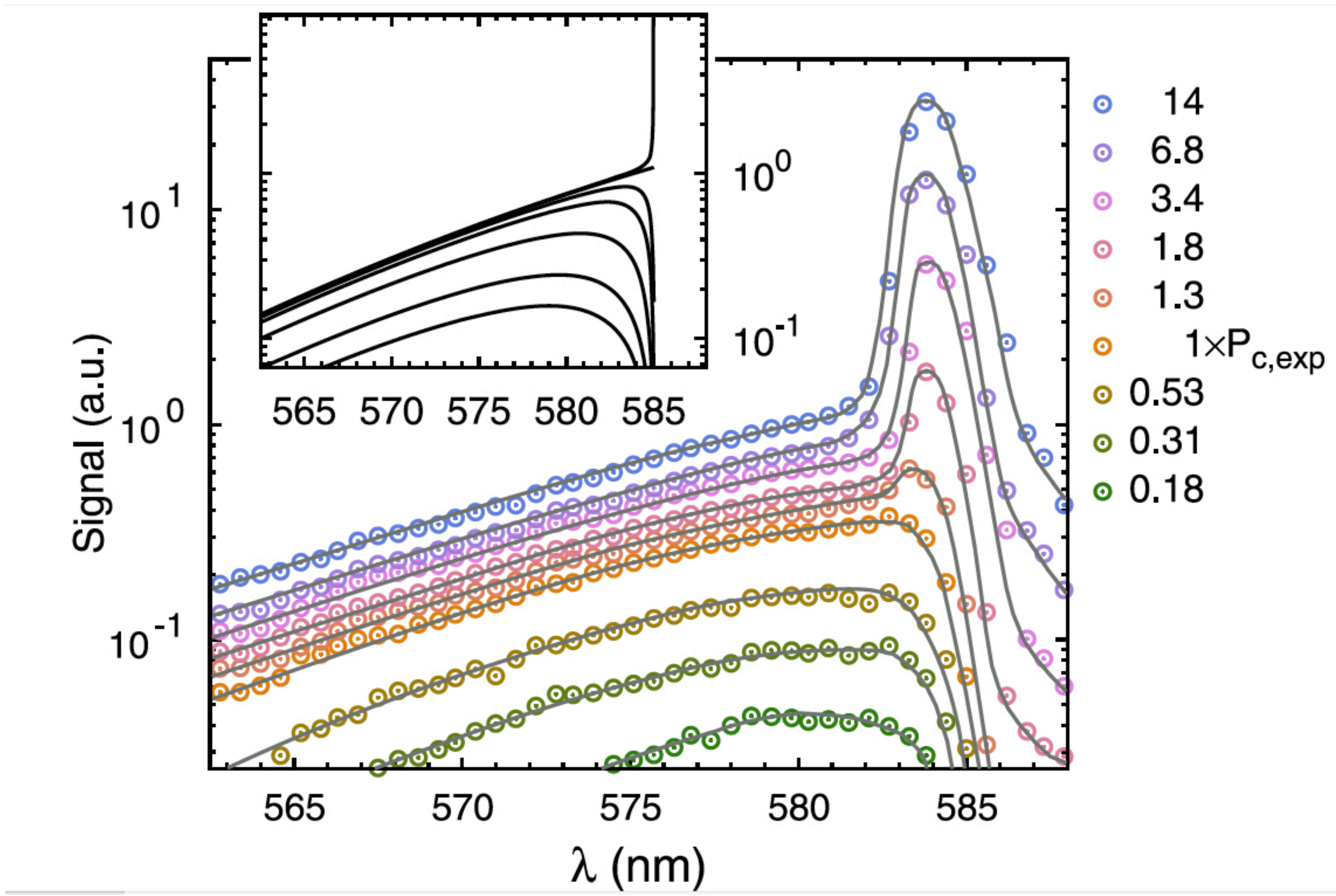
hence the energy of a photon at a distance *r* from optical axis

$$\hbar\omega_0(r) = \hbar\omega_0(0) + \frac{1}{2}m_{\mathrm{ph}}\Omega^2 r^2$$

with : $\Omega = \dfrac{c}{n_0\sqrt{L_0 R/2}}$ $\qquad \Omega/(2\pi) \approx 40\,\mathrm{GHz}$

Summary of this particle-like analysis: $E(\boldsymbol{r}, \boldsymbol{k}_\perp) = \dfrac{\hbar^2 \boldsymbol{k}_\perp^2}{2m_{\mathrm{ph}}} + \dfrac{1}{2}m_{\mathrm{ph}}\Omega^2\boldsymbol{r}^2$

2D photon condensation observed in Bonn



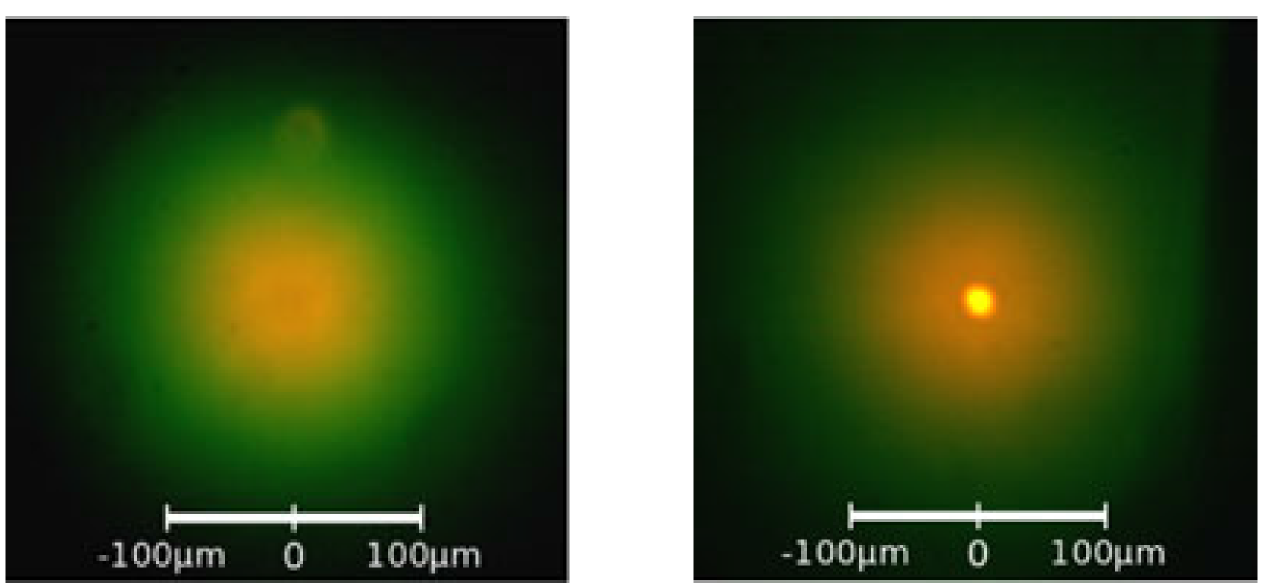
Spectrum of the light for increasing pump power

Fit with a Bose-Einstein law T=300 K and an adjustable chemical potential

Condensation expected for

$$N = \frac{\pi^2}{3} \left(\frac{k_{\mathrm{B}}T}{\hbar\Omega}\right)^2$$

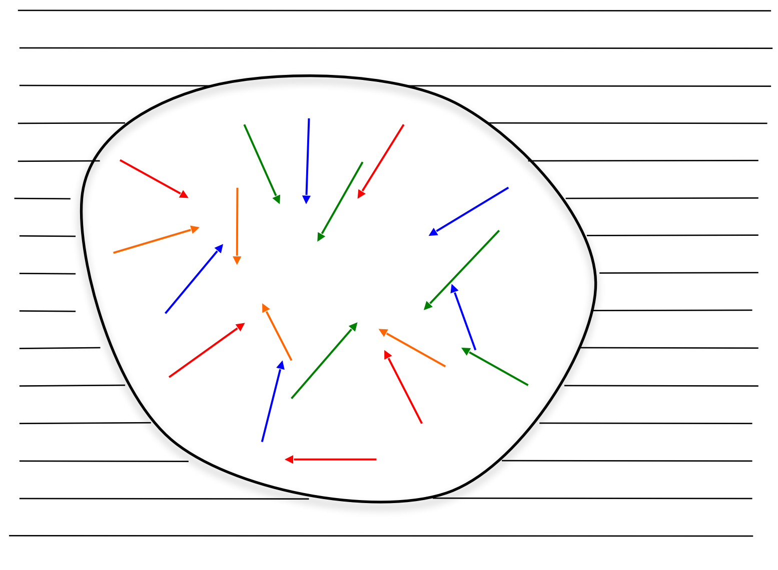
$$\approx 60\,000$$



Klaers et al, Nature **468** 545 (2010)

Good agreement with experiments

Difference with black-body radiation



To derive Planck's law one imposes μ = 0 for photons

1 photon $\rightleftharpoons$ One excitation of the walls $\rightleftharpoons$ 2 photons

Non conservation of total excitation number:

$$\mu = 2\mu \;\Rightarrow\; \mu = 0$$

Here we have the conservation law for the total excitation number (RWA):

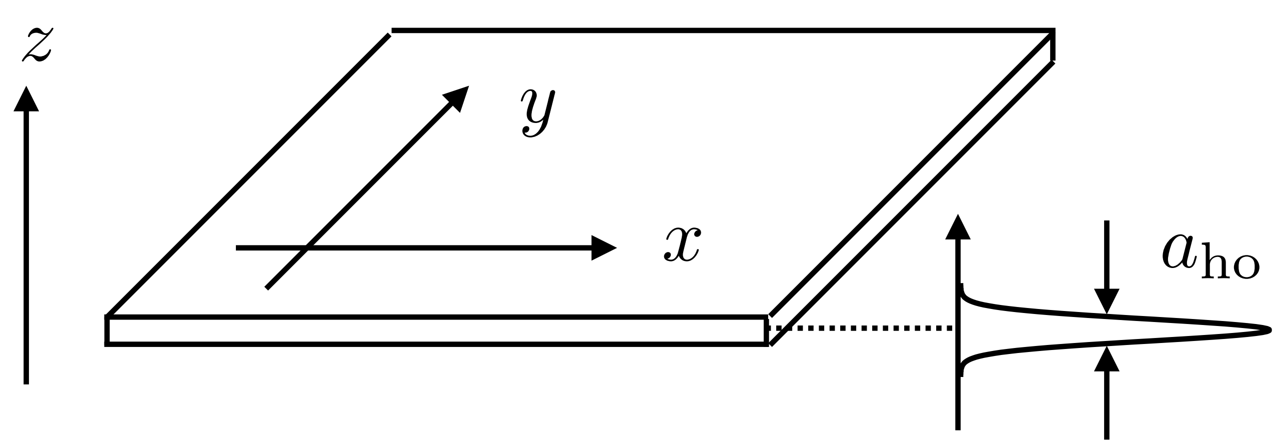
1 photon + 1 molecule g $\rightleftharpoons$ 1 molecule e

Chemical potential: Lagrange parameter associated to this conservation

3.

The Gross-Pitaevskii approach for the interacting 2D gas

- Phase and density fluctuations
- Quasi-long range order
- Bogoliubov spectrum and sound propagation



Contact interactions described by the 3D scattering length a

The 2D Gross-Pitaevskii energy

Description of the state of the gas by the classical field $\psi(x,y)$: $\quad E = E_{\mathrm{kin}} + E_{\mathrm{int}}$

$$E_{\mathrm{kin}} = \frac{\hbar^2}{2m} \int |\boldsymbol{\nabla}\psi|^2 \; \mathrm{d}^2 r$$

$$E_{\mathrm{int}} = \frac{\hbar^2}{2m} \, \tilde{g} \int |\psi(\boldsymbol{r})|^4 \; \mathrm{d}^2 r \qquad\qquad \tilde{g} = \sqrt{8\pi}\,\frac{a}{a_{\mathrm{ho}}}$$

Dimensionless parameter describing the strength of contact interactions

Ground state in a box L x L: $\quad \psi(\boldsymbol{r}) = \sqrt{\rho_0} \qquad \rho_0 = \frac{N}{L^2}$

At non-zero temperature, phase and density fluctuations: $\quad \psi(\boldsymbol{r}) = \sqrt{\rho(\boldsymbol{r})}\; \mathrm{e}^{\mathrm{i}\theta(\boldsymbol{r})}$

Simplest approach at $T \neq 0$: Only phase fluctuations

Freezing of density fluctuations due to repulsive atomic interactions $\psi(\boldsymbol{r}) \approx \sqrt{\rho_0}\, \mathrm{e}^{\mathrm{i}\theta(\boldsymbol{r})}$

⟶ Kinetic energy: $E_{\mathrm{kin}} \approx \frac{\hbar^2}{2m}\, \rho_0 \int (\boldsymbol{\nabla}\theta)^2 \ \mathrm{d}^2 r$

⟶ Interaction energy: $E_{\mathrm{int}} \approx \frac{\hbar^2}{2m}\, \tilde{g} \int \rho_0^2 \ \mathrm{d}^2 r \ = \frac{\hbar^2}{2m}\, \tilde{g}\, L^2\ \rho_0^2$ Frozen!

Valid for large phase space densities $\mathcal{D} \gg 1$ $\mathcal{D} = \rho_0 \lambda_T^2$

Fourier expansion of the phase $\theta(\boldsymbol{r}) = \sum_{\boldsymbol{q}} c_{\boldsymbol{q}}\, e^{i\boldsymbol{q}\cdot\boldsymbol{r}}$

Valid only for soft variations of the phase (no vortex!)

At thermal equilibrium: $\langle |c_{\boldsymbol{q}}|^2 \rangle \propto k_{\mathrm{B}} T$

⟶ After some algebra: $\langle [\theta(\boldsymbol{r}) - \theta(0)]^2 \rangle \approx \frac{2}{\mathcal{D}} \ln(r/\lambda_T)$ cf. Peierls!

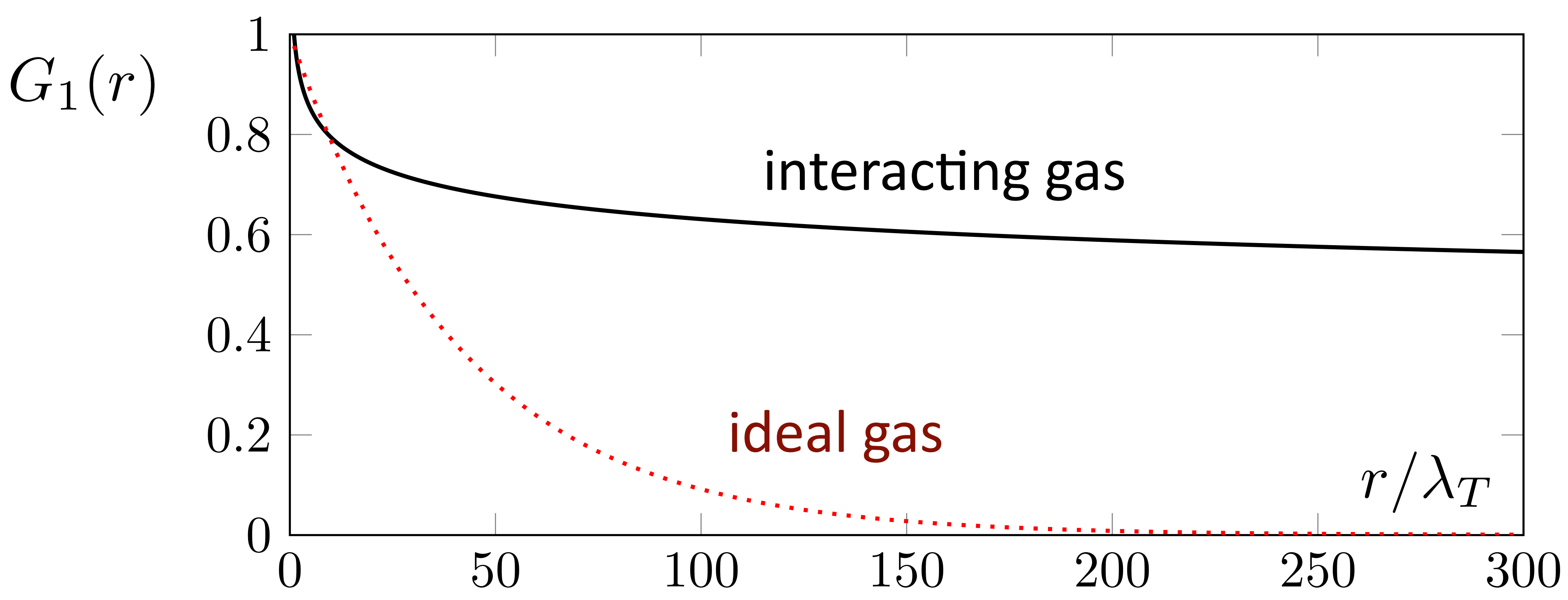
Phase coherence in an interacting 2D Bose gas

One-body correlation function $G_1(r) = \langle \psi(\boldsymbol{r})\ \psi^*(0) \rangle \ \approx\ \rho_0\ \langle \mathrm{e}^{\mathrm{i}[\theta(\boldsymbol{r})-\theta(0)]} \rangle$

- Interacting gas: $G_1(r) \propto 1/r^\alpha$ with $\alpha = 1/\mathcal{D}$

quasi-long range order

- Ideal gas: exponential decrease $G_1(r) \propto \mathrm{e}^{-r/\ell}$



$\mathcal{D} = \rho\lambda_T^2 = 10$

One step beyond: Bogoliubov approach

$$\psi(\boldsymbol{r},t) = \sqrt{\rho_0 \, + \, \delta\rho(\boldsymbol{r},t)} \, \mathrm{e}^{\mathrm{i}\theta(\boldsymbol{r},t)} \qquad\qquad |\delta\rho| \ll \rho_0$$

Again a Fourier expansion (i.e., still no vortices):

$$\rho(\boldsymbol{r},t) = \rho_0 \left[1 \, + \, 2\eta(\boldsymbol{r},t)\right] \qquad\qquad \eta(\boldsymbol{r},t) = \sum_{\boldsymbol{q}} d_{\boldsymbol{q}}(t) \, e^{i\boldsymbol{q}\cdot\boldsymbol{r}}$$

$$\theta(\boldsymbol{r},t) = \sum_{\boldsymbol{q}} c_{\boldsymbol{q}}(t) \, e^{i\boldsymbol{q}\cdot\boldsymbol{r}}$$

Coupled phase-density equations of motion obtained from Gross-Pitaevskii equation

$$\left(\frac{2m}{\hbar}\right) \, \dot{c}_{\boldsymbol{q}} = - \left(q^2 + 4\,\tilde{g}\rho_0\right) \, d_{\boldsymbol{q}} \qquad\qquad \ddot{c}_{\boldsymbol{q}} + \omega_q^2 c_{\boldsymbol{q}} = 0$$

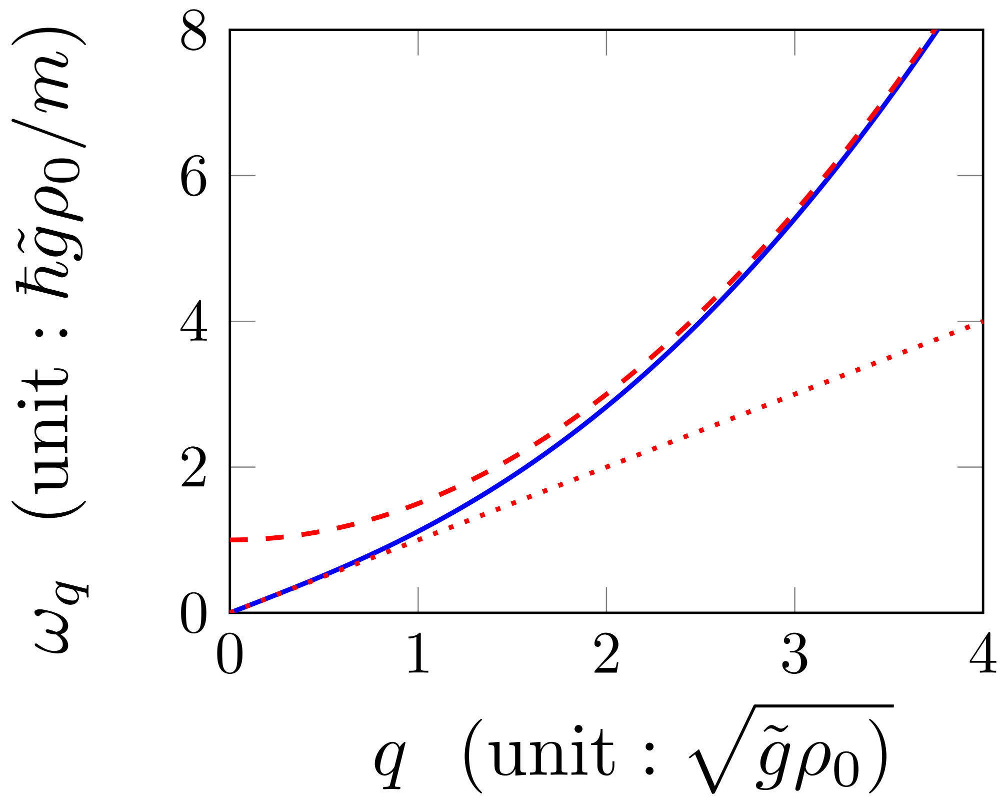
$$\left(\frac{2m}{\hbar}\right) \, \dot{d}_{\boldsymbol{q}} = q^2 \, c_{\boldsymbol{q}} \qquad\qquad \ddot{d}_{\boldsymbol{q}} + \omega_q^2 d_{\boldsymbol{q}} = 0$$

Plane wave solutions with Bogoliubov's dispersion relation:

$$\omega_q = \frac{\hbar}{2m} \left[q^2 \left(q^2 + 4\,\tilde{g}\rho_0\right)\right]^{1/2}$$

Bogoliubov spectrum

$$\omega_q = \frac{\hbar}{2m}\left[q^2\left(q^2 + 4\,\tilde{g}\rho_0\right)\right]^{1/2}$$



- Small wave vector $q^2 \ll 4\,\tilde{g}\rho_0$

$$\omega_q = c_0 q \qquad\qquad c_0 = \frac{\hbar}{m}\sqrt{\tilde{g}\rho_0}$$

Sound waves

Landau criterion: This linear branch ensures superfluidity since an impurity moving at a velocity smaller than c_0 cannot be slowed down

- Large wave vector $q^2 \gg 4\,\tilde{g}\rho_0$ $\qquad \hbar\omega_q = \dfrac{\hbar^2 q^2}{2m}$ up to a constant

Free particle regime

This approach also leads to $G_1(r) \propto 1/r^\alpha$

A 2D uniform gas in the lab

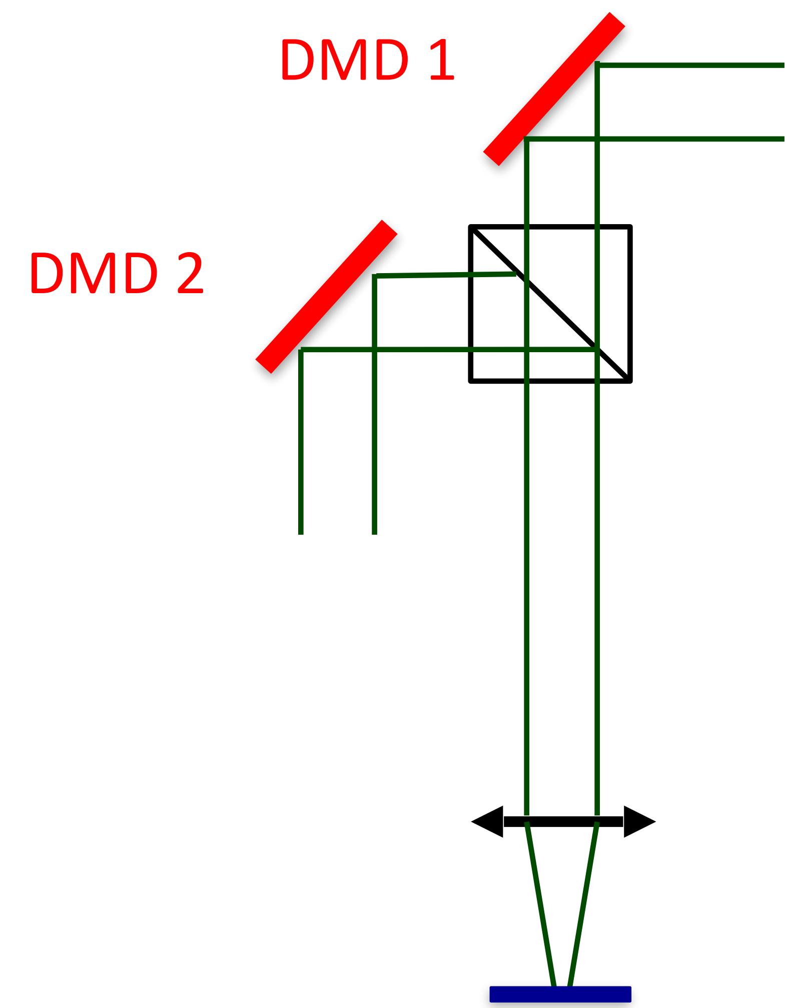
Freezing the vertical direction
by trapping the atoms at
a single node of standing light wave

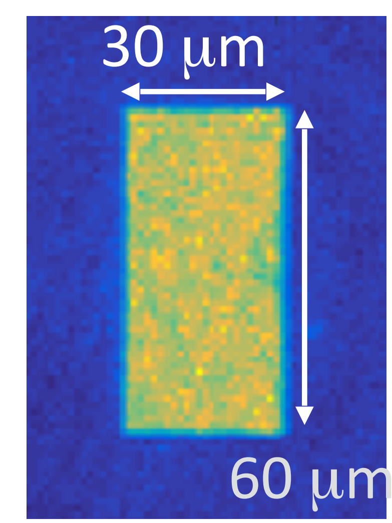
$$k_{\mathrm{B}}T < \hbar\omega_z$$

10 nK

200 nK

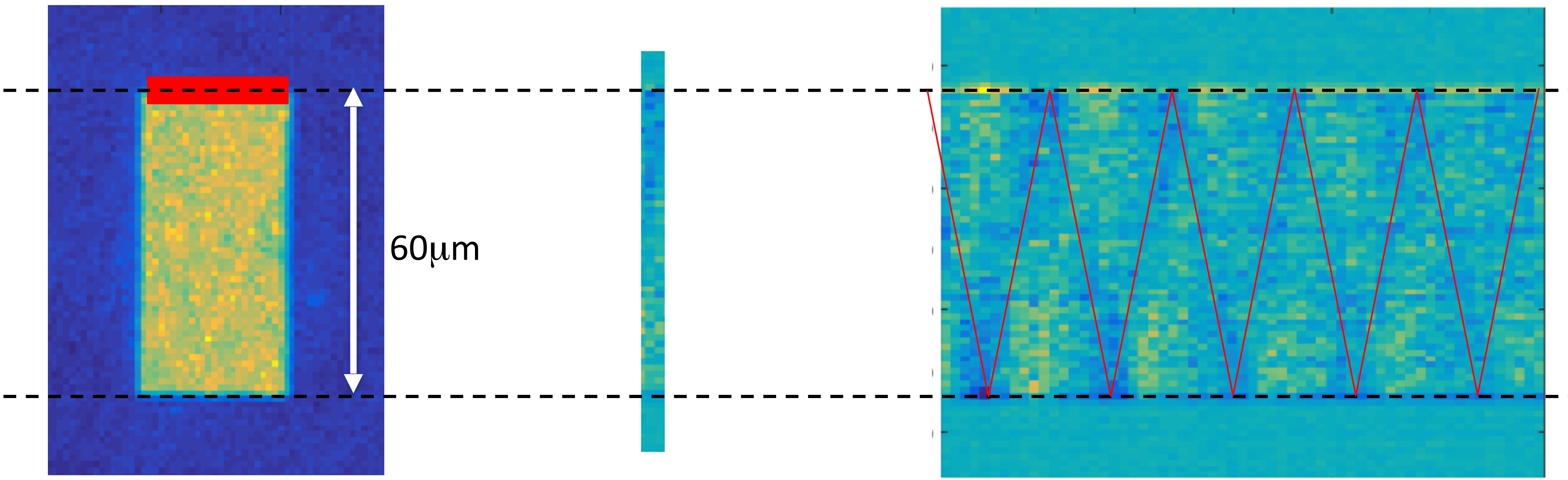
Confinement in the horizontal plane
using a « laser box » imprinted
using digital miroir devices





J.-L. Ville et al., Phys. Rev. Lett. 121, 145301 (2018)

Propagation of sound waves in a 2D gas



Modulation of the atomic density for a short period (20 ms) on one edge

Density modulation right after the excitation

0 100 200 300

Propagation time (ms)

Multiple bounces of the wave packet
Sound velocity 2mm/s

J.-L. Ville et al., Phys. Rev. Lett. 121, 145301 (2018)

Summary for this first part

Using a modelling of the 2D gas in terms of a classical field, we have characterized the dynamics and the thermal equilibrium of the system

- Valid if the phase $\theta(\boldsymbol{r})$ can be Fourier-expanded

Vortices were excluded up to this point

- Leads to quasi-long range order in the presence of interactions

$$G_1(r) = \langle \psi(\boldsymbol{r})\ \psi^*(0) \rangle \ \propto \ 1/r^\alpha \qquad\qquad \alpha = 1/\mathcal{D}$$

Dramatically different from the ideal gas case (exponential decay)

Open questions

What happens in the presence of stronger phase fluctuations?

How can one connect this result to the high temperature case, where one expects to recover an exponential decay of G_1?

Topological matter and
its exploration with quantum gases

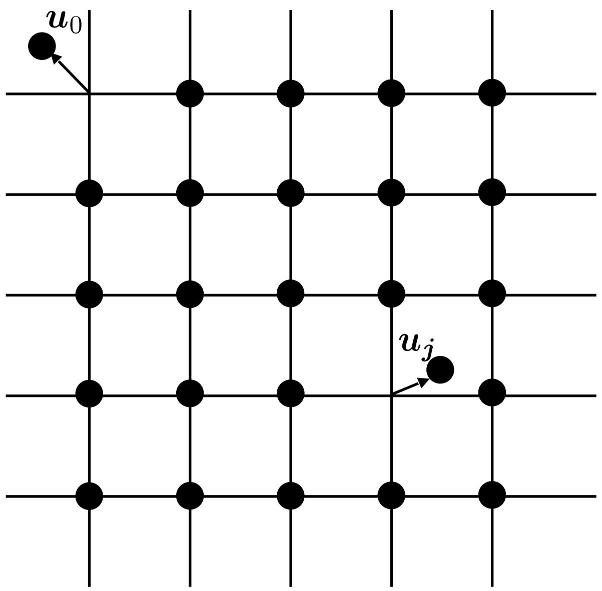
The Kosterlitz-Thouless transition explored with atomic and photonic fluids

Jean Dalibard

Lectures at EPFL
November 2019

COLLÈGE
DE FRANCE
1530

Summary of the first part



What happens to a macroscopic quantum fluid in 2D?
Is there a BEC?

Peierls (1935), Mermin, Wagner, Hohenberg (1966):
No long range order in a uniform system!

We confirmed this result by calculating the one-body correlation function

$$G_1(\boldsymbol{r}, \boldsymbol{r}') = \langle \boldsymbol{r} | \hat{\rho}_1 | \boldsymbol{r}' \rangle$$

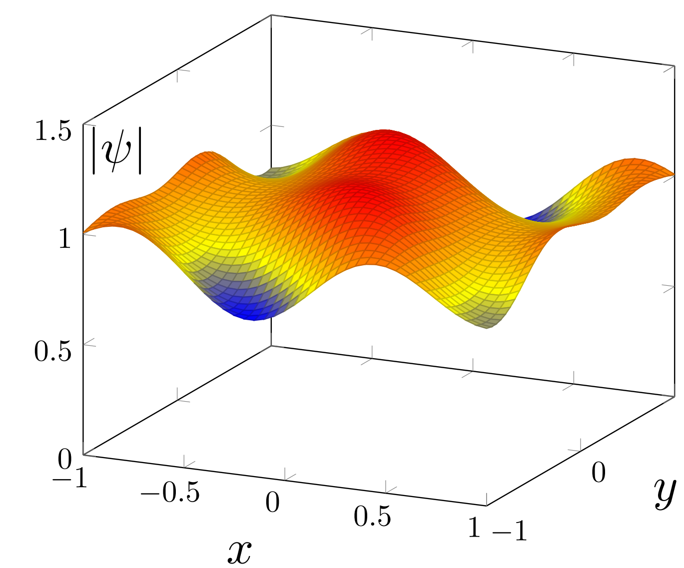
Contrarily to the 3D case, this function in 2D must tend to 0 when $|\boldsymbol{r}\text{-}\boldsymbol{r}'|$ tends to infinity

However, we obtained very different behaviors (both compatible with Mermin-Wagner) for the ideal gas case and for the interacting gas case

Summary of the previous lecture (2)

- Ideal gas: exponential decrease $G_1(r) \propto \mathrm{e}^{-r/\ell}$

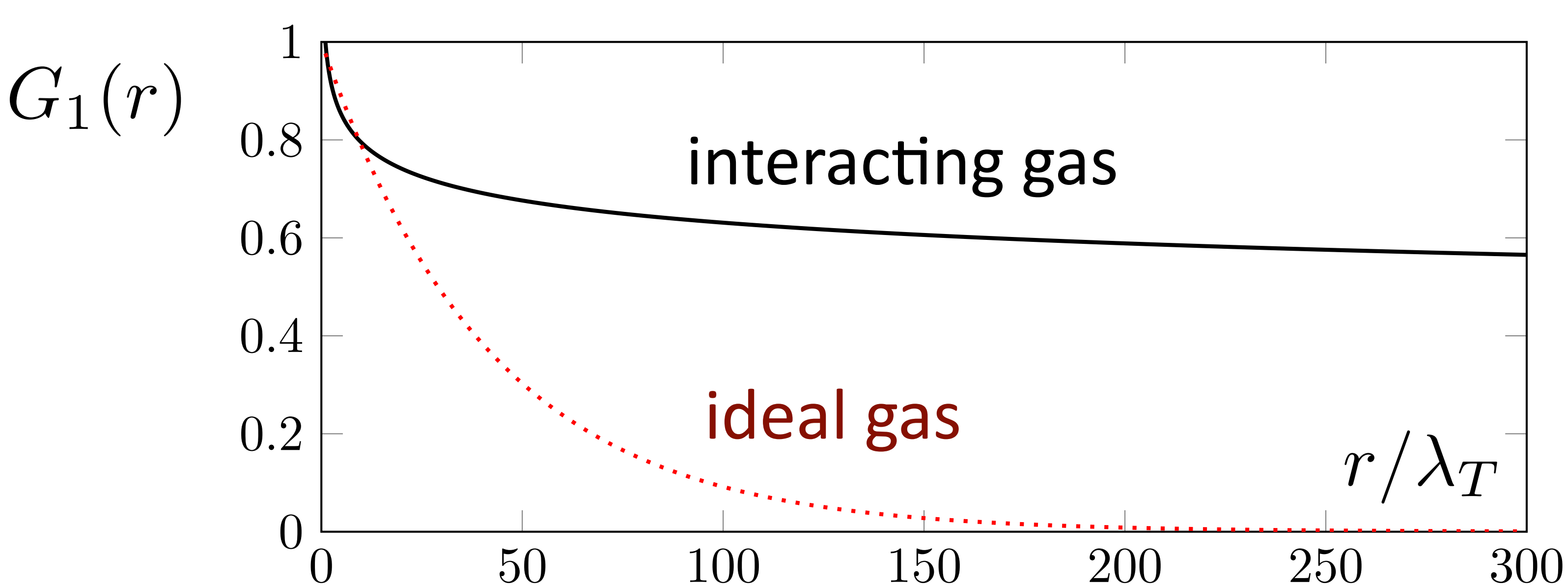
- Interacting gas, assuming reduced density fluctuation and smooth phase fluctuations of the classical field $\psi(\boldsymbol{r}, t)$ describing the state of the gas (phonons):



$$G_1(r) \propto 1/r^\alpha \quad \text{with} \quad \alpha = 1/\mathcal{D}$$

$\mathcal{D} = \rho\lambda_T^2$: phase space density

quasi-long range order



$$\mathcal{D} = \rho\lambda_T^2 = 10$$

Goal of this lecture

The freezing of density fluctuations leading to quasi-long range order is only effective at low temperature

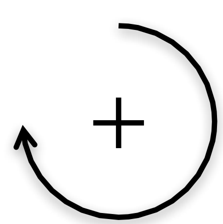
At a larger temperature, interactions should become negligible and one should recover the ideal gas case

How does the transition occur?
Cross-over or unconventional phase transition?

Berezinskii - Kosterlitz - Thouless (1971-74)

The elementary bricks of this transition are quantized vortices

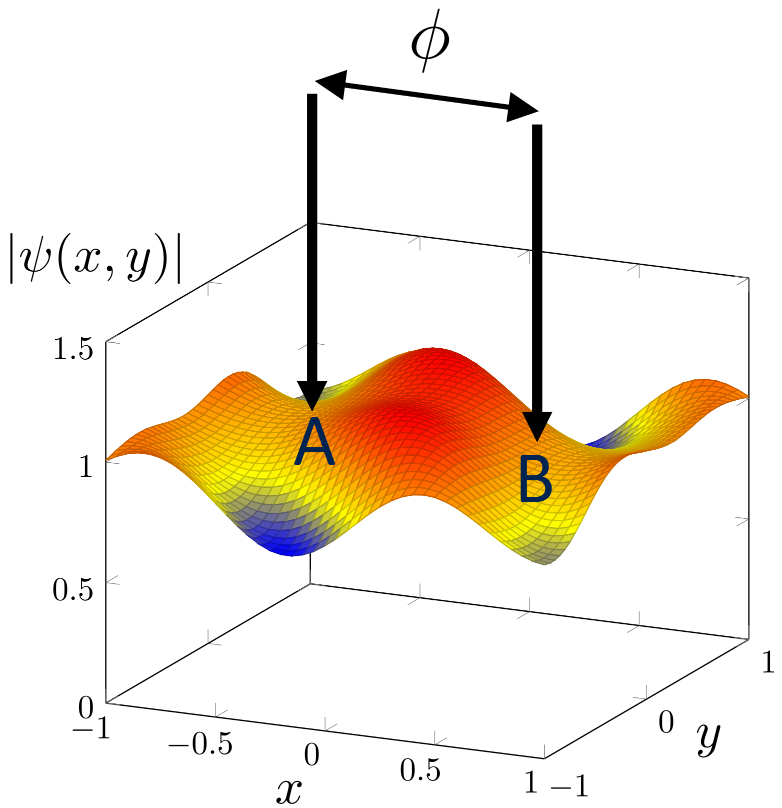
$\psi(x, y)$



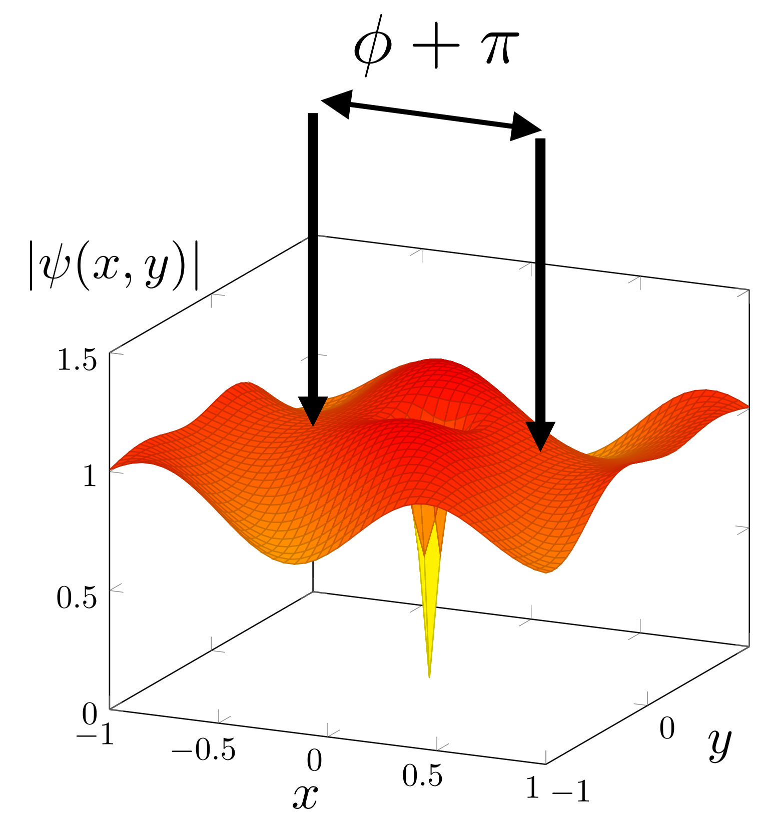
Zero of the spatial density with a phase winding of $n\ 2\pi$ where n is a positive or negative integer

In practice : $\pm\ 2\pi$

Why vortices destroy the quasi-long range order?



Consider two points A and B between which there exists a significant phase coherence if one restricts to phonon excitations



If an isolated vortex has a significant probability to appear in the vicinity of the AB segment, the relative phase will strongly fluctuate:

$$\phi \quad \to \quad \phi+\pi$$

If isolated vortices have a spatial density $\rho_{\rm v}$, one can expect that any phase ordering will be lost over a distance $\sim \rho_{\rm v}^{-1/2}$

Outline of the lecture

1. The BKT transition

Can a single vortex exist in an infinite 2D Bose fluid?

2. Experimental consequences in AMO physics

The critical point

Superfluidity of 2D fluids

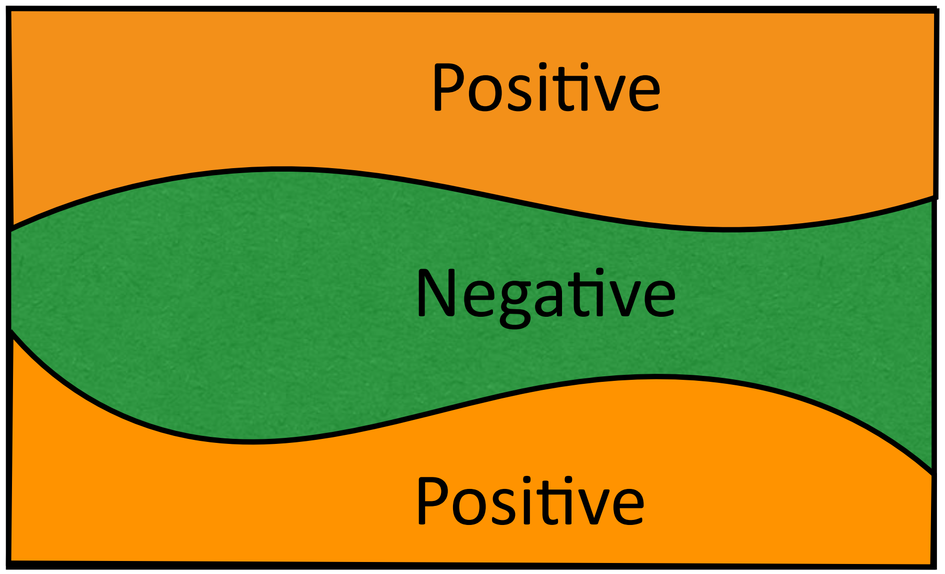
Quasi-long range order

3. Back to crystal order: Experiments with magnetic beads

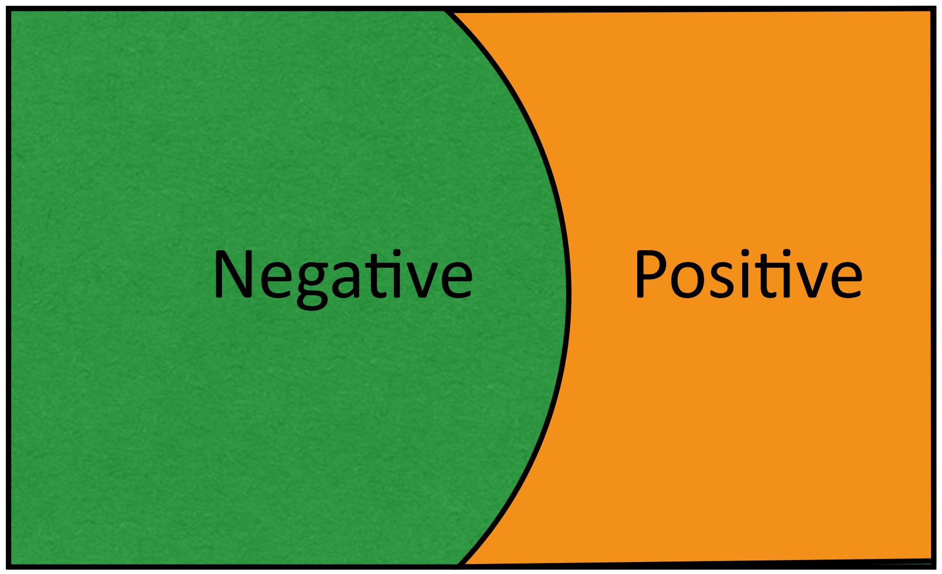
The zeros of the classical field $\psi(x,y)$

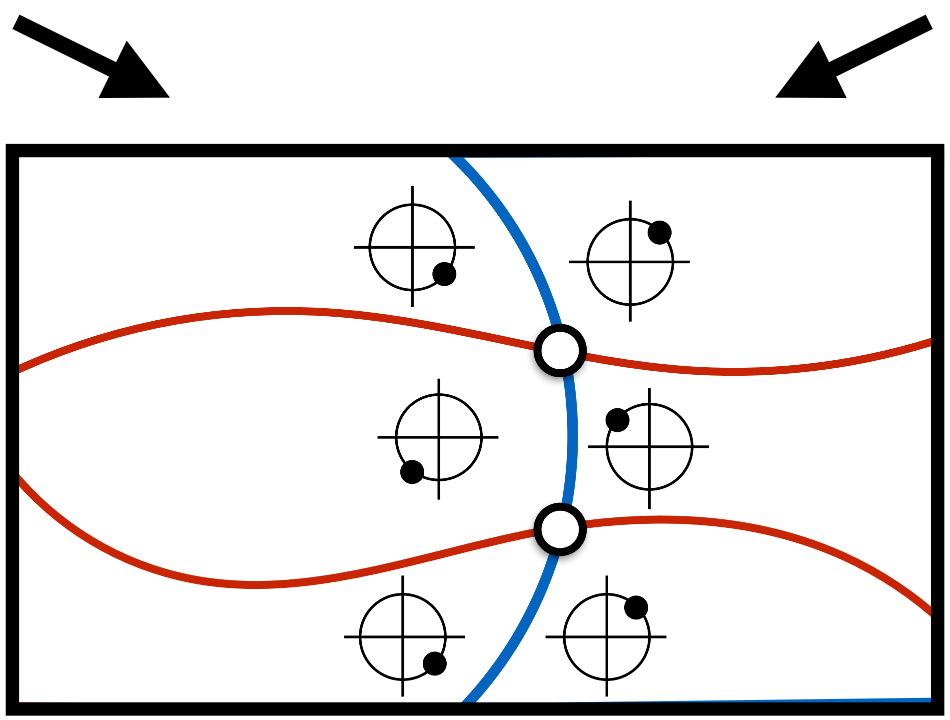
Consider at time t a fluctuating classical field and look at its real and imaginary parts

$\mathrm{Re}\left[\psi(x,y)\right]$



$\mathrm{Im}\left[\psi(x,y)\right]$

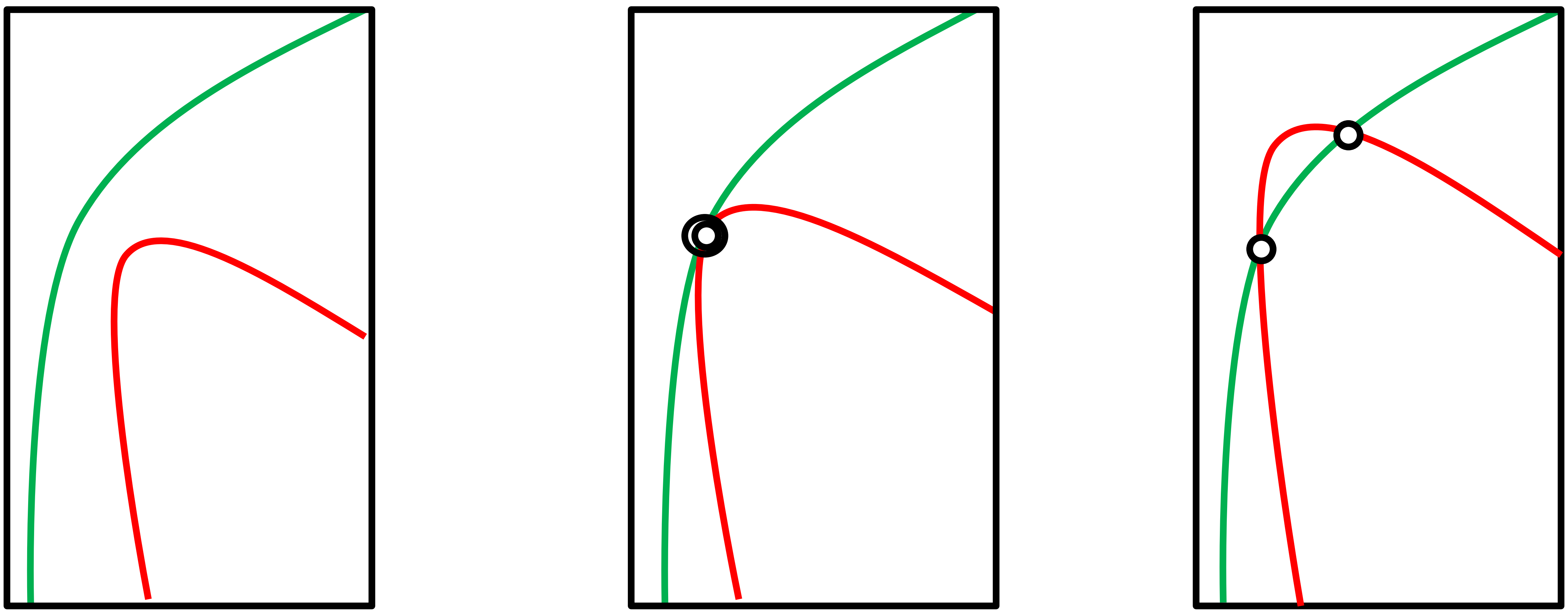




Two vortices of opposite circulations

Apparition and disparition of vortices

$\mathrm{Re}\left[\psi(x,y)\right] = 0 \qquad \mathrm{Im}\left[\psi(x,y)\right] = 0$



No vortex

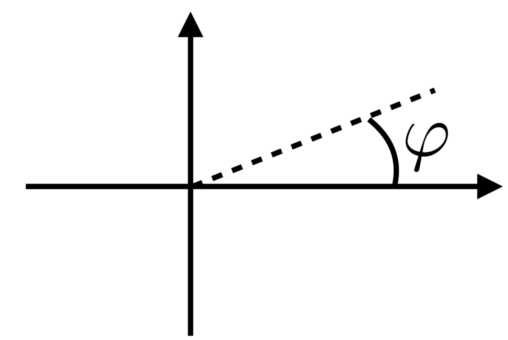
A double zero:
density hole with
no phase winding

Two simple zeros
with opposite circulations:
a vortex pair

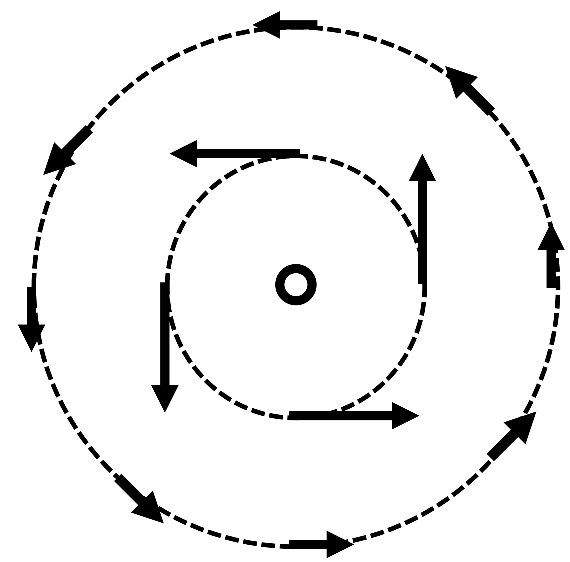
A vortex pair can appear via a significant density fluctuation

Velocity field of a vortex

Example of a vortex in $\boldsymbol{r} = 0$: $\psi(\boldsymbol{r}) = \sqrt{\rho(r)}\, \mathrm{e}^{\mathrm{i}\varphi} \qquad \theta(\boldsymbol{r}) = \varphi$



$$\boldsymbol{v}(\boldsymbol{r}) = \frac{\hbar}{m}\boldsymbol{\nabla}\theta \;=\; \frac{\hbar}{mr}\boldsymbol{u}_\varphi$$



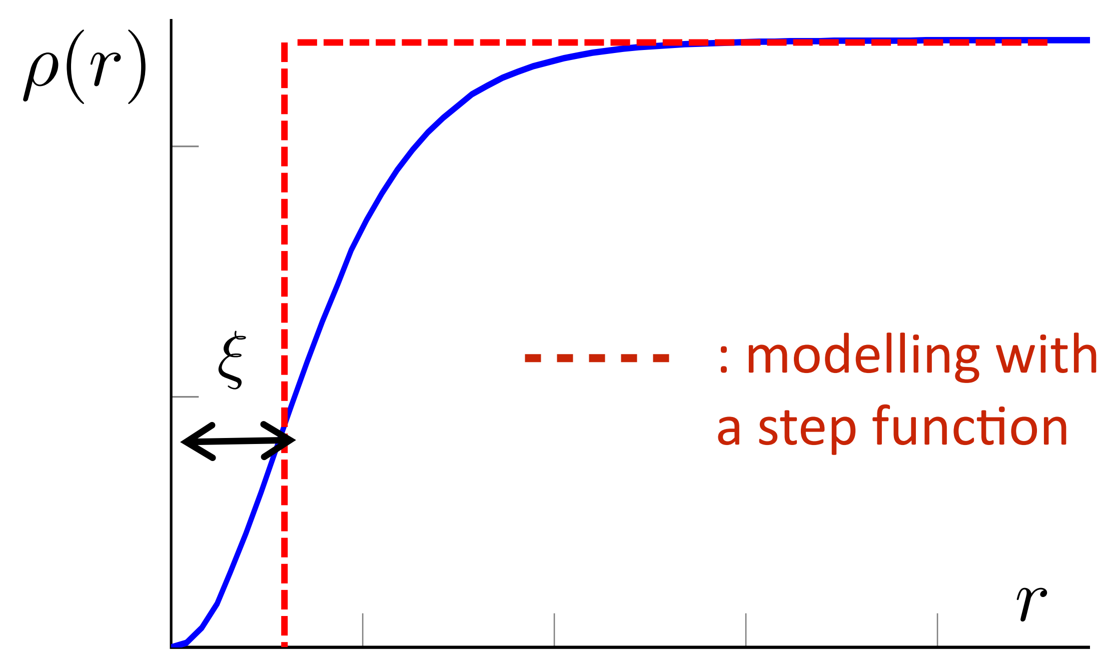
$$\oint \boldsymbol{v}(\boldsymbol{r}) \cdot \mathrm{d}\boldsymbol{r} = 2\pi\frac{\hbar}{m}$$

Density profile close to the vortex location

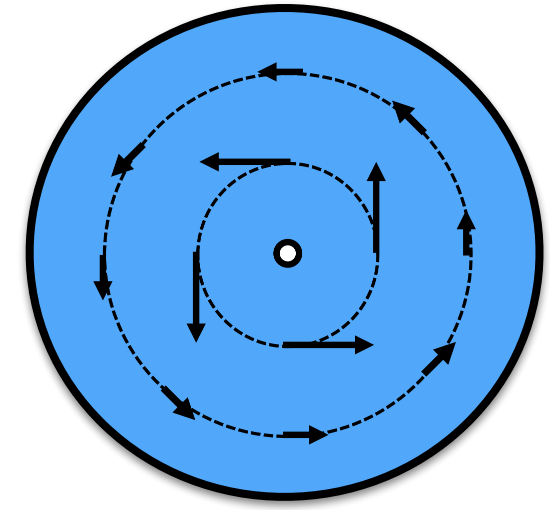
$$\xi = \frac{1}{\sqrt{2\tilde{g}\rho}}$$ *Healing length*

$\tilde{g}$: Dimensionless interaction parameter

$\propto$ 3D scatt. length / thickness



The energy of a vortex



Kinetic energy (vortex at the center of a disk of radius R):

$$E_{\rm cin} = \frac{1}{2} m \int \rho(\boldsymbol{r})\ \boldsymbol{v}^2(\boldsymbol{r})\ \mathrm{d}^2 r \qquad\qquad v(\boldsymbol{r}) = \frac{\hbar}{mr}$$

$$\approx \frac{1}{2} m\ \rho\ \frac{\hbar^2}{m^2} \int_\xi^R \frac{1}{r^2}\ 2\pi r\ \mathrm{d}r$$

$$= \pi\ \frac{\hbar^2 \rho}{m} \ln(R/\xi)$$

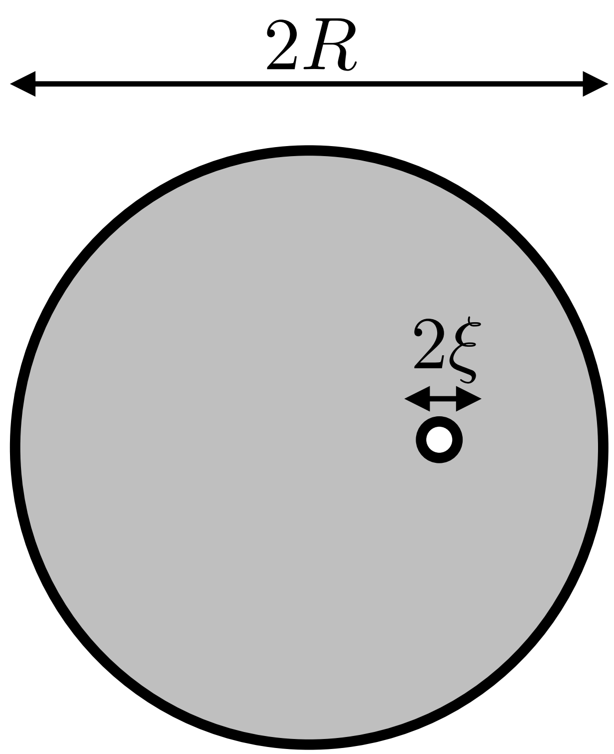
Prefactor : robust
Inside the log : depends on the model for the core

Diverges with system size

Interaction energy: one must create a hole of size ξ in the fluid

$$\epsilon_0 \sim \frac{\hbar^2 \rho}{m} \qquad \ll E_{\rm cin}$$

Is the existence of an isolated vortex likely?



Number of independent « boxes » to place the vortex:

$$W \approx \frac{R^2}{\xi^2}$$

Probability for a vortex to exist in such a box:

$$p \approx \mathrm{e}^{-E_{\mathrm{kin}}/k_{\mathrm{B}}T}$$

Using the expression for the kinetic energy:

$$\frac{E_{\mathrm{kin}}}{k_{\mathrm{B}}T} = \frac{1}{k_{\mathrm{B}}T}\,\frac{\pi\hbar^2\rho}{m}\,\ln(R/\xi) = \frac{\mathcal{D}}{2}\,\ln(R/\xi)$$

$$\mathcal{D} = \rho\lambda_T^2$$

$$\lambda_T^2 = \frac{2\pi\hbar^2}{mk_{\mathrm{B}}T}$$

Probability for a given box: $p \approx \exp\left[-\frac{\mathcal{D}}{2}\log\left(\frac{R}{\xi}\right)\right] = \left(\frac{\xi}{R}\right)^{\mathcal{D}/2}$

Total probability: $\mathcal{P} = Wp \approx \left(\frac{\xi}{R}\right)^{-2+\mathcal{D}/2}$

energetic term

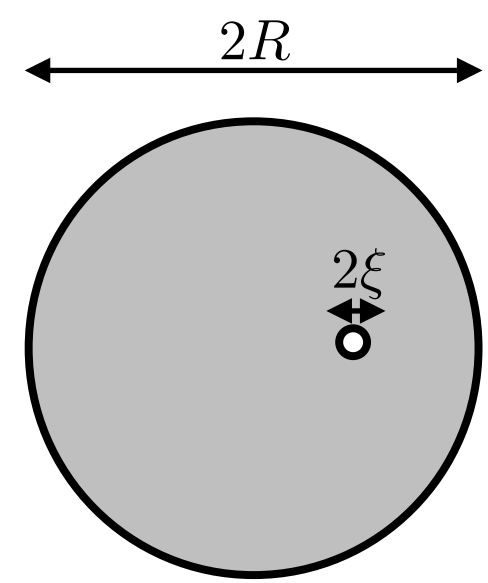
entropic term

Is the existence of an isolated vortex likely (2)?

Renormalization: $\rho, \ \mathcal{D} \longrightarrow \rho_s, \ \mathcal{D}_s$ **superfluid component**

Probability:

$$\mathcal{P} \approx \left(\frac{\xi}{R}\right)^{-2+\mathcal{D}/2} \longrightarrow \mathcal{P} \approx \left(\frac{\xi}{R}\right)^{-2+\mathcal{D}_s/2}$$



- If $-2 + \dfrac{\mathcal{D}_s}{2} > 0$, i.e., $\mathcal{D}_s > 4$, then the probability $\mathcal{P}$ tends to 0 when $R \to \infty$

  The energy cost is larger than the entropy gain: no isolated vortex

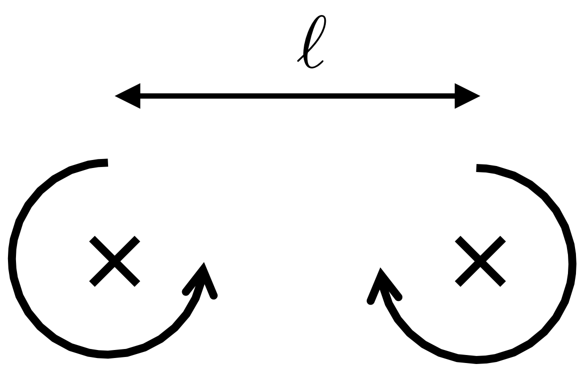
- If $\mathcal{D}_s < 4$, then $\mathcal{P} > 1$: Isolated vortices can proliferate…

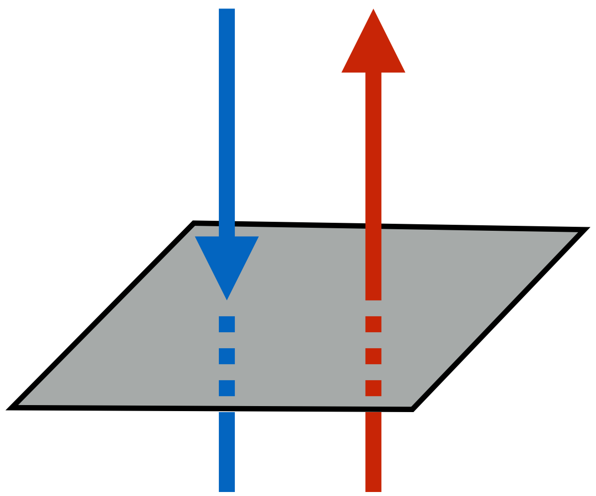
No isolated vortices

Proliferation of isolated vortices, loss of quasi-long range order

The velocity field of a vortex pair

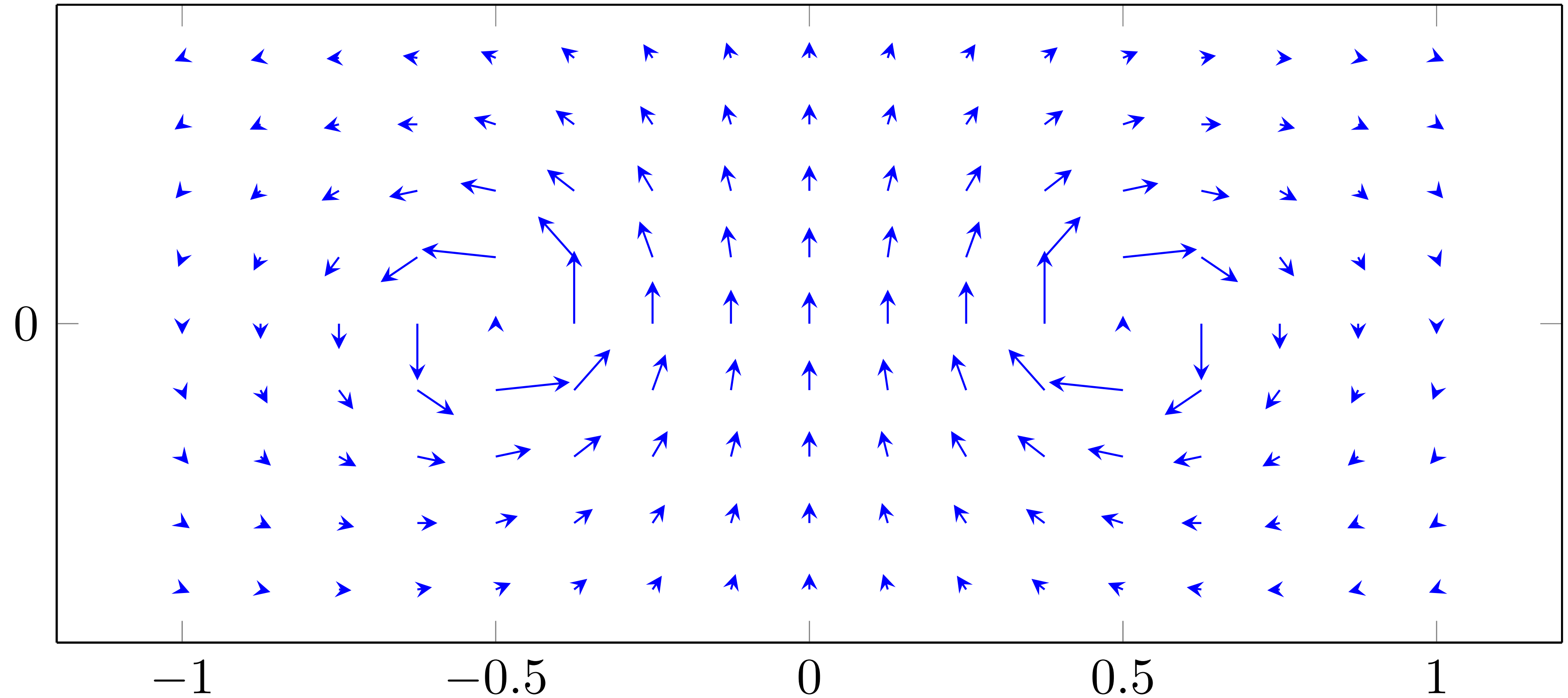
Superposition of the velocity fields created by each vortex



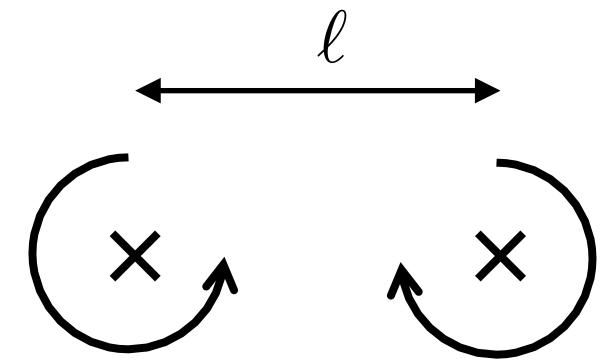


Magnetic analogy:
Field created by parallel wires
with opposite currents

Dipolar field: Decreases as $1/r^2$ at infinity instead of $1/r$ for an isolated vortex



Energy of a vortex pair



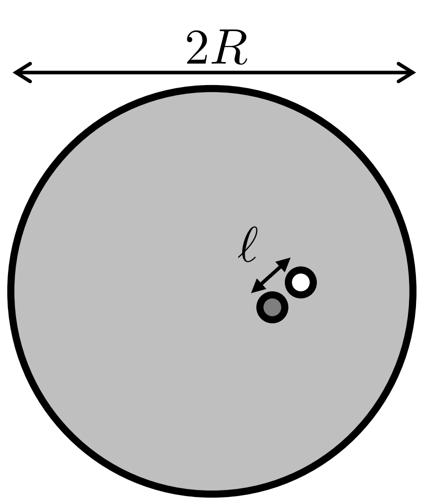
The kinetic energy is now given by a converging integral

r large: $\quad v \propto \dfrac{1}{r^2} \qquad v^2 \propto \dfrac{1}{r^4} \qquad \displaystyle\int v^2(\boldsymbol{r})\,\mathrm{d}^2 r$ converges at ∞

Quantitative result: $\quad E_{\mathrm{cin}}(\ell) \approx 2\pi\,\dfrac{\hbar^2\rho_s}{m}\,\ln\left(\dfrac{\ell}{\xi}\right)$

Interaction energy: creates two holes of size $\xi \quad E_{\mathrm{int}} = 2\,\epsilon_0 \qquad \epsilon_0 \sim \dfrac{\hbar^2\rho_s}{m}$

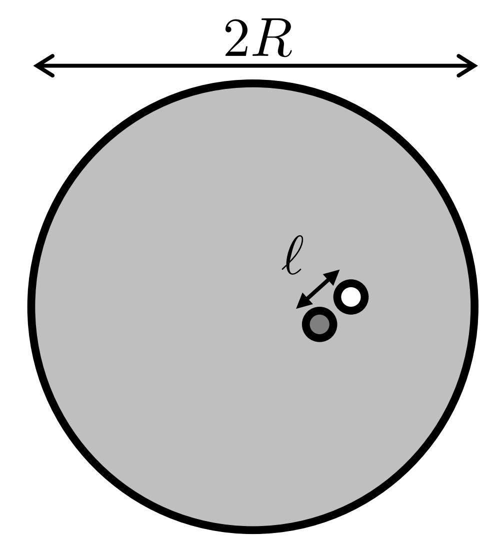
The two energies are comparable for $\ell \sim \xi$



Probability for observing a vortex pair:

$$\mathcal{P}(\ell) = \exp\left\{-\frac{2\epsilon_0 + E_{\mathrm{cin}}(\ell)}{k_{\mathrm{B}}T}\right\} \longrightarrow \propto \left(\frac{\xi}{\ell}\right)^{\mathcal{D}_s}$$

Average size of a vortex pair



Probability distribution for the size

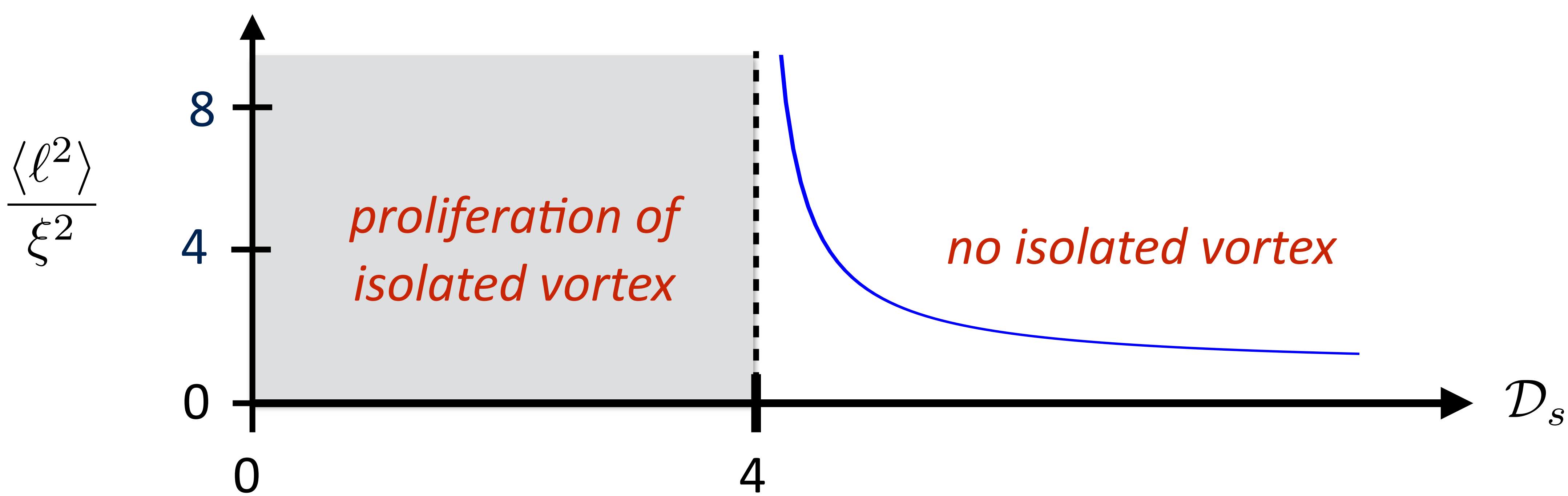
$$\mathcal{P}(\ell) \propto \left(\frac{\xi}{\ell}\right)^{\mathcal{D}_s}$$

Average: $\langle \boldsymbol{r}_a - \boldsymbol{r}_b \rangle = 0$

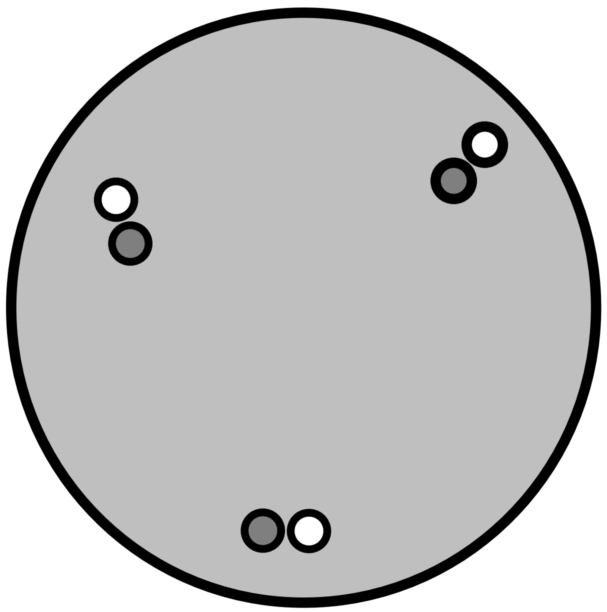
Variance : $\langle (\boldsymbol{r}_a - \boldsymbol{r}_b)^2 \rangle = ?$

$$\langle \ell^2 \rangle = \frac{\int_\xi^{+\infty} \ell^2 \, \mathcal{P}(\ell) \, 2\pi\ell \, \mathrm{d}\ell}{\int_\xi^{+\infty} \mathcal{P}(\ell) \, 2\pi\ell \, \mathrm{d}\ell} \quad = \xi^2 \, \frac{\mathcal{D}_s - 2}{\mathcal{D}_s - 4}$$

diverges for $\mathcal{D}_s \to 4_+$



Vortex pair density



Probability that a pair is present in a given area πR^2 :

$$p(R) = \iint \mathcal{P}\left(|\boldsymbol{r}_a - \boldsymbol{r}_b|\right)\ \frac{\mathrm{d}^2 r_a}{\pi\xi^2}\ \frac{\mathrm{d}^2 r_b}{\pi\xi^2}$$

with $\quad \mathcal{P}(\ell) \approx y_0^2\ \left(\frac{\xi}{\ell}\right)^{\mathcal{D}_s}$

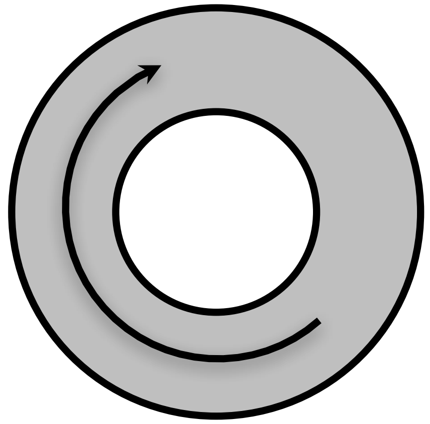
Average distance between neighboring pairs: $\quad d \approx \xi\ \mathrm{e}^{\epsilon_0/k_\mathrm{B}T}\ \sqrt{\mathcal{D}_s}$

For a strongly degenerate gas, $\mathcal{D}_s \gg 1$, vortex pairs form a dilute gas

When $\mathcal{D}_s$ gets closer to the limiting value $\mathcal{D}_s = 4$, the distance between pairs becomes comparable to the pair size: one must refine the analysis

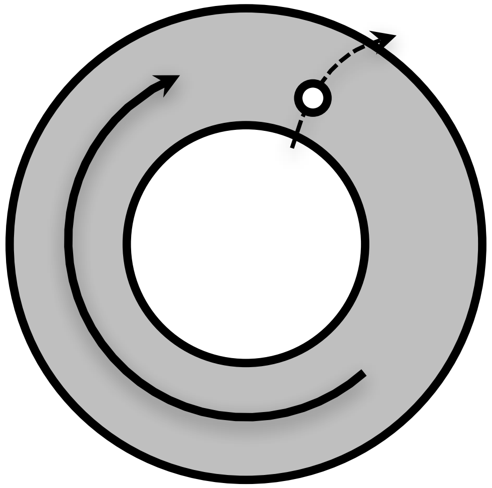
Isolated vortices and loss of superfluidity

Current in a ring, corresponding to a phase winding $2\pi N$ of the field $\psi(\boldsymbol{r})$



Is this current metastable ?

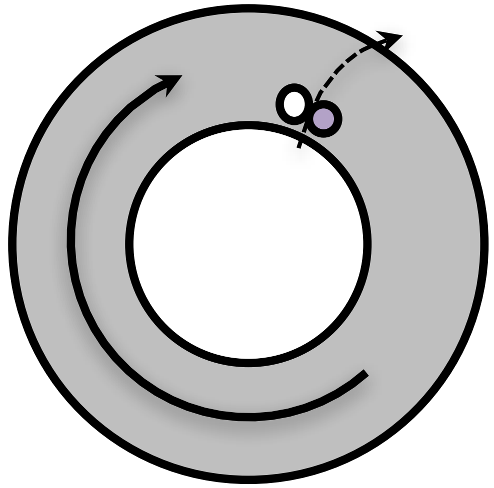
If isolated vortices exist in the ring, they may cross it:



$$N \to N \pm 1$$

Fluctuations of the current, which will thus be damped and will tend to zero

A pair of vortices of opposite charges has no effect on the current



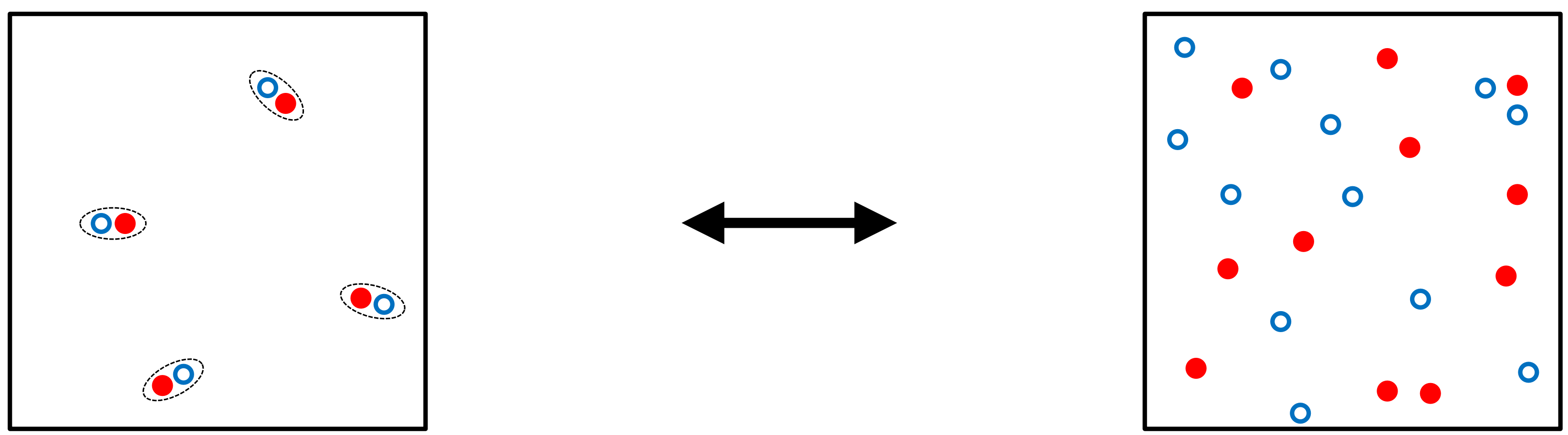
To summarize

The BKT transition occurs between two different types of states

Superfluid state $\mathcal{D}_s > 4$ — $G_1(r) \propto 1/r^{\alpha}$, $\alpha = 1/\mathcal{D}_s$

Normal state $\mathcal{D}_s = 0$ — $G_1(r) \propto \mathrm{e}^{-r/\ell}$

0 — T_c — Temperature



Universal law for the critical value of the superfluid density: $\mathcal{D}_{\mathrm{s,crit}} = 4 \qquad \alpha_{\mathrm{crit}} = 1/4$

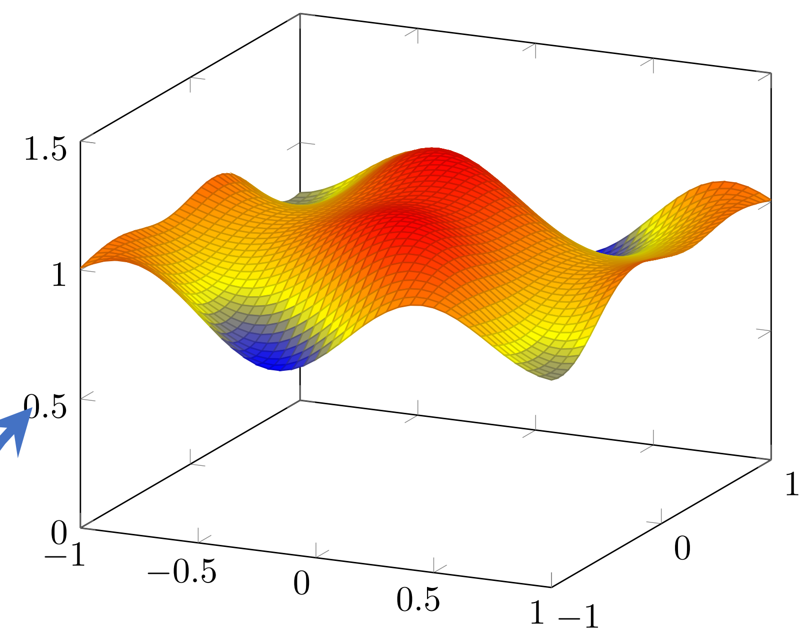
Total (superfluid+normal) phase-density at the critical point: $\mathcal{D}_{\mathrm{total}} \approx \ln\left(\dfrac{380}{\tilde{g}}\right) > 4$

$\tilde{g}$: Dimensionless interaction parameter

Prokofev and Svistunov

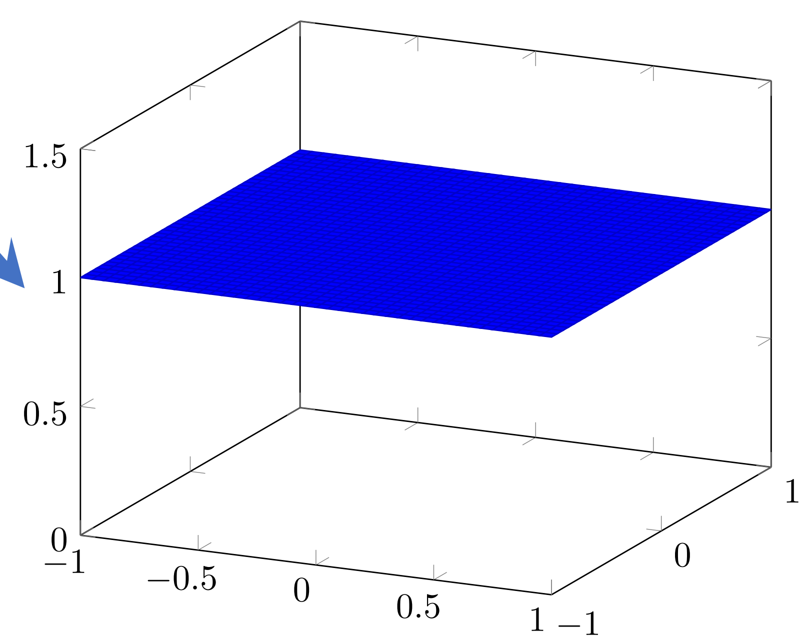
To summarize (2)

Low T: disordered (as required by Mermin-Wagner), but in a “gentle” way that can be “ironed-out”

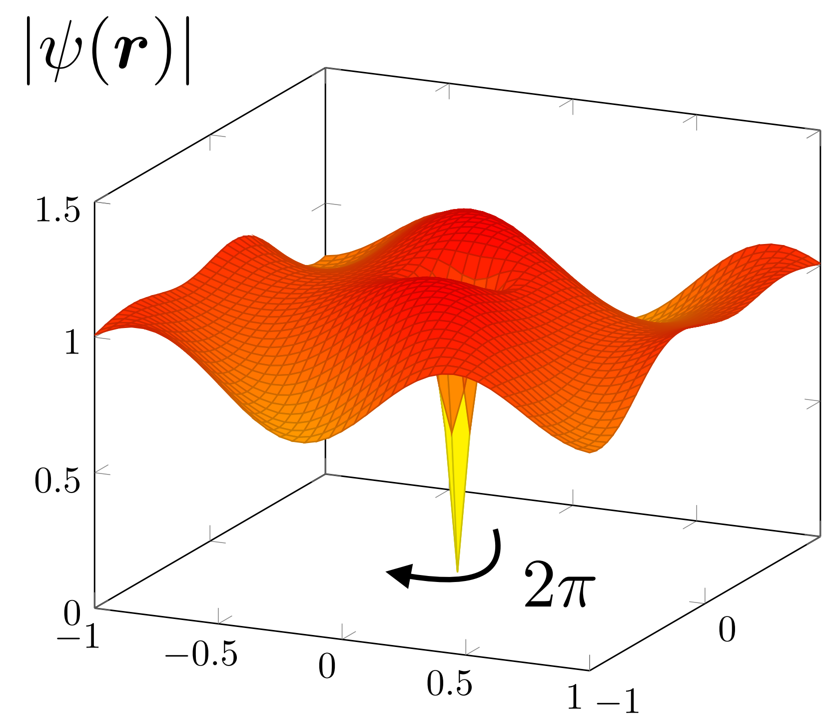


topologically équivalent

no



Large T: the presence of isolated vortices make the state qualitatively different from the low T version

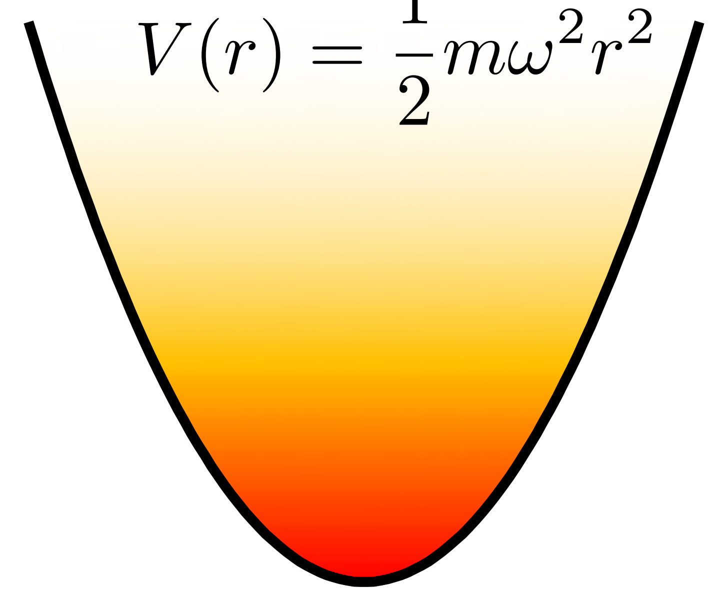


with a vortex

2.

Experimental consequences in AMO physics

The critical point in a trap (Cambridge)



$$V(r) = \frac{1}{2} m \omega^2 r^2$$

$$r^2 = x^2 + y^2$$

Reminder: Ideal gas BEC in a 2D harmonic trap

Saturation of the excited states of the trap for

$$N > N_{\mathrm{c,ideal}} \qquad\qquad N_{\mathrm{c,ideal}} = \frac{\pi^2}{6}\left(\frac{k_{\mathrm{B}}T}{\hbar\omega}\right)^2$$

In the thermodynamic limit $N \to \infty \quad \omega \to 0 \quad N\omega^2 = \mathrm{constant}$

this point is reached for a density at center $\rho(0) = \infty$

Impossible to achieve for a gas with repulsive interactions

The BKT critical point

How many atoms should one put in a trap in order to reach the superfluid threshold at the center of the trap?

Local density approximation: $\rho(0)\lambda_T^2 \;=\; \mathcal{D}_{\mathrm{c,BKT}}$

Finite quantity, by contrast to the threshold for BEC in a 2D trap

Using the equation of state of an interacting 2D gas, one finds (Holzmann et al.) :

$$\frac{N_{\mathrm{c,BKT}}}{N_{\mathrm{c,ideal}}} \approx 1 + \frac{3\,\tilde{g}}{\pi^3}\ln^2\left(\frac{\tilde{g}}{16}\right) + \frac{3\,\tilde{g}}{8\pi^2}\left[15 + \ln\left(\frac{\tilde{g}}{16}\right)\right]$$

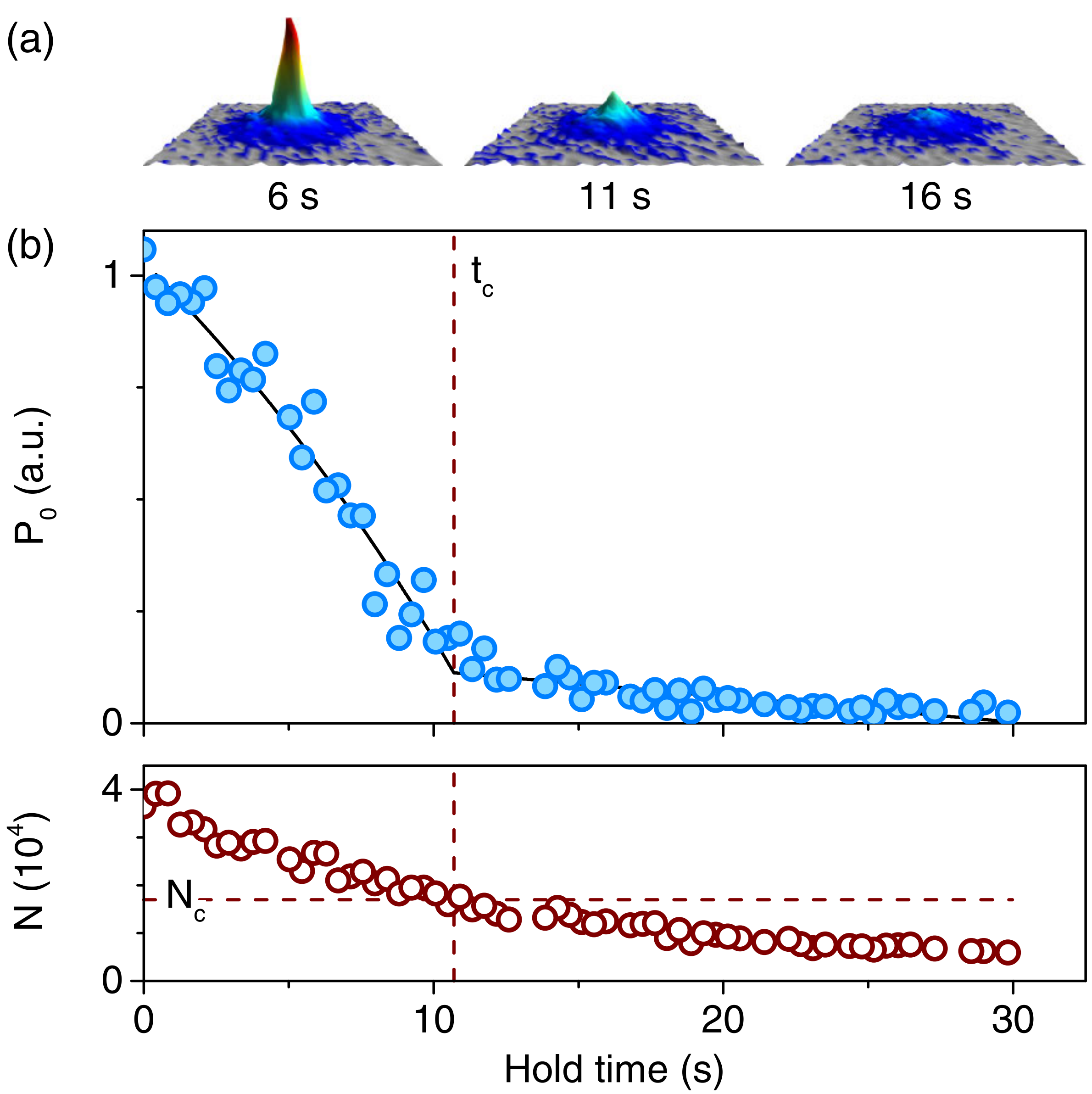
When the interaction parameter $\tilde{g} \to 0$, one finds $\dfrac{N_{\mathrm{c,BKT}}}{N_{\mathrm{c,ideal}}} \to 1$

Ideal gas condensation in a 2D harmonic potential can be viewed as the limiting case of the superfluid transition of an interacting gas in the same trap

Cambridge experiment (Z. Hadzibabic's group)

Detection of the central superfluid component via momentum distribution

Fletcher et al.
Phys. Rev. Lett. **114**, 255302 (2015)

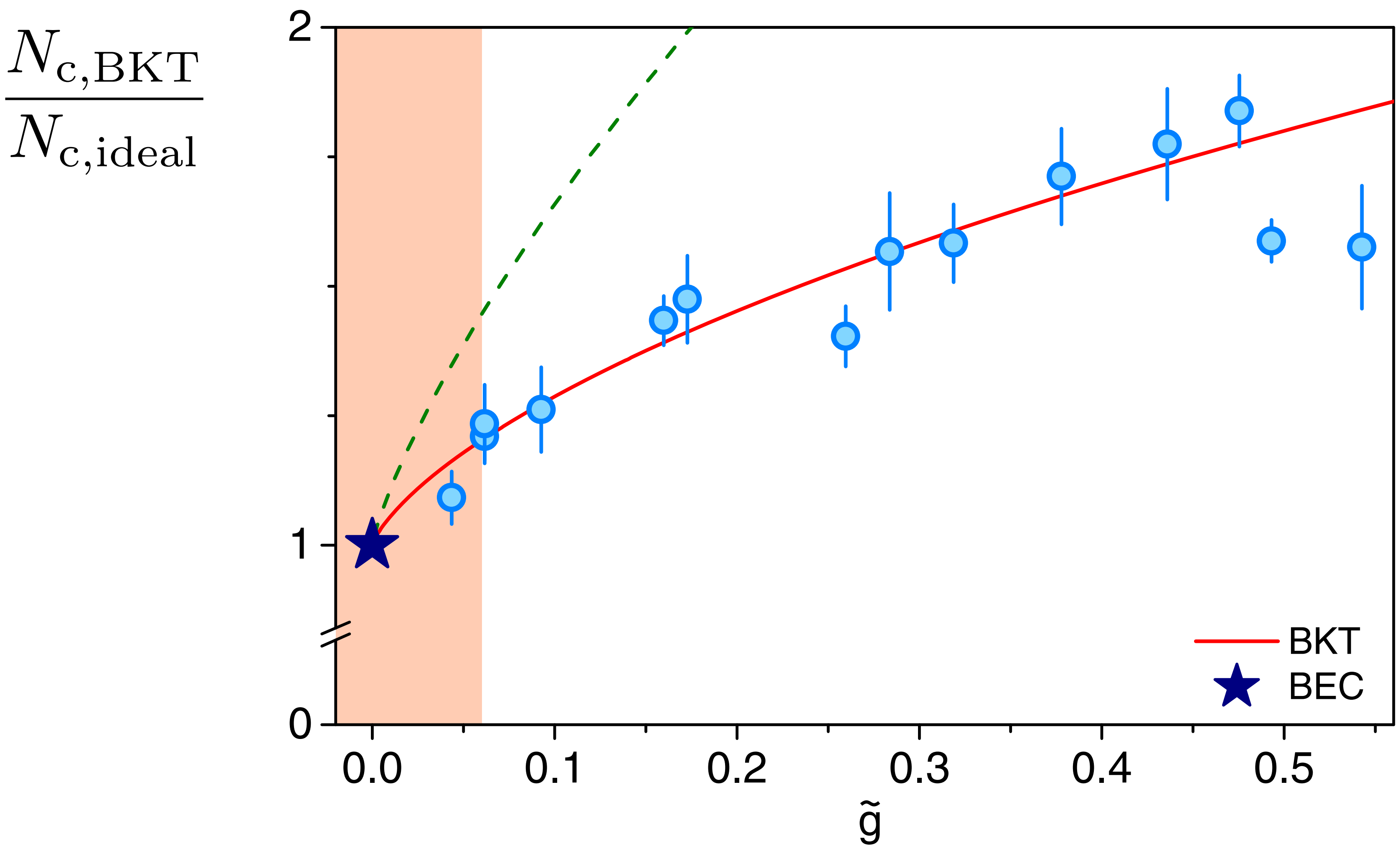


$$G_1(r) \propto \frac{1}{r^\eta}$$

Narrow peak in momentum distribution

Experiment with ^{39}K :
A Feschbach resonance allows one to vary the 3D scattering length, hence $\tilde{g}$

Cambridge experiment (2)



Fletcher et al.
Phys. Rev. Lett. **114**
255302 (2015)

Good agreement with the expected law

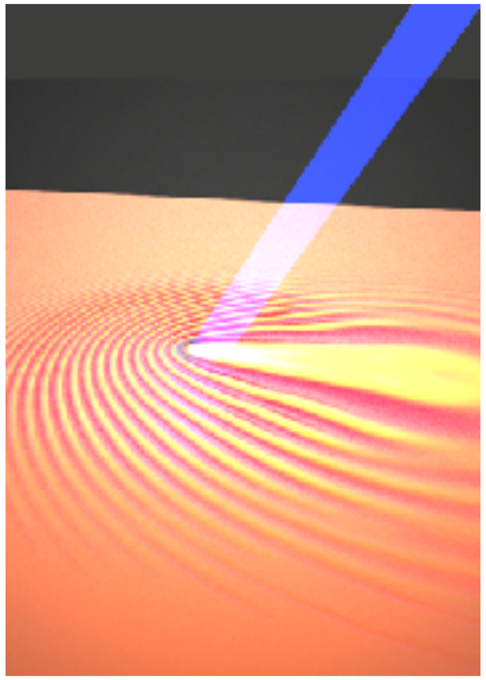
$$\frac{N_{\mathrm{c,BKT}}}{N_{\mathrm{c,ideal}}} \approx 1 + \frac{3\,\tilde{g}}{\pi^3} \ln^2\left(\frac{\tilde{g}}{16}\right) + \frac{3\,\tilde{g}}{8\pi^2}\left[15 + \ln\left(\frac{\tilde{g}}{16}\right)\right]$$

Confirms the statement that ideal gas BEC in a 2D harmonic trap is the (singular) limit of the BKT transition of an interacting gas

2.

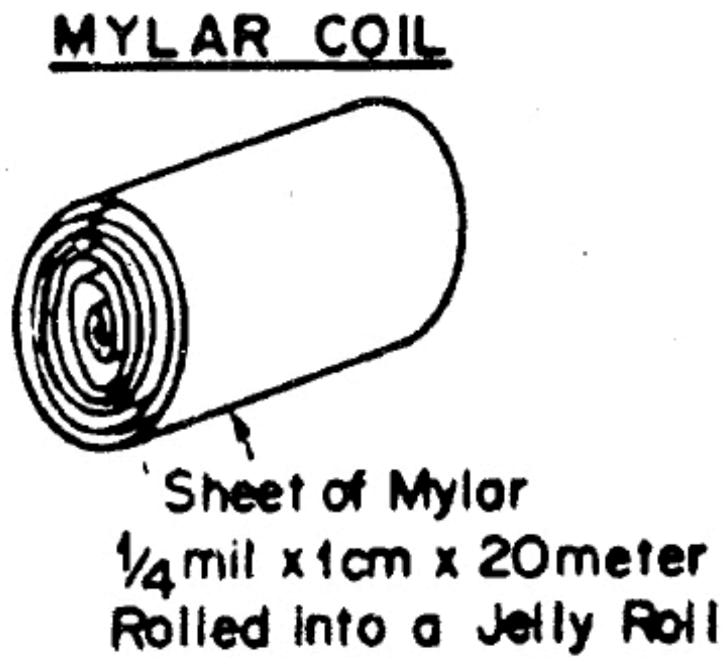
Experimental consequences in AMO physics

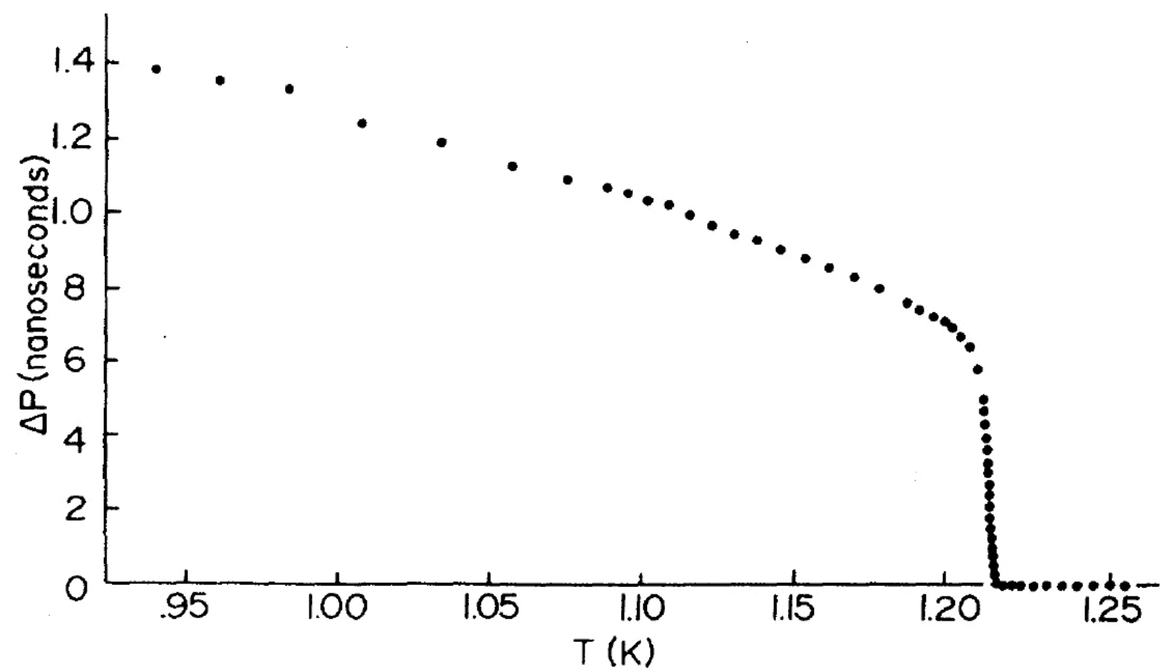
Superfluidity of a 2D gas



The BKT transition observed with a Helium film

Bishop & Reppy,
Phys. Rev. Lett. 40, 1727 (1978)
Phys. Rev. B 22, 5171 (1980)





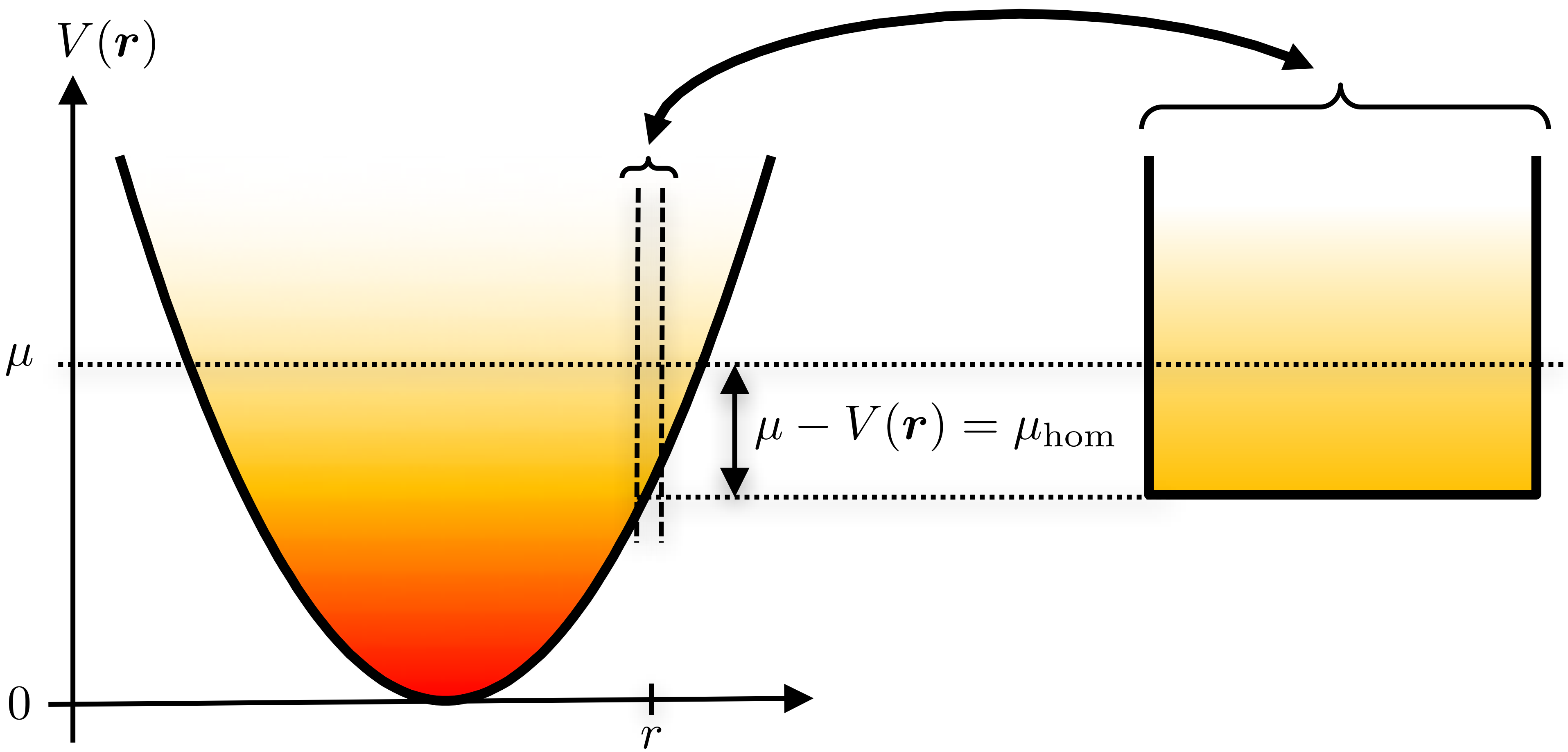
Confirmation of the universal jump $\Delta \mathcal{D}_s = 4$

Trapped atomic gases and local density approximation

Gas at equilibrium in a trap with temperature T and chemical potential μ

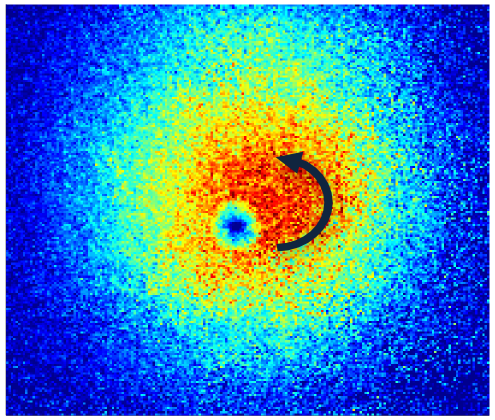
Link between the density at one point in the trap and that of a homogeneous system

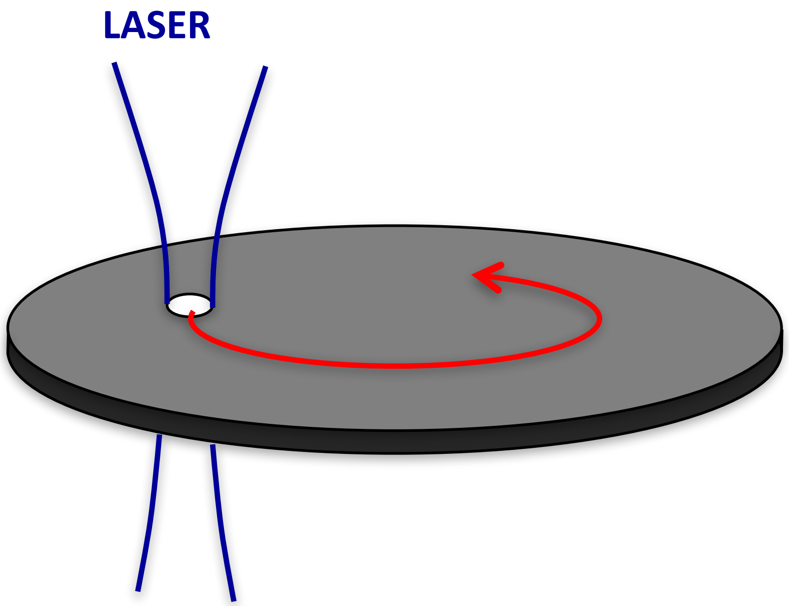
$$T_{\text{hom.}} = T \qquad \mu_{\text{hom.}} = \mu - V(\boldsymbol{r})$$



Validity : mean free path, healing length << size of the gas

Testing superfluidity with a Rb atomic gas

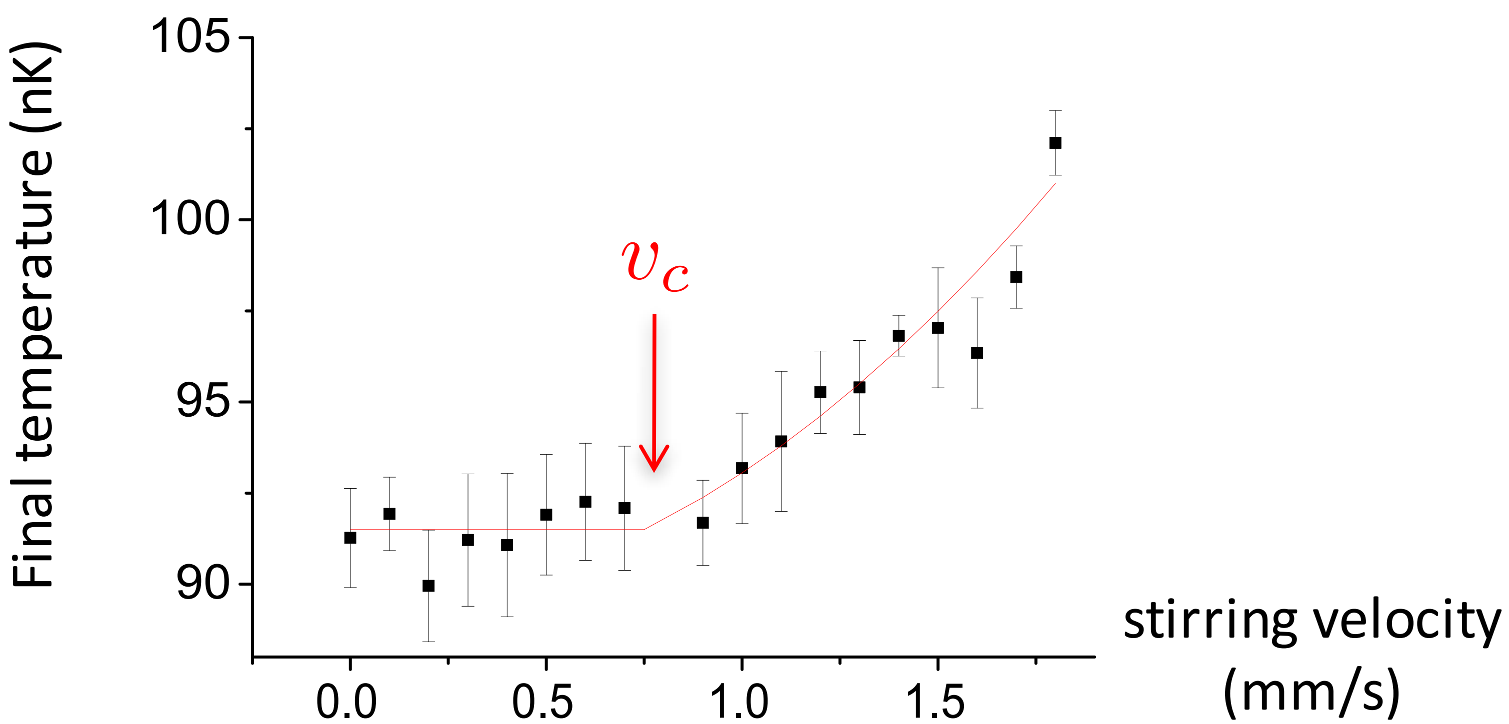




Does a moving impurity “heat” the sample?

Impurity: focused laser beam that repels the atoms

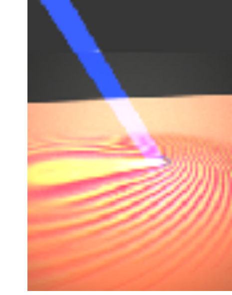
For given μ, T, we stir for 200 ms and measure the slight increase of temperature



Desbuquois et al.,
Nature Physics **8** 645 (2012)

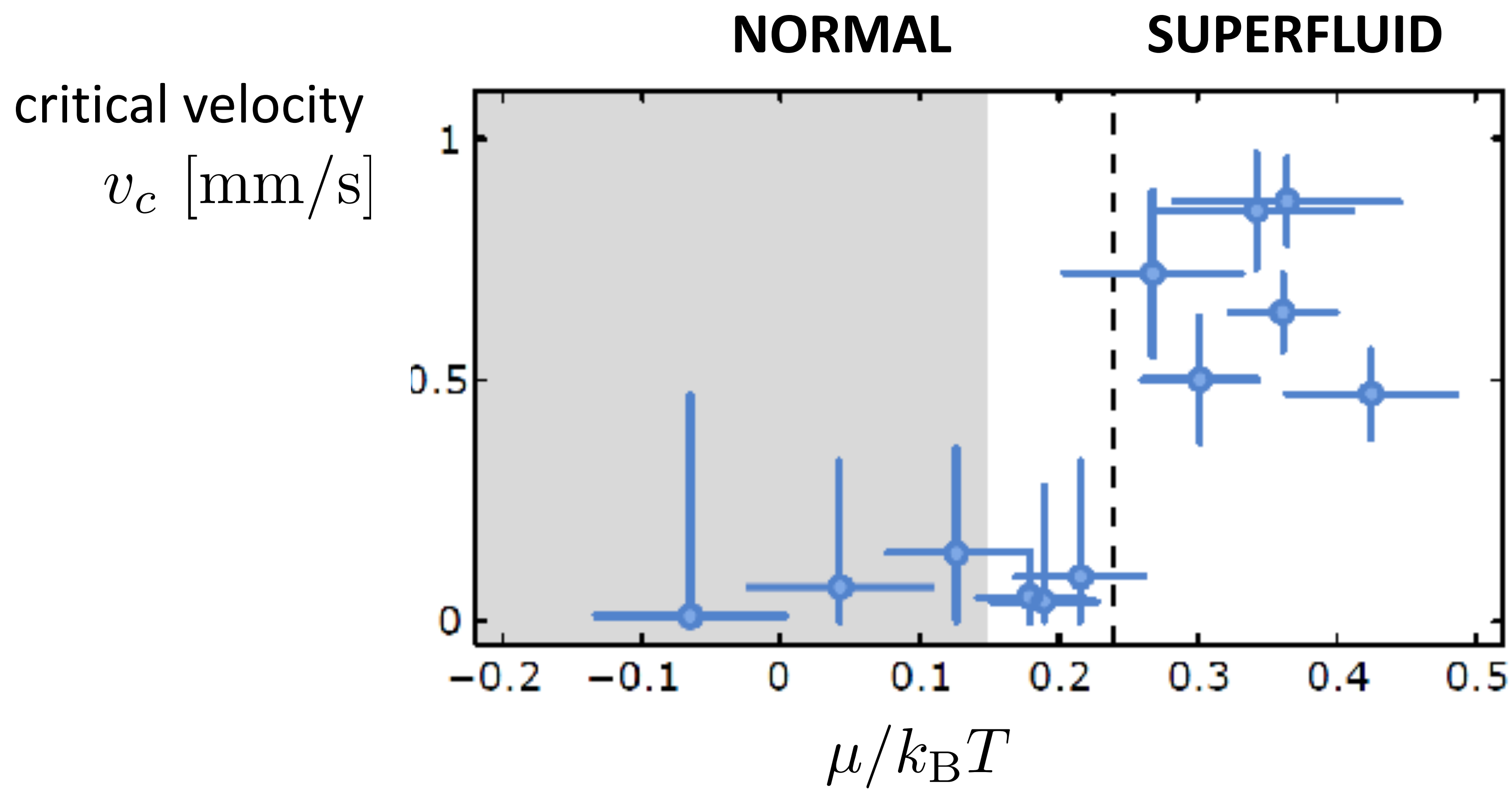
Related experiment in Seoul (2015, Yong-Il Shin’s group)

The critical velocity in 2D



Critical velocity measured for various μ, T

$\tilde{g} = 0.1$



$v_c \approx 0.3$ to $0.6\ c_s$

c_s : sound velocity

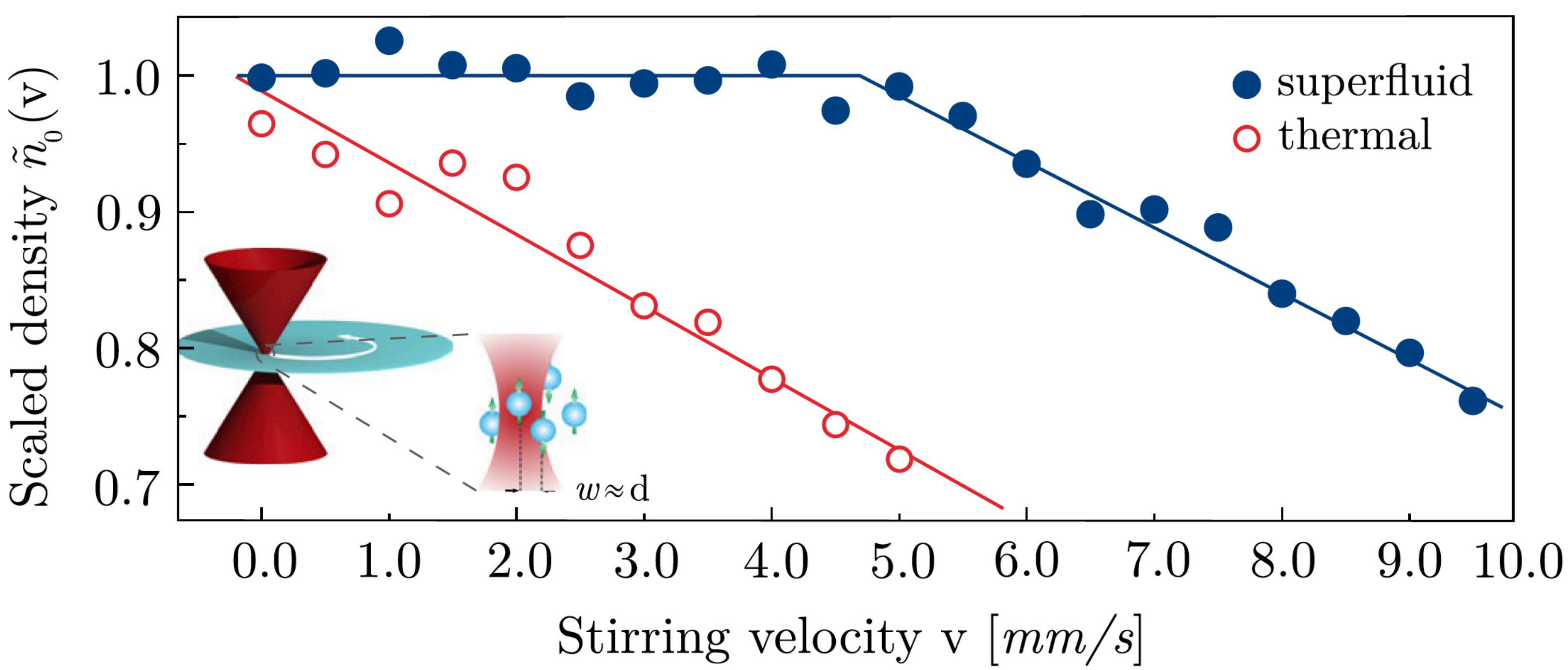
Critical $\mu/k_{\mathrm{B}}T$ in excellent agreement with classical field simulations (Mathey's team)

Desbuquois et al.,
Nature Physics **8** 645 (2012)

Vijay Pal Singh et al.,
Phys. Rev. A **95**, 043631 (2017)

Testing superfluidity with boson molecules

Hambourg 2015 (Moritz's group): strongly interacting $^6Li_2$



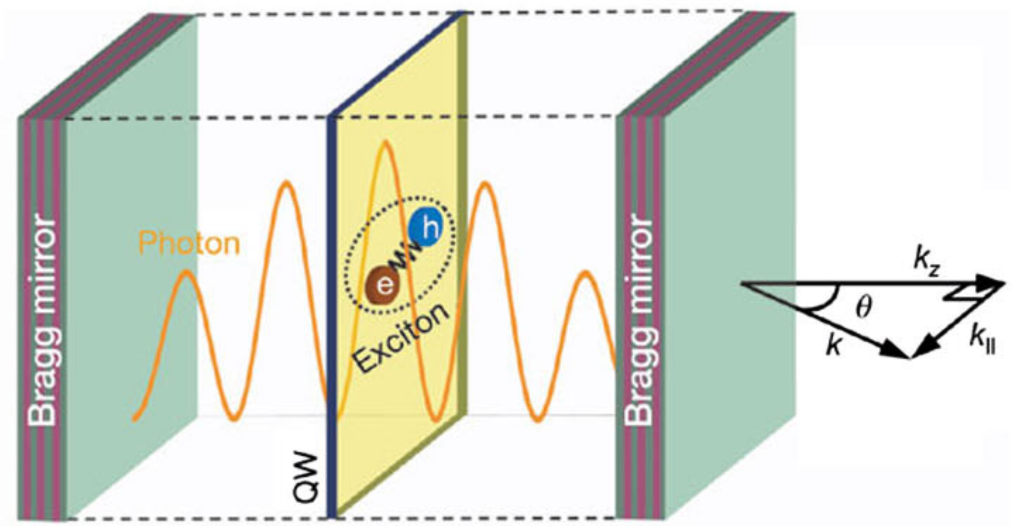
$\tilde{g} \approx 1$

Weimer et al.
Phys. Rev. Lett.
114 095301 (2015)

Here also excellent agreement with classical field simulations (Mathey's team)

Superfluidity of polariton fluids

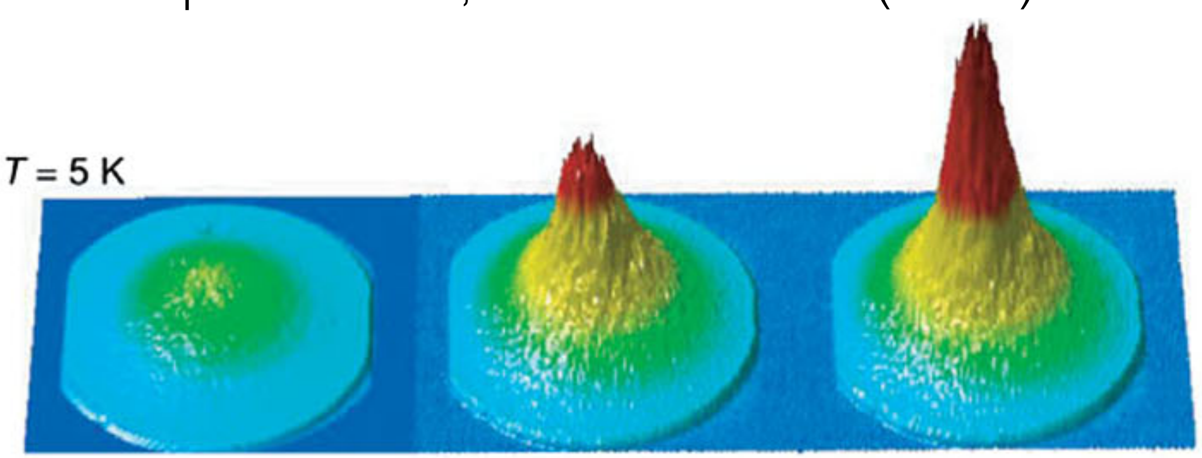
Hybrid objects in 2D, 1/2 photon, 1/2 exciton (electron-hole pair in a quantum well)



- Mass similar to the 2D photon mass derived in the first lecture
- Interactions due to the exciton part

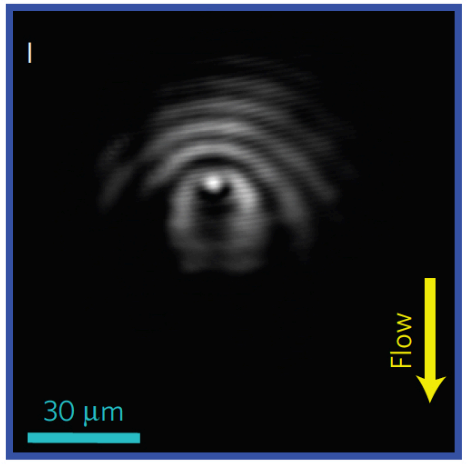
Quasi-condensation

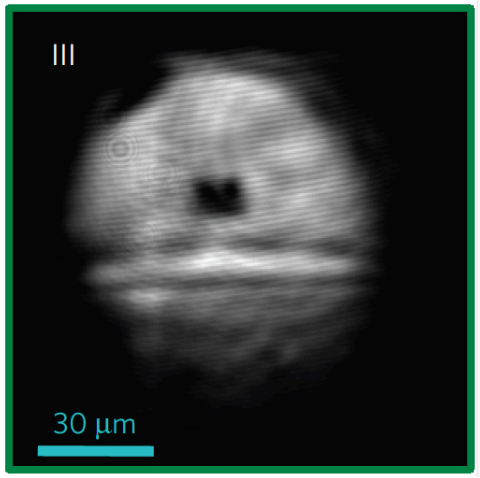
Kasprzak et al.,Nature **443** 409 (2006)



Flow around a static defect

Amo et al., Nat. Phys. **5** 805 (2009)



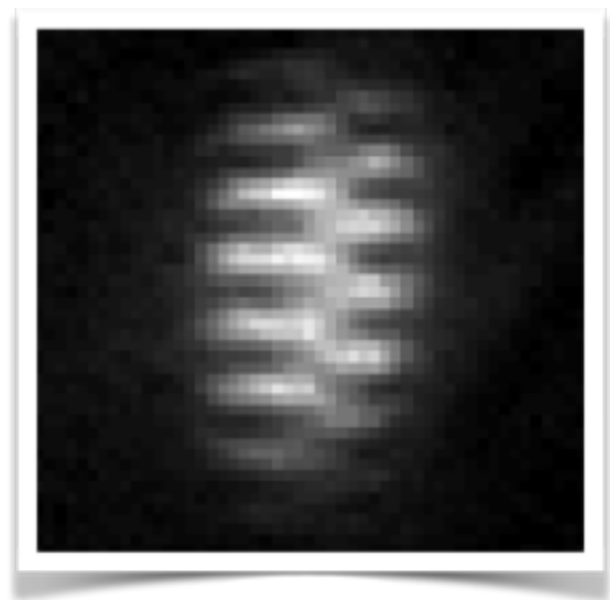


low dens: 1 μm^{-1} large dens: 40 μm^{-1}

2.

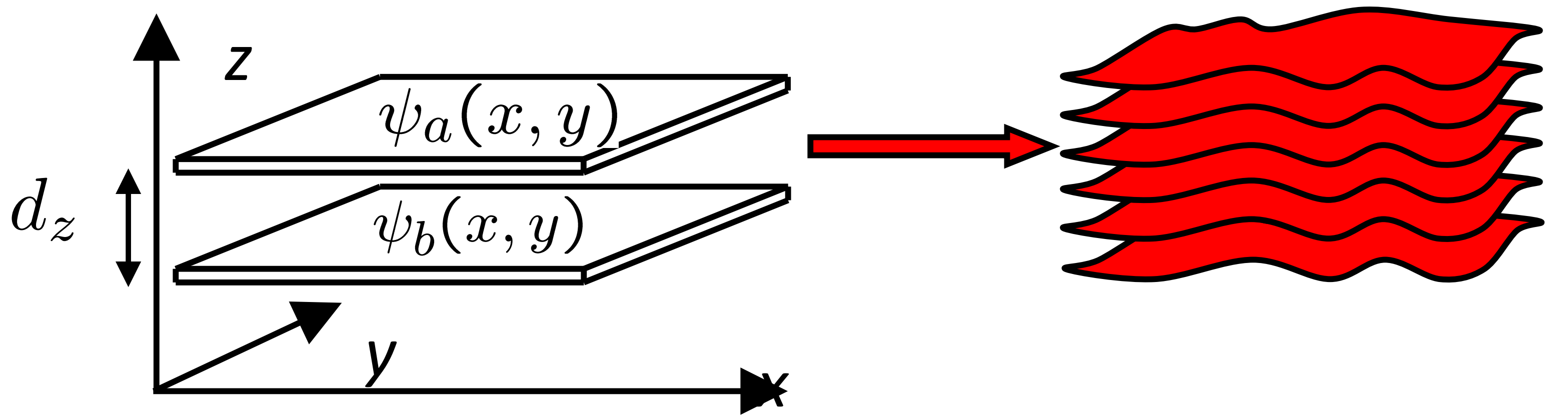
Experimental consequences in AMO physics

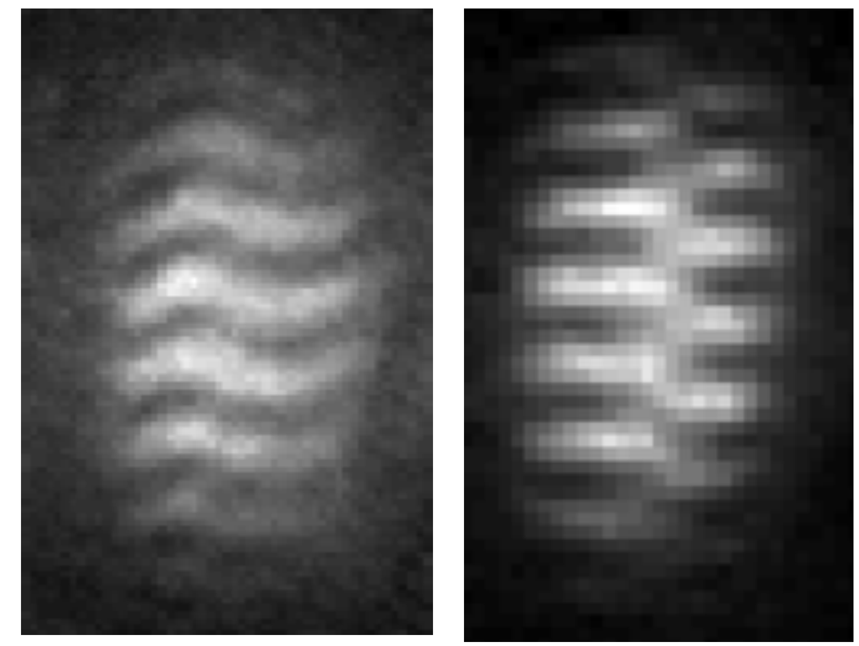
Observation of vortices and measurement of $G_1(r)$

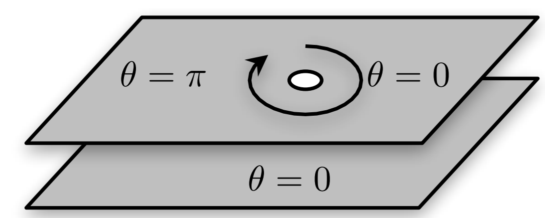


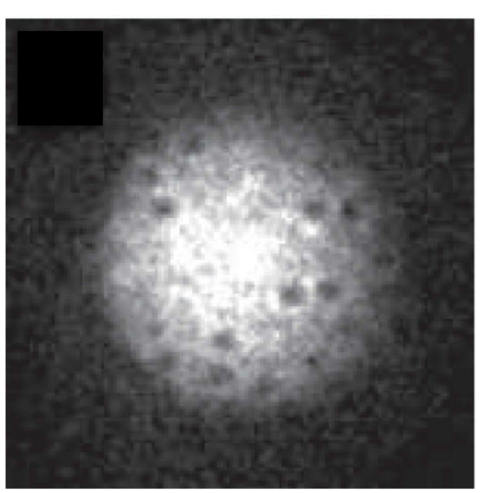
An early experiment (Paris, 2006)

Hadzibabic et al., Nature **441** 1118 (2006)









Observation of density holes in the cloud

Example from the group of J.-I. Shin (Seoul)

PRL **110**, 175302 (2013)

The function $G_1(\boldsymbol{r}, \boldsymbol{r}')$

Expected result for a 2D homogeneous superfluid

$$G_1(\boldsymbol{r}, \boldsymbol{r}') \propto \frac{1}{|\boldsymbol{r} - \boldsymbol{r}'|^{\alpha}} \qquad \alpha = \frac{1}{\mathcal{D}_s} = \frac{1}{\rho_s \lambda_T^2}$$

$$\alpha \le \alpha_{\mathrm{crit}} = 1/4$$

So far, published results have been obtained with harmonic trapping in the xy plane, and the spatial variation of the density complicates the analysis

Only semi-quantitative agreement …

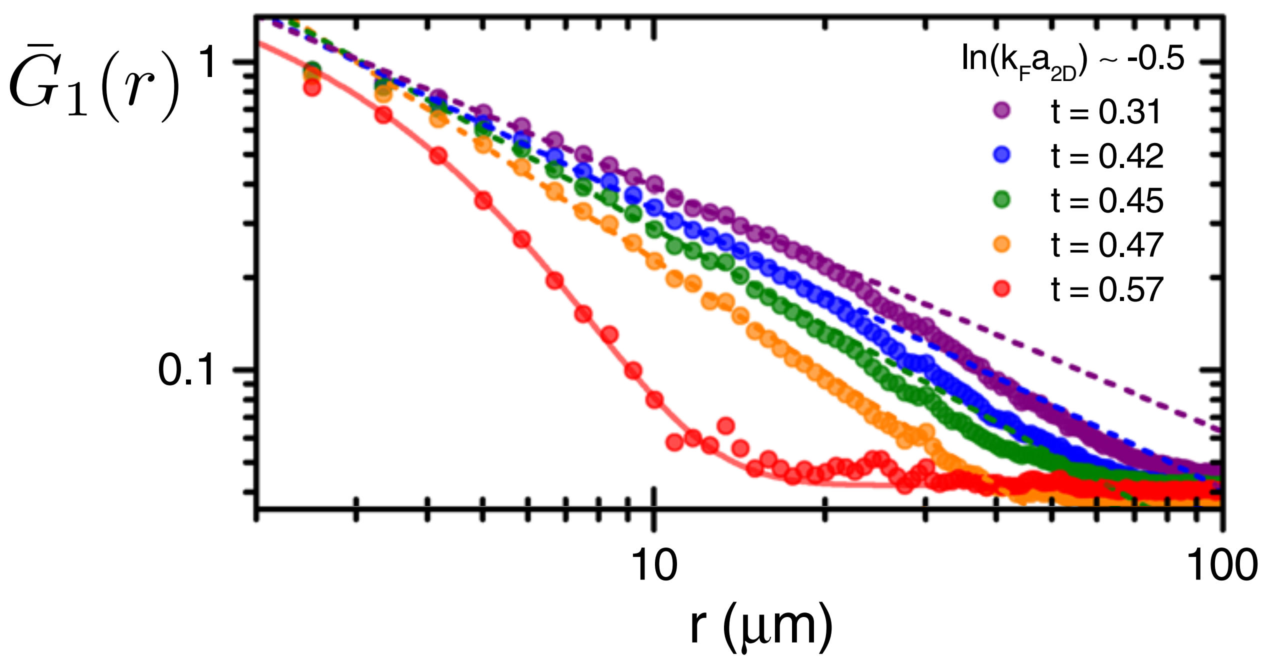
The G_1 function and momentum distribution

Starting from $G_1(\boldsymbol{r}, \boldsymbol{r}')$ in a non-uniform fluid, define:

$$\bar{G}_1(\boldsymbol{r}) = \int G_1\left(\boldsymbol{R} + \frac{\boldsymbol{r}}{2}\ ,\ \boldsymbol{R} - \frac{\boldsymbol{r}}{2}\right)\ \mathrm{d}^2 R$$

$$\bar{G}_1(\boldsymbol{r}) \overset{\text{T.Fourier}}{\longleftrightarrow} N(\boldsymbol{p})$$

Heidelberg (Jochim's group): $^6Li_2$ molecules close to a Feshbach resonance $\tilde{g} \approx 3$



Murthy et al.
Phys. Rev. Lett. 115 010401 (2015)

Algebraic or exponential decay?

Algebraic at low T but:

$$\alpha_{\text{crit}}^{\text{mes.}} \sim 1.5 \gg \frac{1}{4}$$

Boettcher & Holzmann 2016: Crucial role of thermal non-superfluid wings in the trap

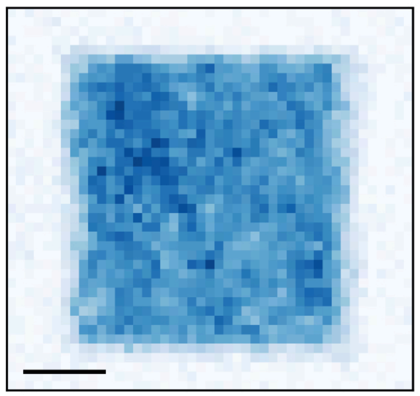
Remedy: Switch to uniform systems...

Quasi-long range order in a uniform system

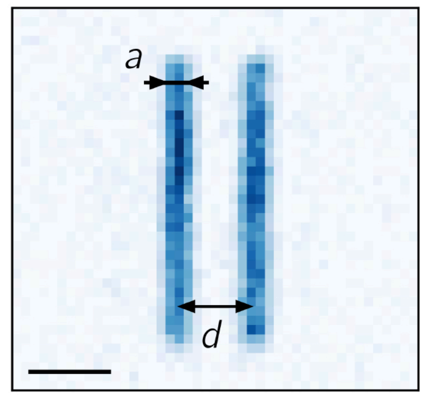
Paris group, to be published

Investigation via a “Young slit” experiment

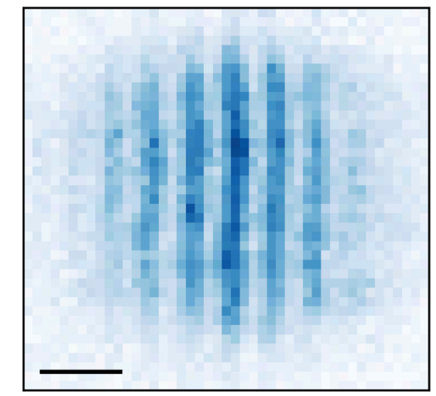
Initial state



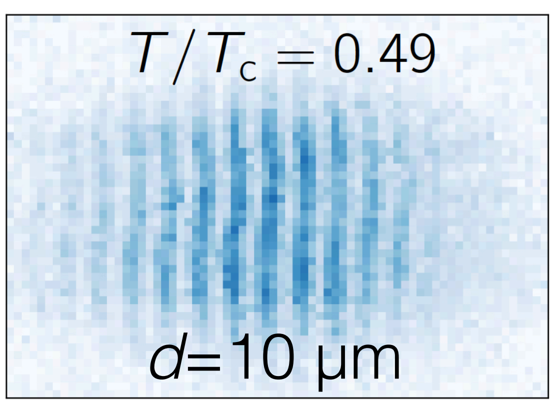
Isolate two slits using a shaped laser beam →

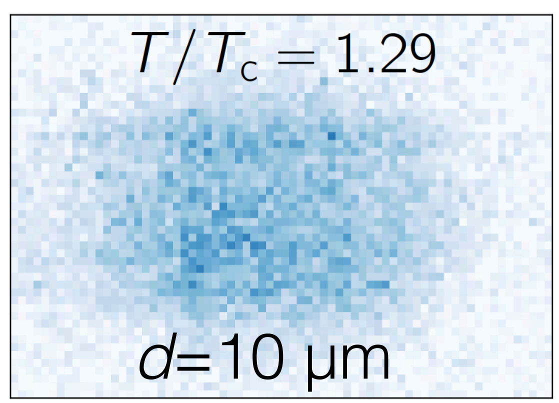


Interference after time-of-flight reveals the coherence between the slits →

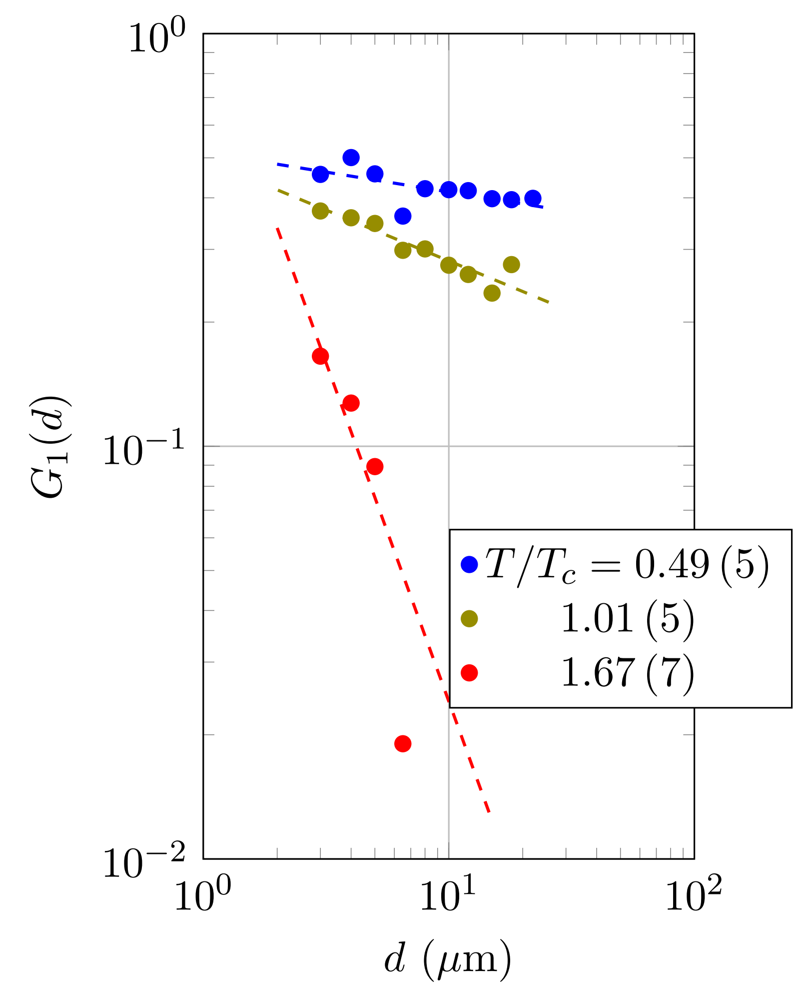


Average over several shots prepared in identical conditions:





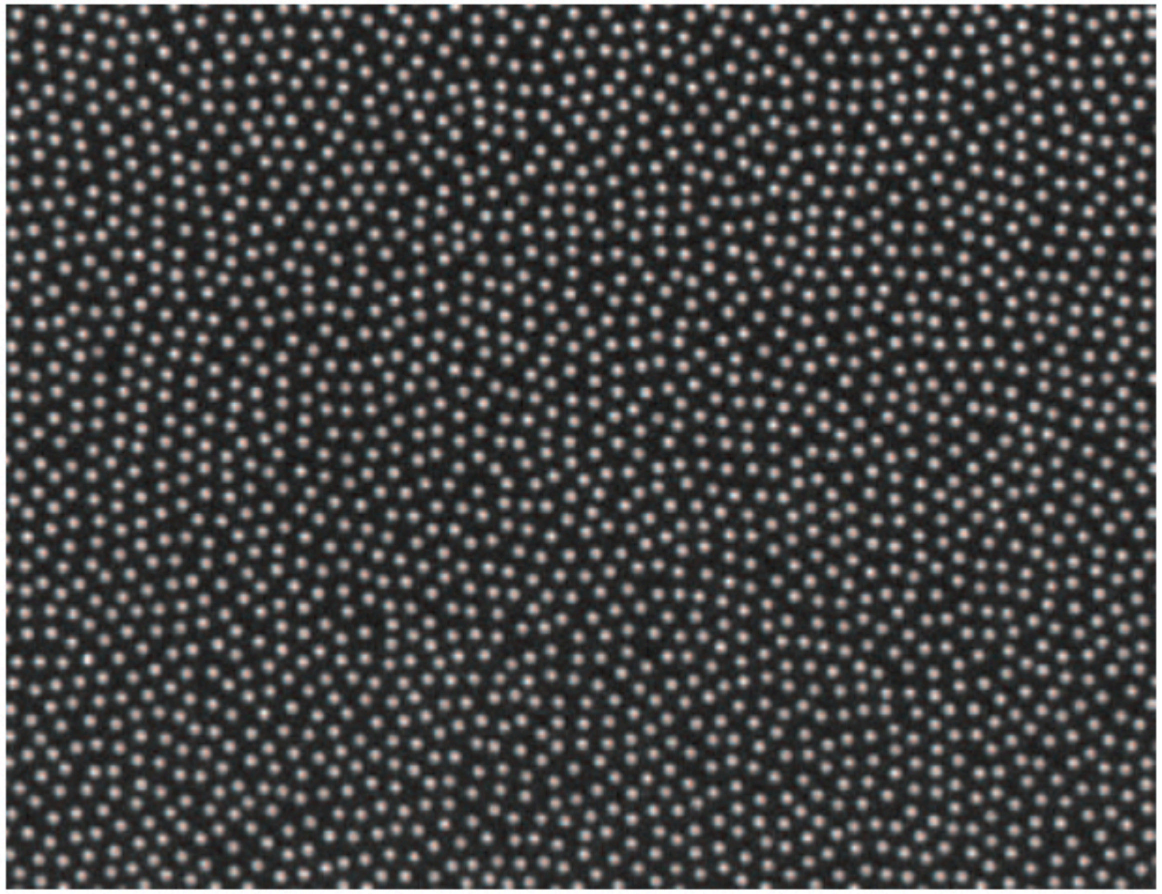
The variation of the contrast with the distance d gives access to $G_1(d)$



For $T \approx T_c$ we obtain $G_1(d) \sim d^{-1/4}$:
expected universal law for BKT transition

3.

2D classical fluids and crystals in the lab



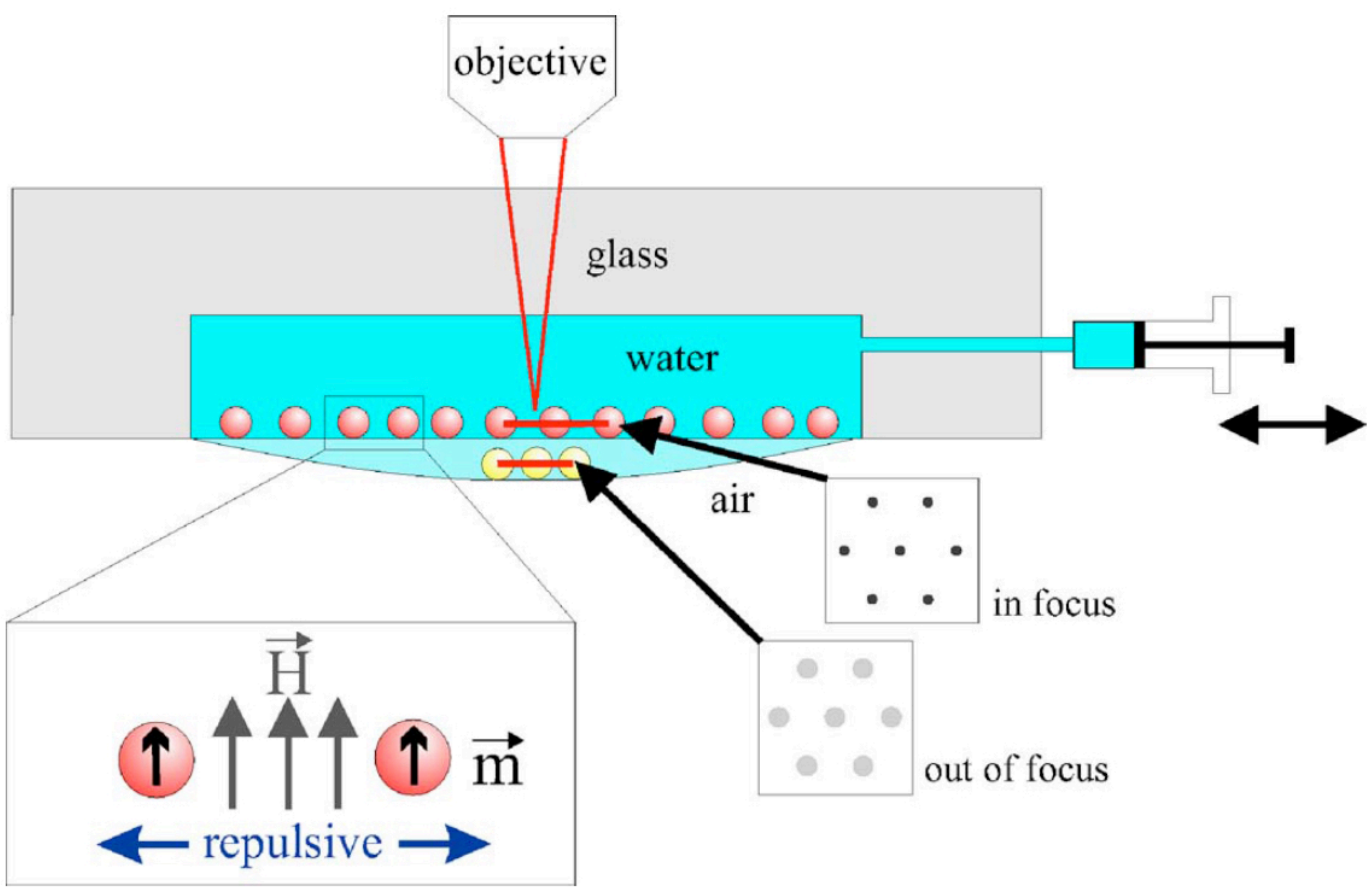
Georg Maret & Peter Keim
(Constance) with colloidal crystals

Experimental setup in Constance

- Water droplet (8 mm diameter) suspended by surface tension

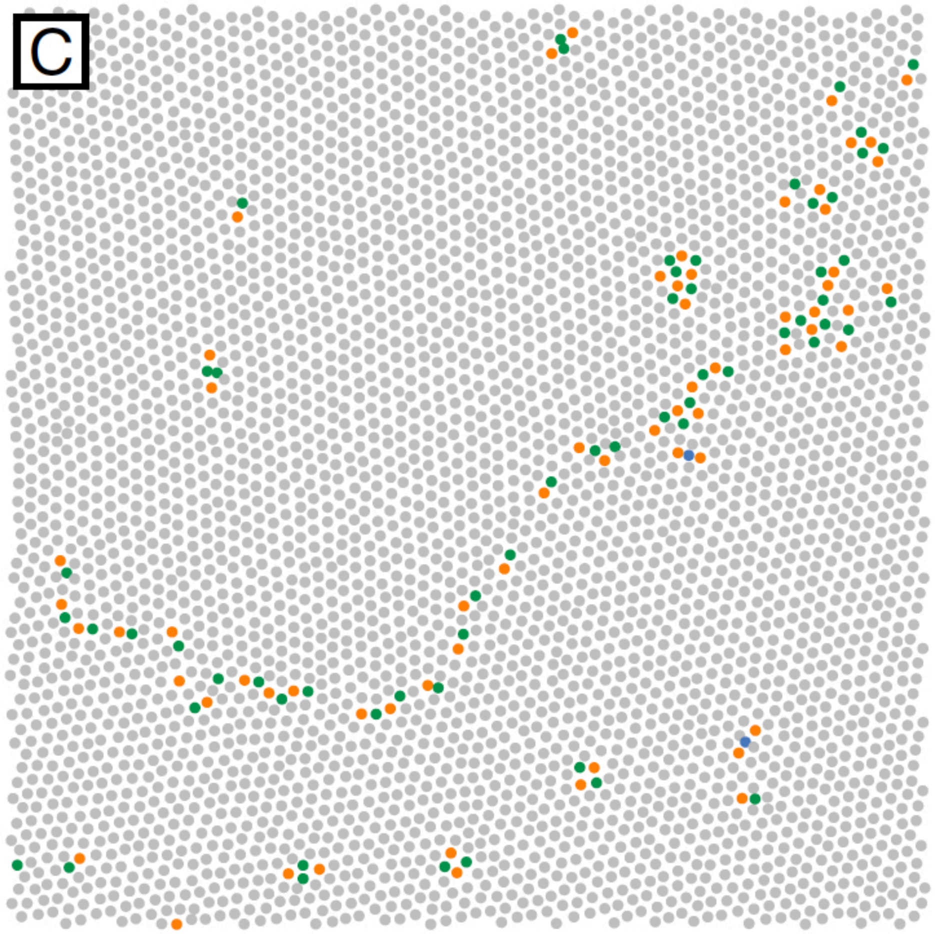
- 10^5 polystyrene beads with 4.5 μm diameter (density 1.5) at the water-air interface

- The beads are doped with nanoparticles with iron oxyde. In the presence of a magnetic field, repulsive dipole-dipole interaction between beads.



Keim et al, Phys. Rev. E **75**, 031402 (2007)

Experimental setup in Constance



Deutschlander et al., 2015

A 1 mm x 1mm window is selected: around 4000 beads, with a 15 µm average distance between them

The position of each bead is recorded every second, with sub-micron precision

Time constant to reach equilibrium > 1 month!

Experimental control parameter : $\Gamma = \dfrac{E_{\mathrm{mag}}}{k_{\mathrm{B}}T}$

Different phases for the assembly of particles

KTHNY : Kosterlitz, Thouless, Halperin, Nelson, Young (1973-1979) $\quad \Gamma = \dfrac{E_{\mathrm{mag}}}{k_{\mathrm{B}}T}$

Scenario validated by experiment and numerical simulations

- $T = 0$: Perfect triangular lattice (if one neglects quantum fluctuations)
- Very low T: logarithmic fluctuations *à la Peierls*

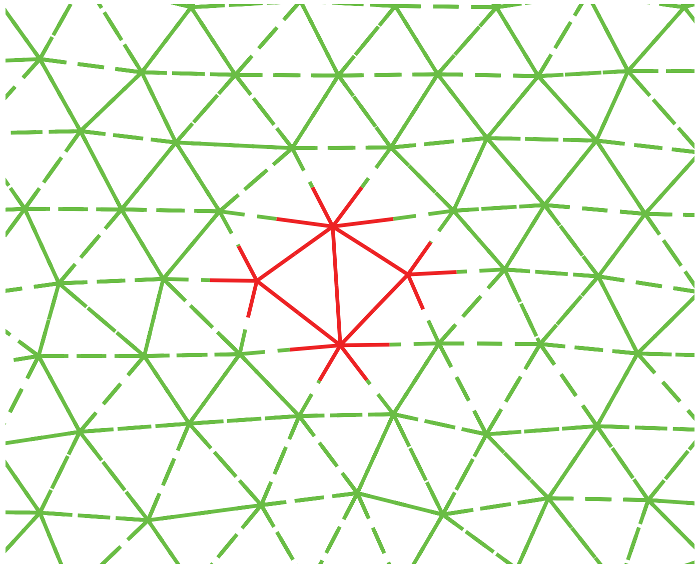
Translational quasi-order : $\langle (\boldsymbol{u_j} - \boldsymbol{u}_0)^2 \rangle \;\propto\; \log(R_{\boldsymbol{j}}/a)$

Local defects that do not affect the quasi-long range order

7

5 **5**

7



Wierschem & Manousakis (2011)

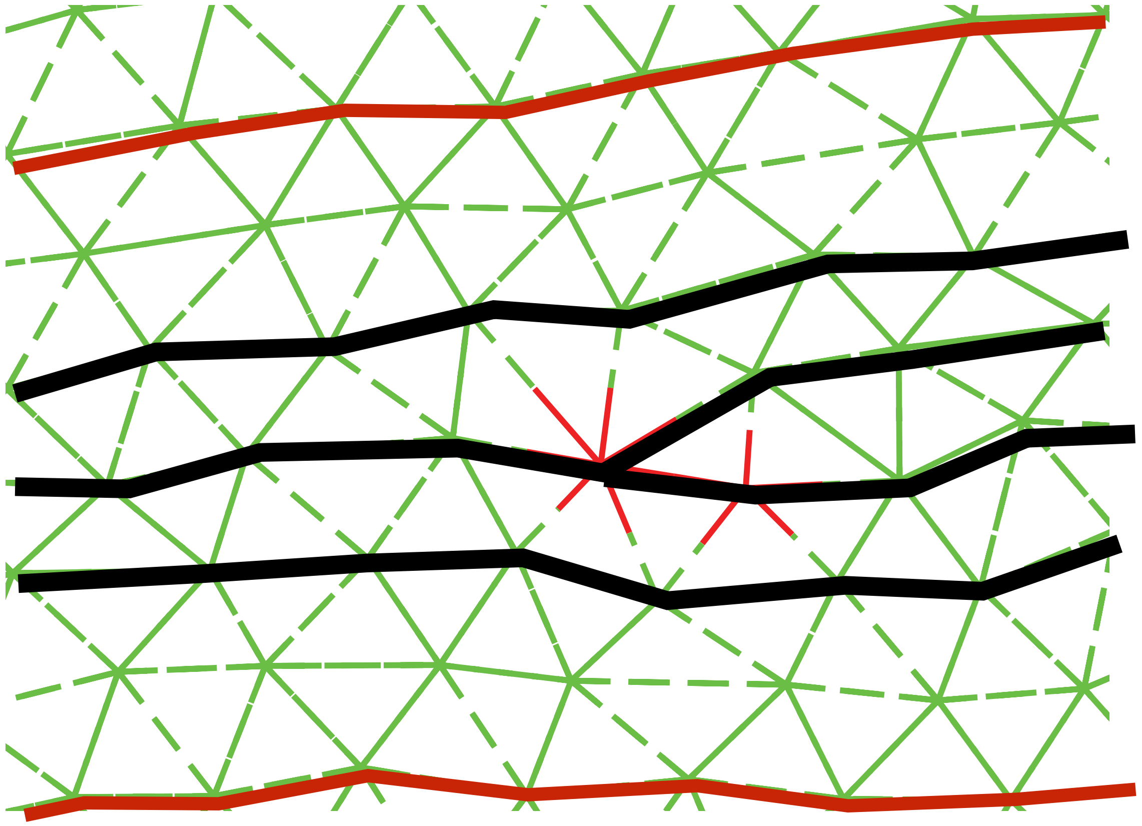
True orientational order: $\langle (\theta_{\boldsymbol{j}} - \theta_0)^2 \rangle$ does not tend to zero at infinity

Different phases for the assembly of particles (2)

First phase transition at a temperature T_m such that

$$\Gamma_m = \frac{E_{\rm mag}}{k_{\rm B} T_m} = 70.3$$

Apparition of "dislocations" : defects of type 7-5 corresponding to the addition of a half-line of particles



Wierschem & Manousakis (2011)

- Destroys the translational quasi-order
- The orientational quasi-order subsists

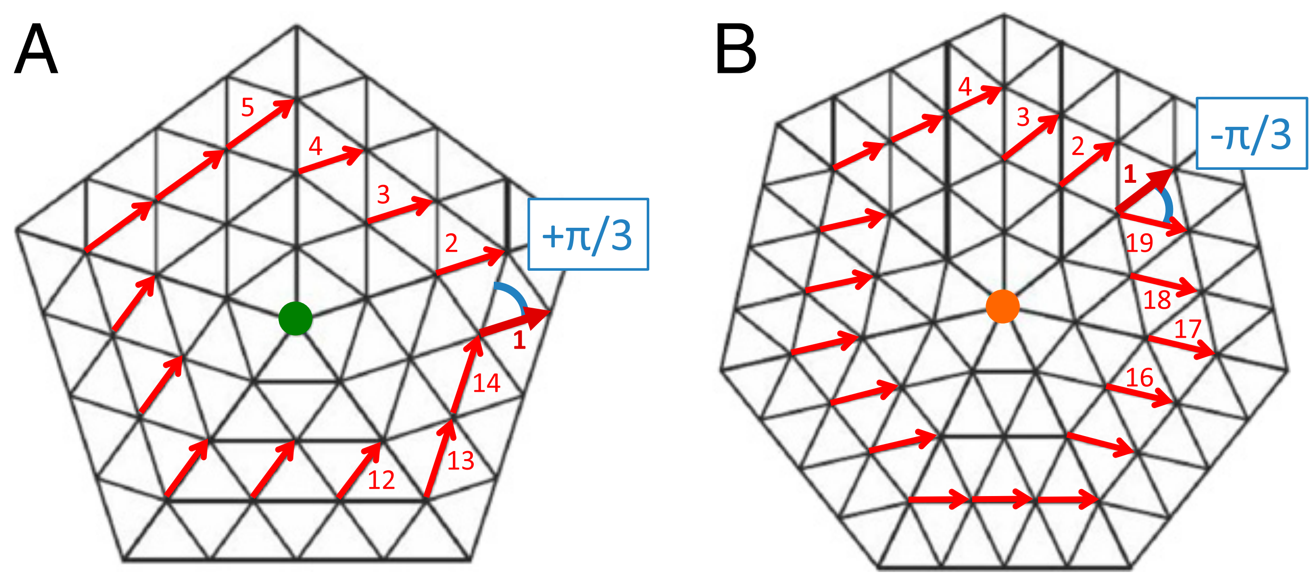
Hexatic phase

Different phases for the assembly of particles (2)

Second phase transition at temperature T_i such that

$$\Gamma_i = \frac{E_{\mathrm{mag}}}{k_{\mathrm{B}} T_i} = 67.3$$

Apparition of “disclinations”: isolated defects of type 7 or type 5



Deutschlander et al., 2015

Loss of orientational order: no long range or quasi-long range remains

Summary

From Peierls to Berezinskii - Kosterlitz - Thouless

R. Peierls 1907-95

VADIM L'VOVIČH BEREZINSKIĬ (1935-1980)

J.M. Kosterlitz

D.J. Thouless

No breaking of a continuous symmetry in a 2D system at $T \neq 0$

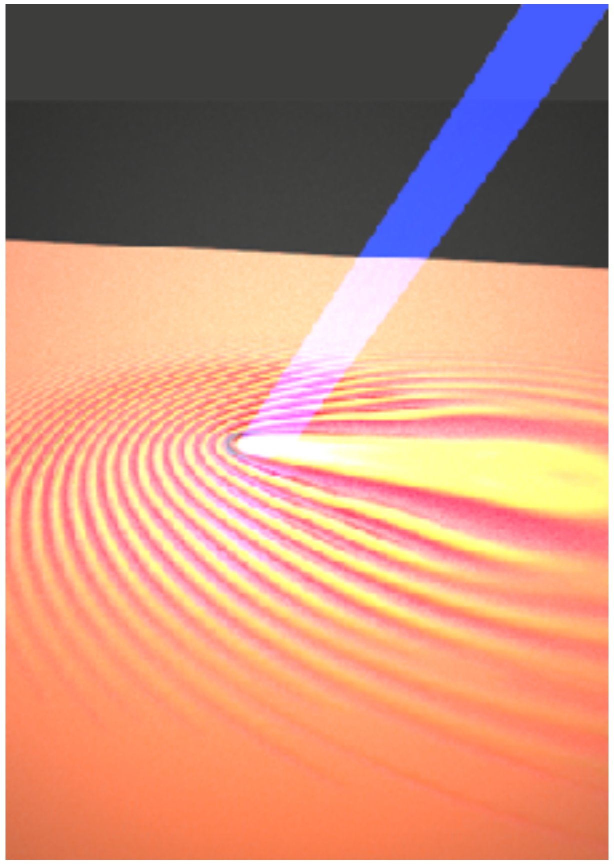
No crystal, no BEC, no long-range order

BKT : A non conventional phase transition is still possible

Superfluid transition (infinite order)

The role of AMO systems

Quantum fluids with atoms, molecules, photons, polaritons, have provided a unique insight in several aspects of BKT physics



- Superfluid behavior and critical point
- Visualisation of vortices
- Sound propagation
- Evidence for algebraic decay: $G_1(r) \propto r^{-\alpha}$ with $\alpha \approx 1/4$ at the critical point

Current and future developments

- Influence of disorder
- Dynamics across the phase transition: revisiting the Kibble-Zurek mechanism