Topological matter and
its exploration with quantum gases

Topological bands in 2D and edge channels: From the Haldane model to Landau levels

Jean Dalibard

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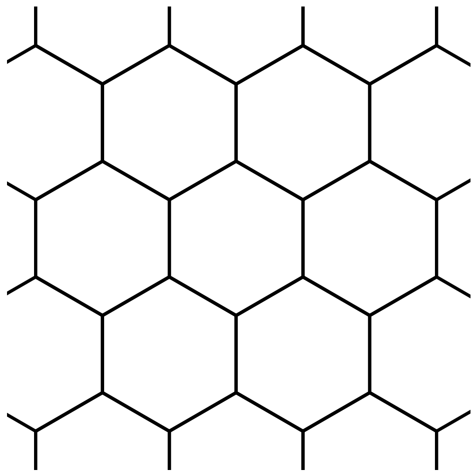
1530

Where we stand

Periodic lattices in 2D



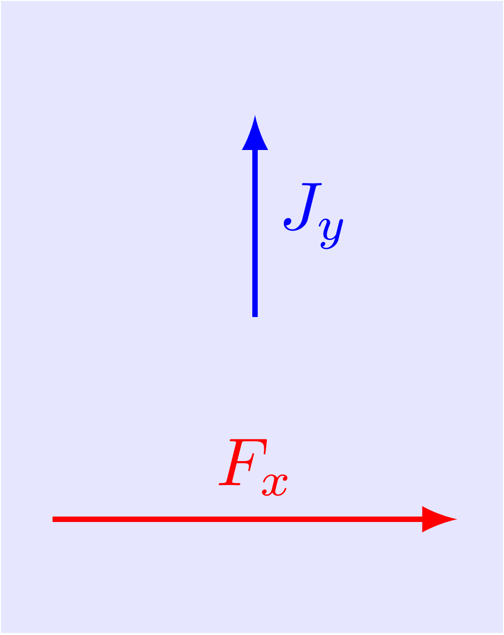
Link between the topology of an energy band and Hall conductivity

Example of a filled lowest band, all other bands empty

External force along the x axis

Particle current along the y axis

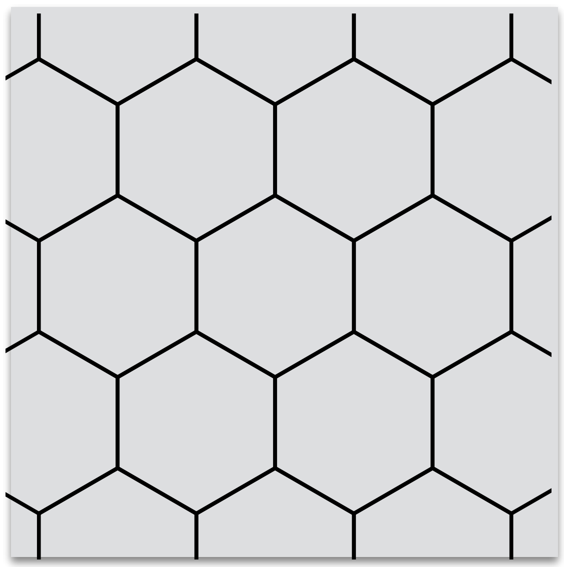
$$J_y = \sigma_{yx} F_x \qquad \text{with} \qquad \sigma_{yx} = \frac{\mathcal{C}}{2\pi\hbar}$$



$\mathcal{C}$: Chern number (integer) associated with the occupied band

Where we stand (2)

Topology and Berry curvature



Periodic lattice: Bloch theorem

Hamiltonian eigenstates: $\mathrm{e}^{\mathrm{i}\boldsymbol{q}\cdot\boldsymbol{r}}\, u_{\boldsymbol{q}}(\boldsymbol{r})$ $\qquad u_{\boldsymbol{q}}$: **periodic**

The Bloch momentum ***q*** is chosen in the Brillouin zone

Berry connection: $\boldsymbol{\mathcal{A}}_{\boldsymbol{q}} = \mathrm{i}\,\langle u_{\boldsymbol{q}}\,|\,\boldsymbol{\nabla}_{\boldsymbol{q}} u_{\boldsymbol{q}}\rangle$ real vector in the plane (q_x, q_y)

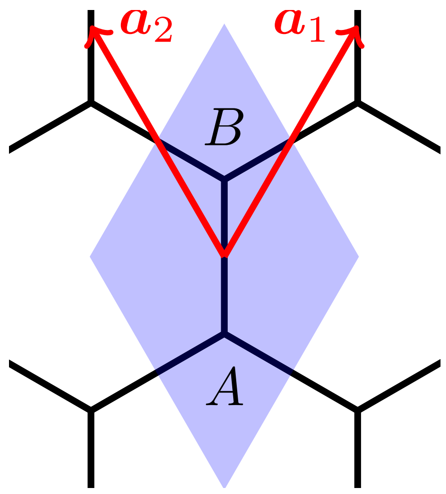
Berry curvature: $\boldsymbol{\Omega}_{\boldsymbol{q}} = \boldsymbol{\nabla}_{\boldsymbol{q}} \times \boldsymbol{\mathcal{A}}_{\boldsymbol{q}}$ oriented along *z* (gauge invariant)

The Chern number reads: $$\mathcal{C} = \frac{1}{2\pi}\iint_{\mathrm{ZB}} \Omega_{\boldsymbol{q}}\ \mathrm{d}^2 q$$

How to obtain $\Omega_{\boldsymbol{q}} \neq 0$ or (even better) $\mathcal{C} \neq 0$?

Where we stand (3)

Tight-binding regime for a bi-partite lattice



Example of a graphene-type lattice, with two sites/ unit cell
Periodic function on the lattice:

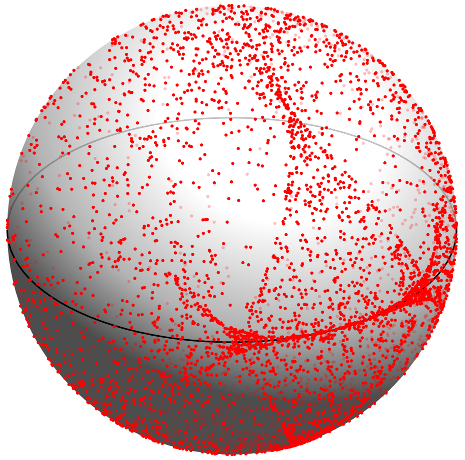
$$|u_{\boldsymbol{q}}\rangle = \alpha_{\boldsymbol{q}} \left(\sum_{\boldsymbol{j}} |A_{\boldsymbol{j}}\rangle\right) + \beta_{\boldsymbol{q}} \left(\sum_{\boldsymbol{j}} |B_{\boldsymbol{j}}\rangle\right) = \begin{pmatrix} \alpha_{\boldsymbol{q}} \\ \beta_{\boldsymbol{q}} \end{pmatrix}$$

pseudospin 1/2

Periodic Hamiltonian for $|u_{\boldsymbol{q}}\rangle$: 2 x 2 matrix

$$\hat{H}_{\boldsymbol{q}} = E_0(\boldsymbol{q})\hat{1} - \boldsymbol{h}(\boldsymbol{q}) \cdot \hat{\boldsymbol{\sigma}}$$

- Energies : $E_0 \pm |\boldsymbol{h}|$
- Eigenstates determined by $\boldsymbol{n} = \dfrac{\boldsymbol{h}}{|\boldsymbol{h}|}$



$$\mathcal{C} = -\frac{1}{4\pi} \iint_{\mathrm{ZB}} \boldsymbol{n} \cdot \left[(\partial_{q_x}\boldsymbol{n}) \times (\partial_{q_y}\boldsymbol{n})\right] \, \mathrm{d}q_x \, \mathrm{d}q_y$$

Wrapping number of the sphere

First goal for today

→ Study an example of a topological lattice — Haldane 1988

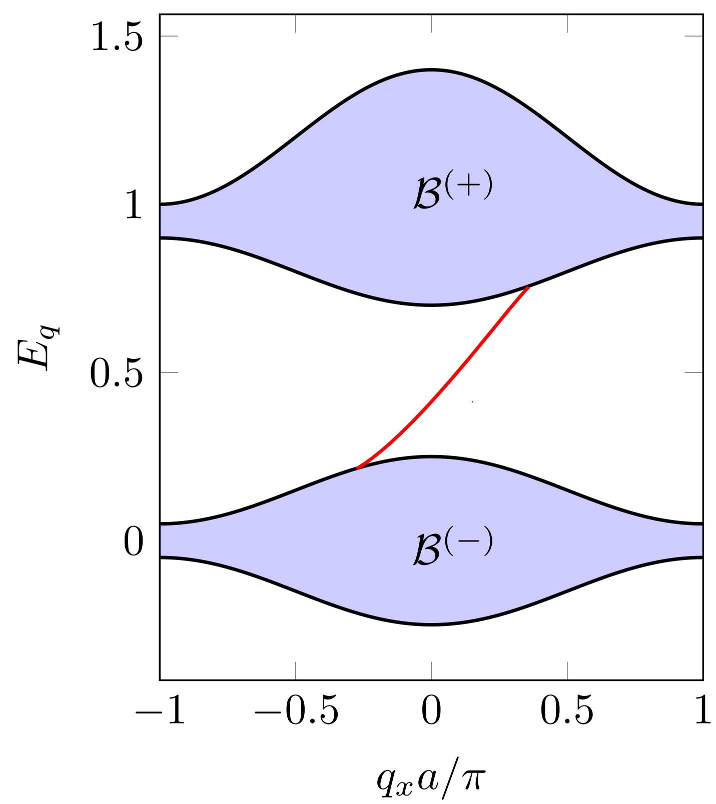
Tight-binding, two-site unit cell: "enriched" graphene

→ Implementation of Haldane model using a time-modulated Hamiltonian

Effective Hamiltonian for a fast modulation
Zurich experiment (cold atoms)

→ Link between topology and edge channels

Finite-size systems
Technion experiment (photonic)



1.

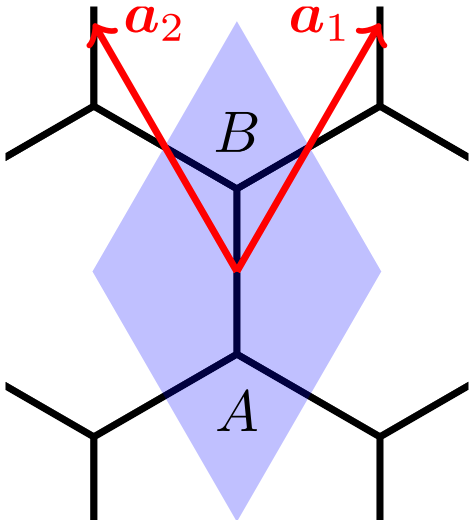
The Haldane model

F. D. M. Haldane

Phys. Rev. Lett. 61, 2015 (1988)

Model for a Quantum Hall Effect without Landau Levels :
Condensed-Matter Realization of the "Parity Anomaly"

The hexagonal lattice (reminder)



Same energy for A and B sites: $E_A = E_B = 0$

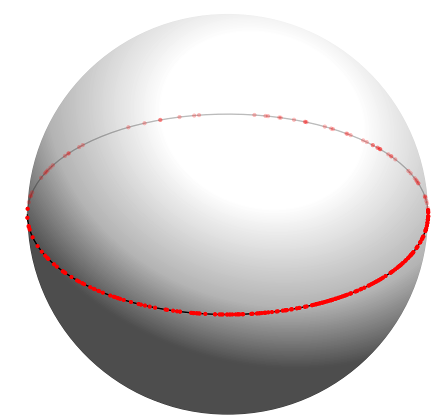
Only nearest-neighbor couplings:

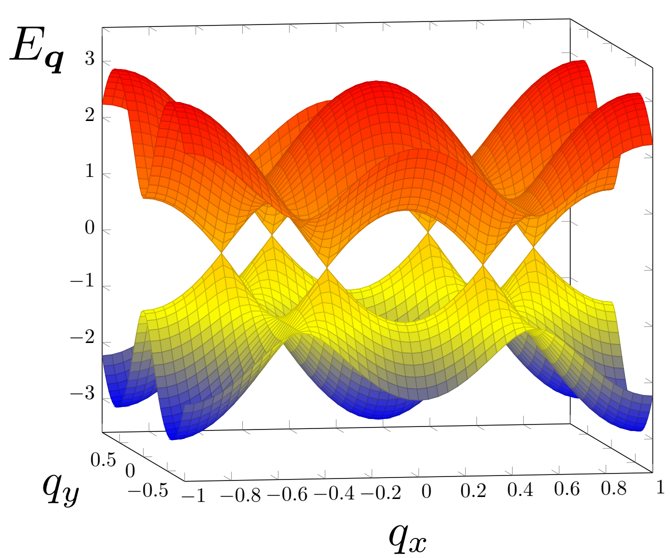
The A site is coupled to three B sites

The B site is coupled to three A sites

$$\hat{H}_{\boldsymbol{q}} = -J \begin{pmatrix} 0 & 1 + \mathrm{e}^{-\mathrm{i}\boldsymbol{q}\cdot\boldsymbol{a}_1} + \mathrm{e}^{-\mathrm{i}\boldsymbol{q}\cdot\boldsymbol{a}_2} \\ 1 + \mathrm{e}^{\mathrm{i}\boldsymbol{q}\cdot\boldsymbol{a}_1} + \mathrm{e}^{\mathrm{i}\boldsymbol{q}\cdot\boldsymbol{a}_2} & 0 \end{pmatrix} = -\boldsymbol{h}(\boldsymbol{q}) \cdot \hat{\boldsymbol{\sigma}}$$

with: $\boldsymbol{h}(\boldsymbol{q}) = J \begin{pmatrix} 1 + \cos(\boldsymbol{q}\cdot\boldsymbol{a}_1) + \cos(\boldsymbol{q}\cdot\boldsymbol{a}_2) \\ \sin(\boldsymbol{q}\cdot\boldsymbol{a}_1) + \sin(\boldsymbol{q}\cdot\boldsymbol{a}_2) \\ 0 \end{pmatrix}$





Singularity at Dirac points such that

$$1 + \mathrm{e}^{\mathrm{i}\boldsymbol{q}\cdot\boldsymbol{a}_1} + \mathrm{e}^{\mathrm{i}\boldsymbol{q}\cdot\boldsymbol{a}_2} = 0$$

How to enrich the graphene lattice

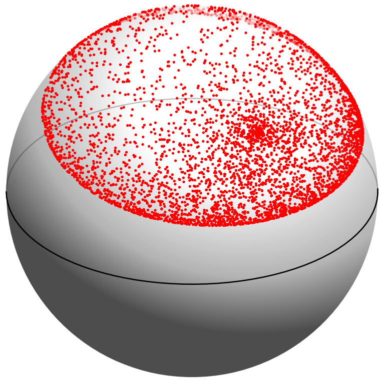
We look for a Hamiltonian $\hat{H}_{\boldsymbol{q}} = E_0(\boldsymbol{q})\hat{1} - \boldsymbol{h}(\boldsymbol{q}) \cdot \hat{\boldsymbol{\sigma}}$ where $\boldsymbol{h}(\boldsymbol{q})$ fully wraps the Bloch sphere

First strategy:
disymmetrize the A and B sites by allowing them to have different energies

$$E_A = -\Delta \qquad\qquad E_B = +\Delta$$

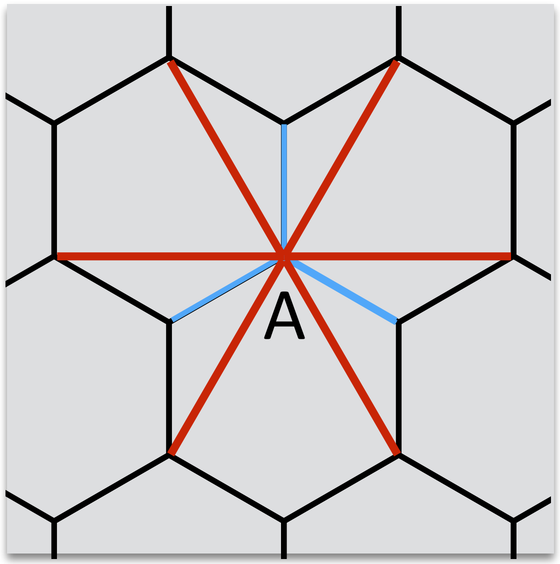
$$\hat{H}_{\boldsymbol{q}} = -\begin{pmatrix} \Delta & h_x(\boldsymbol{q}) - \mathrm{i}h_y(\boldsymbol{q}) \\ h_x(\boldsymbol{q}) + \mathrm{i}h_y(\boldsymbol{q}) & -\Delta \end{pmatrix} \qquad h_z(\boldsymbol{q}) = \Delta$$

The sign of $h_z(\boldsymbol{q})$ is constant over the Brillouin zone: one covers at most one hemisphere of the Bloch sphere



$[\hat{H}(\boldsymbol{q})]_{AA}$ and $[\hat{H}(\boldsymbol{q})]_{BB}$ must depend on $\boldsymbol{q}$: next-to-nearest neighbor couplings

Next-to-nearest neighbor (NNN) coupling for graphene



Six NNN for a site ***A*** : couplings $A \to A$

Same for ***B***

We also know that we have to break time-reversal symmetry. Otherwise:

$$\Omega_{\boldsymbol{q}} = -\Omega_{-\boldsymbol{q}} \qquad \longrightarrow \qquad \mathcal{C} = \frac{1}{2\pi} \iint_{\mathrm{ZB}} \Omega_{\boldsymbol{q}} \, \mathrm{d}^2 q = 0$$

Tunnel coupling $A \to A$ and $B \to B$ with a complex matrix element

Graphical representation: $\boxed{A_1} \xrightarrow{J' \, \mathrm{e}^{\mathrm{i}\phi_0}} \boxed{A_2}$

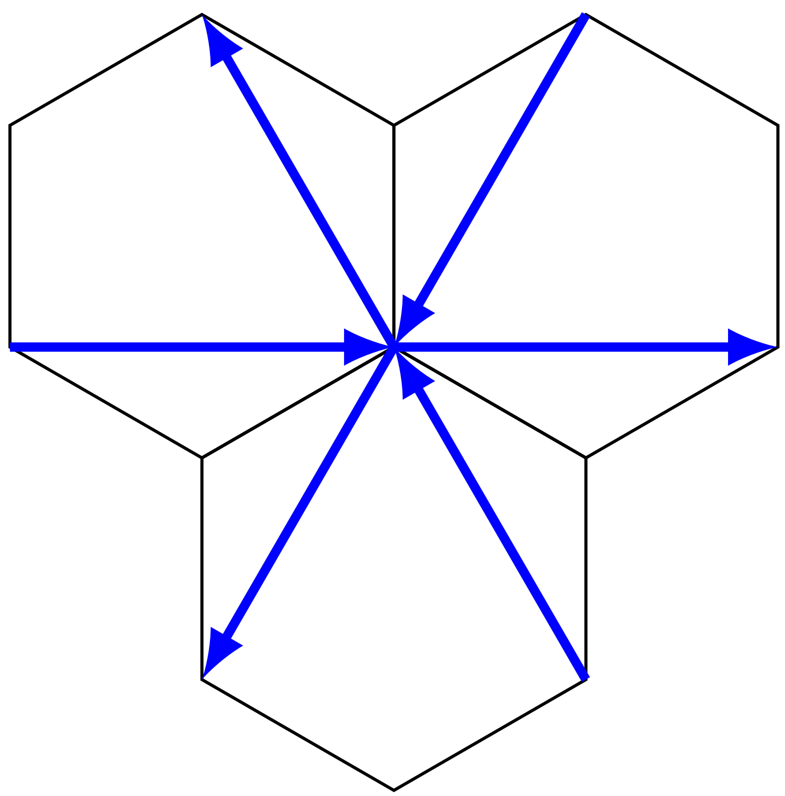
$$-J' \left(\mathrm{e}^{+\mathrm{i}\phi_0} |A_2\rangle\langle A_1| \; + \; \mathrm{e}^{-\mathrm{i}\phi_0} |A_1\rangle\langle A_2| \right)$$

Choice for the NNN coupling

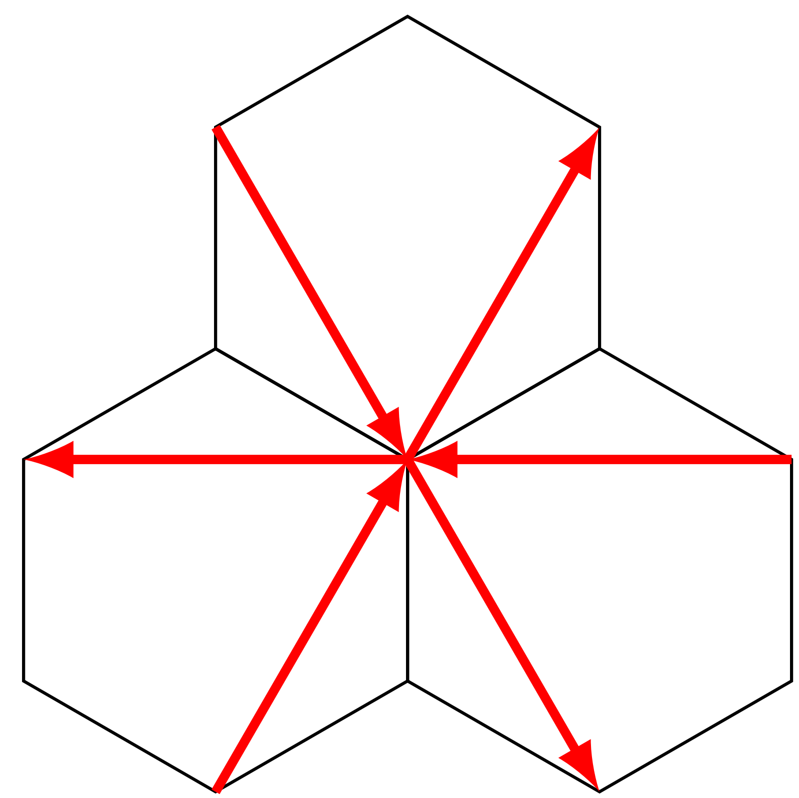
$$A_1 \xrightarrow{J'\, \mathrm{e}^{\mathrm{i}\phi_0}} A_2$$

$$-J' \left(\mathrm{e}^{+\mathrm{i}\phi_0} |A_2\rangle\langle A_1| \; + \; \mathrm{e}^{-\mathrm{i}\phi_0} |A_1\rangle\langle A_2| \right)$$

Coupling of site *A* to its six NNN: $\mathrm{e}^{\pm\mathrm{i}\phi_0}$

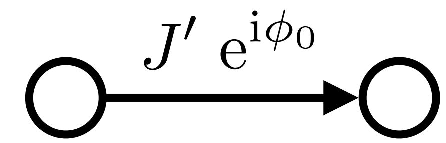


Coupling of site *B* to its six NNN: $\mathrm{e}^{\pm\mathrm{i}\phi_0}$



Keep the initial periodicity with the same unit cell

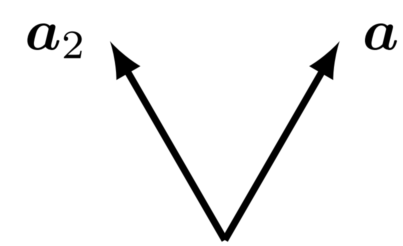
Haldane Hamiltonian



- On-site energies $$\hat{H}_{0,\boldsymbol{q}} = \begin{pmatrix} -\Delta & 0 \\ 0 & \Delta \end{pmatrix}$$

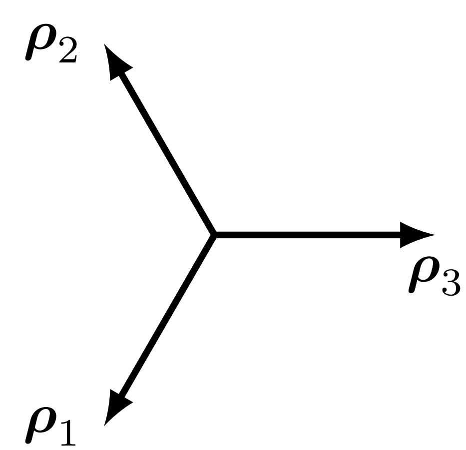
- The three couplings to nearest neighbors:

$$\hat{H}_{1,\boldsymbol{q}} = -J \begin{pmatrix} 0 & 1 + \mathrm{e}^{-\mathrm{i}\boldsymbol{q}\cdot\boldsymbol{a}_1} + \mathrm{e}^{-\mathrm{i}\boldsymbol{q}\cdot\boldsymbol{a}_2} \\ 1 + \mathrm{e}^{\mathrm{i}\boldsymbol{q}\cdot\boldsymbol{a}_1} + \mathrm{e}^{\mathrm{i}\boldsymbol{q}\cdot\boldsymbol{a}_2} & 0 \end{pmatrix}$$



- The six additional couplings to NNN: $J' \, \mathrm{e}^{\pm \mathrm{i}\phi_0}$

$$\hat{H}_{2,\boldsymbol{q}} = -J' \begin{pmatrix} \sum_{\alpha=1}^{3} 2\cos(\boldsymbol{q}\cdot\boldsymbol{\rho}_\alpha - \phi_0) & 0 \\ 0 & \sum_{\alpha=1}^{3} 2\cos(\boldsymbol{q}\cdot\boldsymbol{\rho}_\alpha + \phi_0) \end{pmatrix}$$



Haldane Hamiltonian and Bloch sphere

$$\bigcirc \xrightarrow{J'\,\mathrm{e}^{\mathrm{i}\phi_0}} \bigcirc$$

Canonical form for a (pseudo-) spin 1/2:

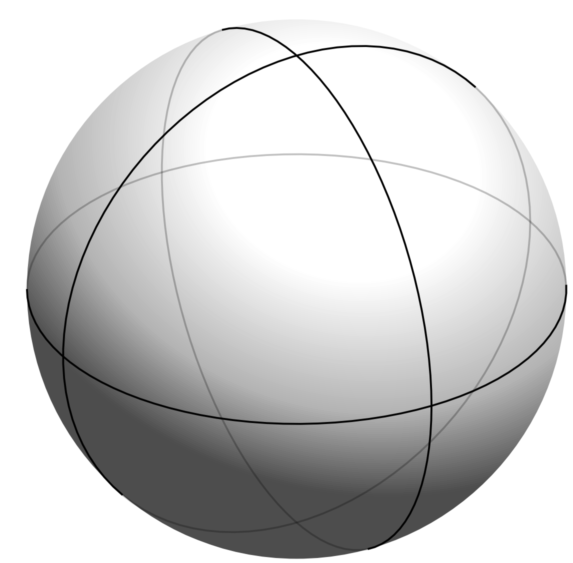
$$\hat{H}_{\boldsymbol{q}} = E_0(\boldsymbol{q})\,\hat{1} - \boldsymbol{h}(\boldsymbol{q}) \cdot \hat{\boldsymbol{\sigma}}$$

with:

$$\left.\begin{aligned} h_x(\boldsymbol{q}) &= J\,[1 + \cos(\boldsymbol{q}\cdot\boldsymbol{a}_1) + \cos(\boldsymbol{q}\cdot\boldsymbol{a}_2)] \\ h_y(\boldsymbol{q}) &= J\,[\sin(\boldsymbol{q}\cdot\boldsymbol{a}_1) + \sin(\boldsymbol{q}\cdot\boldsymbol{a}_2)] \end{aligned}\right\} \text{\textit{unchanged with respect to graphene}}$$

$$h_z(\boldsymbol{q}) = \Delta + 2J'\sin\phi_0\,\mathcal{S}_{\boldsymbol{q}} \qquad \text{avec} \quad \mathcal{S}_{\boldsymbol{q}} = \sum_{\alpha=1}^{3} \sin(\boldsymbol{q}\cdot\boldsymbol{\rho}_\alpha)$$

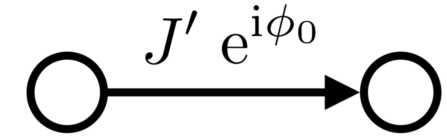
Necessary condition to wrap the Bloch sphere: reach the poles



$h_x(\boldsymbol{q}) = h_y(\boldsymbol{q}) = 0$: Dirac points location

$h_z(\boldsymbol{q}) > 0$: North pole $\qquad$ $h_z(\boldsymbol{q}) < 0$: South pole

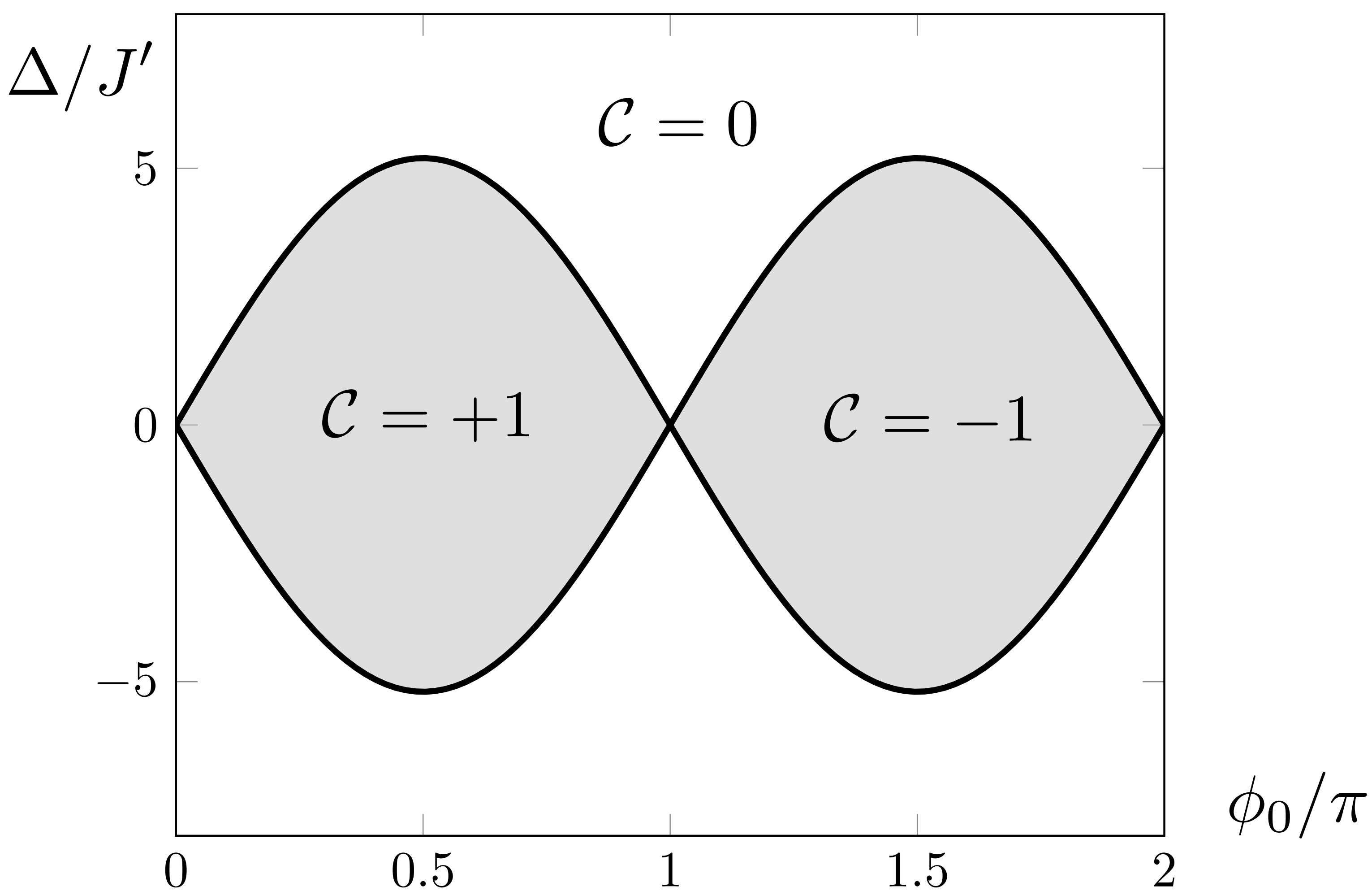
Phase diagram for Haldane model



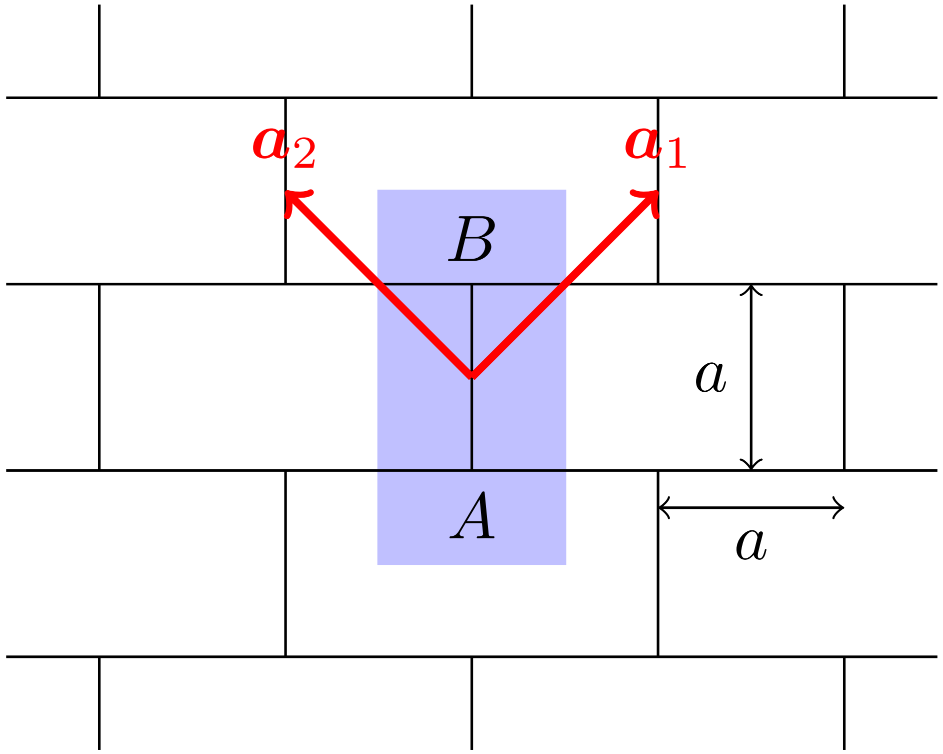
The condition "reach the poles" is actually sufficient to ensure the full wrapping of the Bloch sphere

$$h_z(\boldsymbol{q}) = \Delta + 2J' \sin\phi_0 \, \mathcal{S}_{\boldsymbol{q}}$$ with at the Dirac points $$\mathcal{S}_{\boldsymbol{q}} = \pm\frac{3\sqrt{3}}{2}$$

Therefore it is sufficient that $\Delta \pm 3\sqrt{3}J' \sin\phi_0$ have opposite signs



A clone of the graphene lattice: the brick-wall lattice

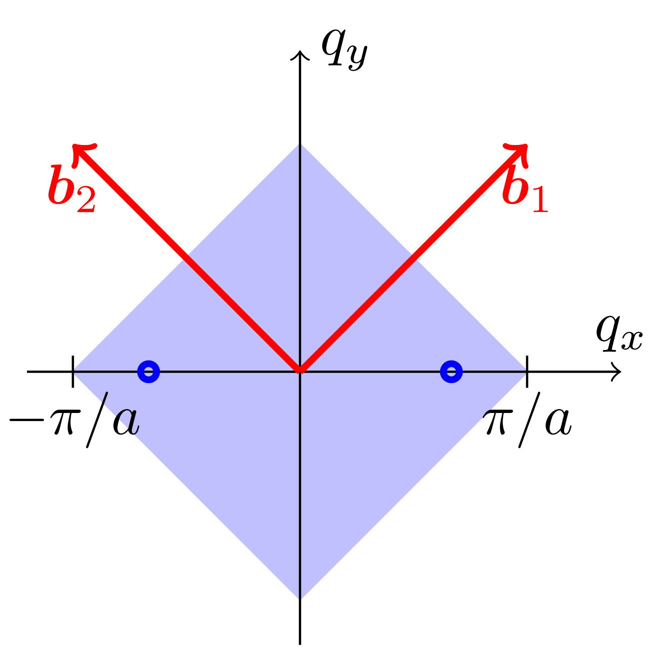


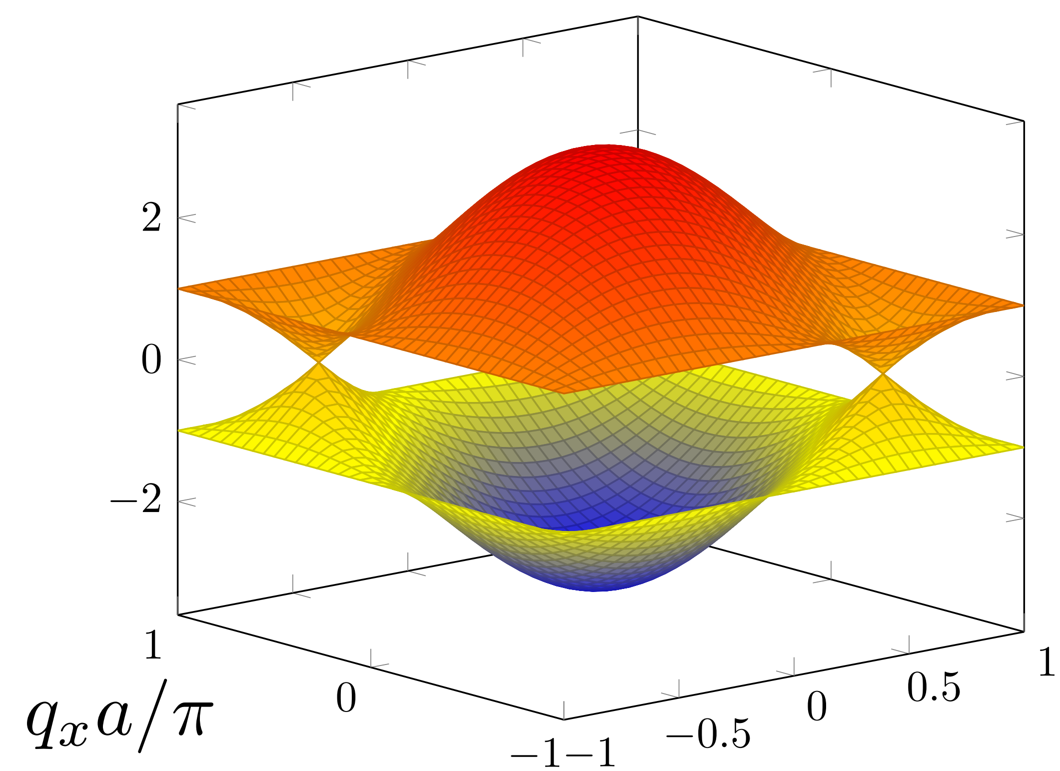
Simpler calculations

Can be achieved with laser standing waves at right angles

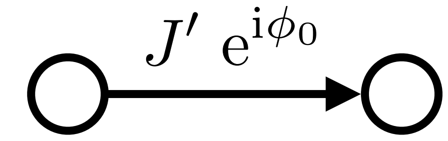
Zurich, 2012

Square Brillouin zone with two Dirac points as for graphene



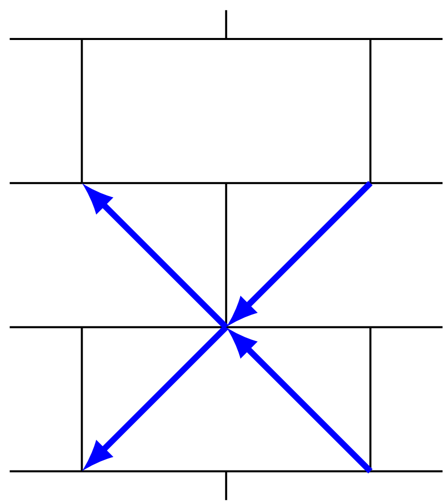


Phase diagram for the brick-wall lattice

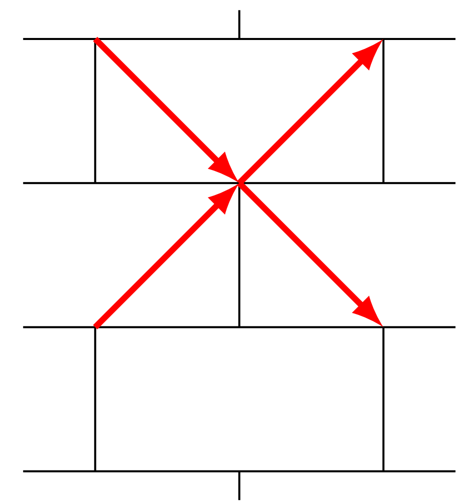


Same structure as for the Haldane model, with 4 NNN instead of 6

site A :



site B :



Non topological example

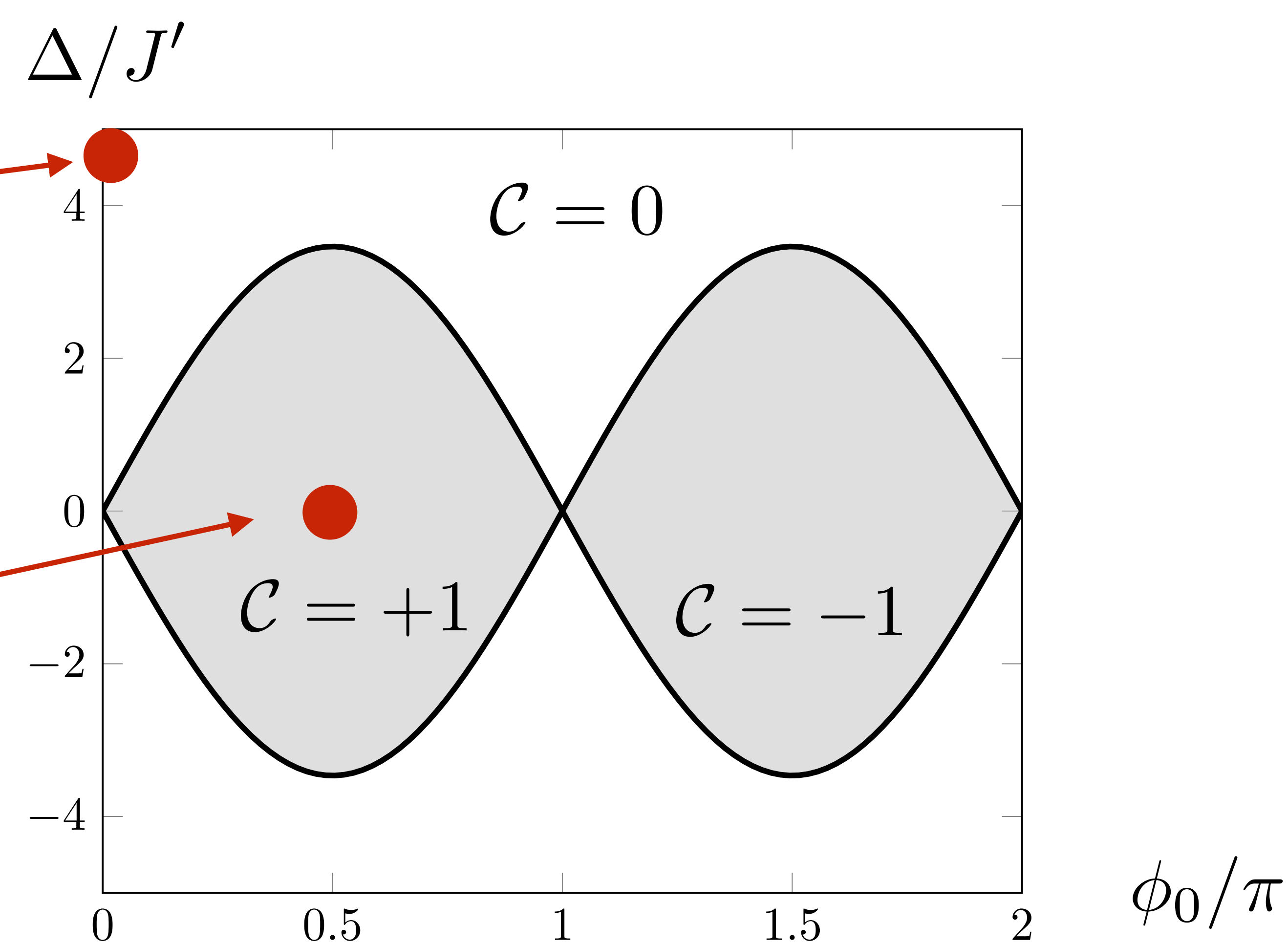
$\phi_0 = 0$

$\Delta \gg J'$

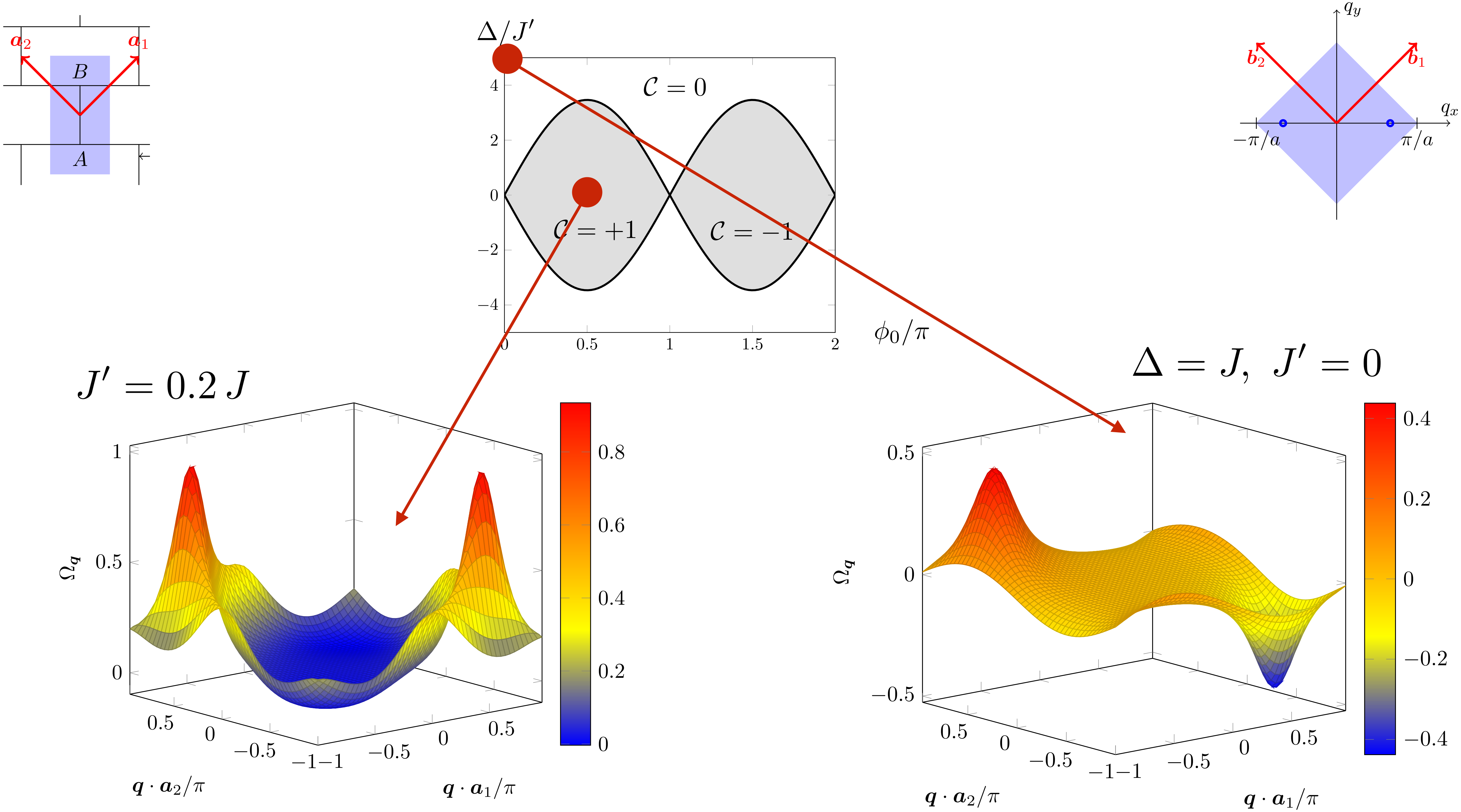
Topological example

$\phi_0 = \pi/2$

$\Delta = 0$



Berry curvature of the brick-wall lattice



$$\mathcal{C} = \frac{1}{2\pi} \iint_{\mathrm{ZB}} \Omega_{\boldsymbol{q}}\, \mathrm{d}^2 q = 1$$

$$\mathcal{C} = 0$$

Berry curvature in the tight-binding regime

Unit cell with n sites

- One finds n energy bands $E_{\boldsymbol{q}}^{(j)},\ j=1,\ldots,n$
- One calculate Berry curvature for each band $\boldsymbol{\Omega}_{\boldsymbol{q}}^{(j)},\ j=1,\ldots,n$

One can then show:

$$\sum_{j=1}^{n}\boldsymbol{\Omega}_{\boldsymbol{q}}^{(j)}=0$$ for each $\boldsymbol{q}$ and thus: $$\sum_{j=1}^{n}\mathcal{C}^{(j)}=0$$

For a unit cell with two sites: $\mathcal{C}^{(+)}=-\mathcal{C}^{(-)}$

Hint for the proof:

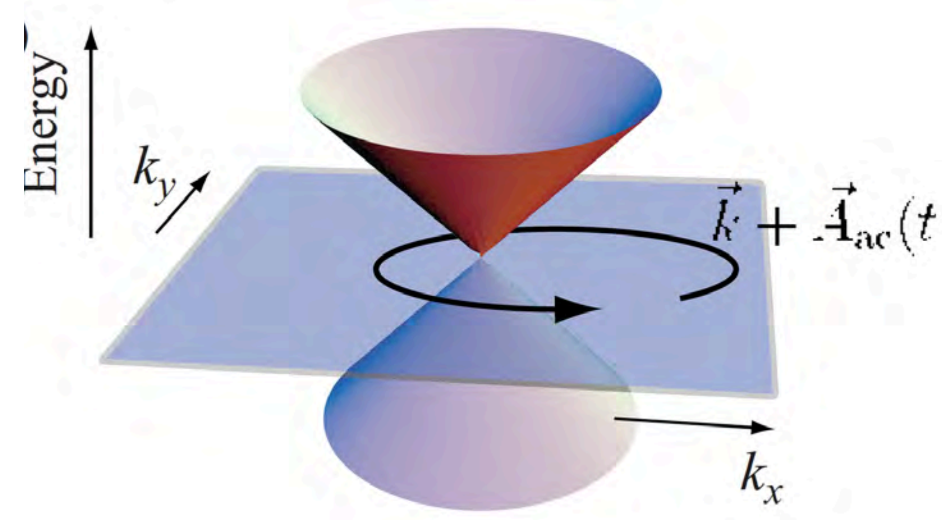
use $$\boldsymbol{\Omega}_{\boldsymbol{q}}^{(j)}=\mathrm{i}\sum_{j'\neq j}\frac{\langle u_{\boldsymbol{q}}^{(j)}|\,\boldsymbol{\nabla}_{\boldsymbol{q}}\hat{H}_{\boldsymbol{q}}\,|u_{\boldsymbol{q}}^{(j')}\rangle\ \times\ \langle u_{\boldsymbol{q}}^{(j')}|\,\boldsymbol{\nabla}_{\boldsymbol{q}}\hat{H}_{\boldsymbol{q}}\,|u_{\boldsymbol{q}}^{(j)}\rangle}{\left(E_{\boldsymbol{q}}^{(j)}-E_{\boldsymbol{q}}^{(j')}\right)^{2}}$$

2.

Time-modulation and topology

T. Oka & H. Aoki
Phys. Rev. B 79, 081406 (2009)
Photovoltaic Hall effect in graphene

Opening of a gap in a graphene sheet using circularly polarized light

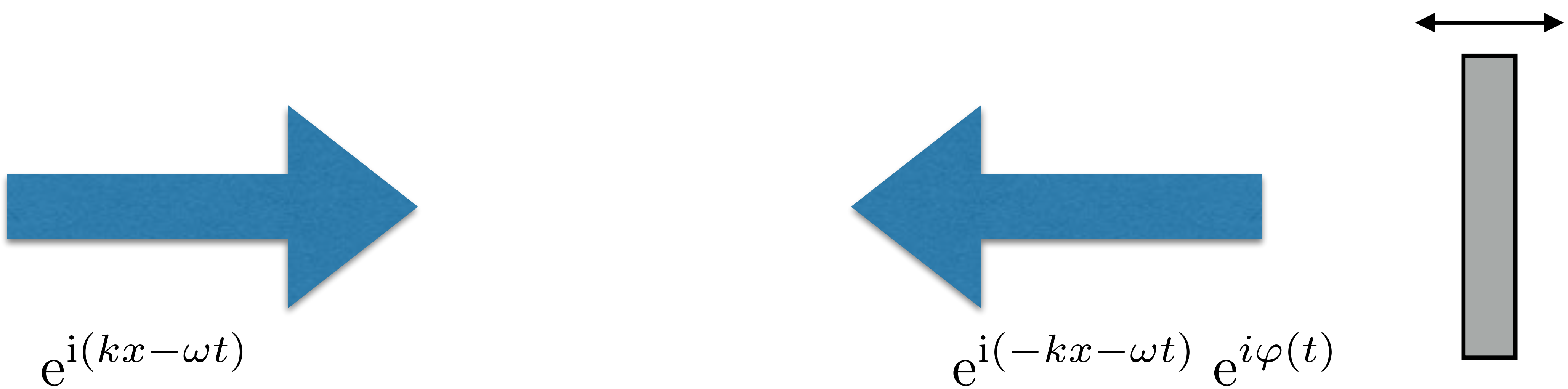


Shaken lattices

How to create a complex tunnel coupling such as $A_1 \xrightarrow{J'\ \mathrm{e}^{\mathrm{i}\phi_0}} A_2$

A possible answer: shake the lattice

For cold atoms, this can be achieved by modulating the phase of the standing waves forming the lattice



lattice nodes in $kx_j(t) = j\pi\ +\ \varphi(t)$

Inertial force in a shaken lattice

In the absence of modulation $$\hat{H}_{\text{rest}} = \sum_\alpha E_\alpha \hat{P}_\alpha - \sum_{\alpha,\beta} J^{(0)}_{\alpha,\beta}\, |\boldsymbol{r}_\alpha\rangle\langle\boldsymbol{r}_\beta|$$

$$\hat{P}_\alpha = |\boldsymbol{r}_\alpha\rangle\langle\boldsymbol{r}_\alpha|$$ $J^{(0)}_{\alpha,\beta}$: real and positive

For a global motion of the lattice with the trajectory $\boldsymbol{\rho}_t$, switch to the moving frame: inertial force on the particles $\boldsymbol{F}_t = -m\ddot{\boldsymbol{\rho}}_t$

Hamiltonian in this frame : $$\hat{H}_t = \hat{H}_{\text{rest}} - \boldsymbol{F}_t \cdot \hat{\boldsymbol{r}}$$

with : $$\hat{\boldsymbol{r}} = \sum_\alpha \boldsymbol{r}_\alpha\, \hat{P}_\alpha$$

Unitary transform (same as in the previous lecture):

$$\hat{U}_t = \exp[-\mathrm{i}\, \hat{\boldsymbol{r}} \cdot \boldsymbol{A}_t] \qquad \boldsymbol{A}_t = \frac{1}{\hbar}\int^t \boldsymbol{F}_{t'}\, \mathrm{d}t' = -\frac{m}{\hbar}\dot{\boldsymbol{\rho}}_t$$

Inertial force in a shaken lattice (2)

Hamiltonian after unitary transform $\hat{U}_t = \exp[-\mathrm{i}\, \hat{\boldsymbol{r}} \cdot \boldsymbol{A}_t]$ with $\boldsymbol{A}_t = -\frac{m}{\hbar}\dot{\boldsymbol{\rho}}_t$

$$\hat{H}_{\text{rest}} = \sum_\alpha E_\alpha |\boldsymbol{r}_\alpha\rangle\langle\boldsymbol{r}_\alpha| - \sum_{\alpha,\beta} J^{(0)}_{\alpha,\beta}\ |\boldsymbol{r}_\alpha\rangle\langle\boldsymbol{r}_\beta|$$

$$\longrightarrow \qquad \hat{\tilde{H}}_t = \sum_\alpha E_\alpha |\boldsymbol{r}_\alpha\rangle\langle\boldsymbol{r}_\alpha| - \sum_{\alpha,\beta} J_{\alpha,\beta}(t)\ |\boldsymbol{r}_\alpha\rangle\langle\boldsymbol{r}_\beta|$$

Effective tunnel coefficients with time-modulation : $J_{\alpha,\beta}(t) = J^{(0)}_{\alpha,\beta}\ \mathrm{e}^{\mathrm{i}m(\boldsymbol{r}_\alpha - \boldsymbol{r}_\beta)\cdot\dot{\boldsymbol{\rho}}/\hbar}$

Modulation of tunneling along a link $\longleftrightarrow$ Vibration of the lattice along this link

Limit of a high-frequency modulation (compared to Bohr frequencies)

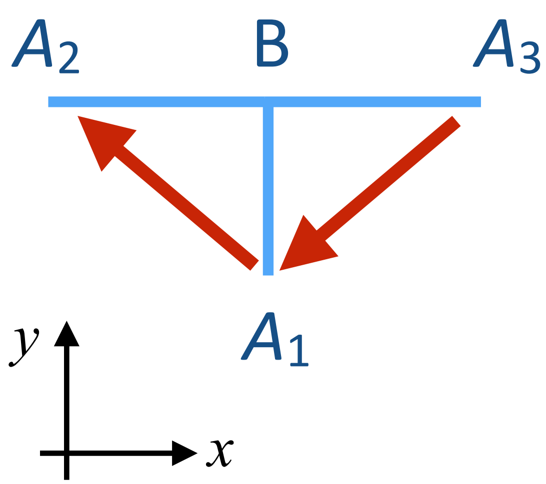
Expand $\hat{\tilde{H}}_t$ in Fourier series: $\hat{H}_t = \hat{H}^{(0)} + \sum_{n>0} \left(\hat{H}^{(n)}\mathrm{e}^{\mathrm{i}n\omega t} + \hat{H}^{(-n)}\mathrm{e}^{-\mathrm{i}n\omega t}\right)$

and keep only terms of order 0 and 1 in the expansion in terms of $1/\omega$:

$$\hat{H}_{\text{eff}} = \hat{H}^{(0)} + \frac{1}{\hbar\omega}\sum_{n>0}\frac{1}{n}\ \left[\hat{H}^{(n)}, \hat{H}^{(-n)}\right] + \mathcal{O}(1/\omega^2)$$

e.g. Goldman & Dalibard 2014

Emergence of a NNN coupling



Couplings in the static lattice: $$-J \sum_{\alpha=1,2,3} |A_\alpha\rangle\langle B| + \mathrm{H.c.}$$

Coupling to A_α with modulation: $$-J\ \mathrm{e}^{-\mathrm{i}\kappa \sin(\omega t+\varphi_\alpha)}|A_\alpha\rangle\langle B| + \mathrm{H.c.}$$

First term in $1/\hbar\omega$ for the effective Hamiltonian : $$\frac{1}{\hbar\omega}\ \left[\hat{H}^{(1)}, \hat{H}^{(-1)}\right]$$

with (among others) : $$\hat{H}^{(1)} = \mathrm{e}^{\mathrm{i}\phi_y}\ |A_1\rangle\langle B| + \ldots$$
$$\hat{H}^{(-1)} = \mathrm{e}^{-\mathrm{i}\phi_x}\ |B\rangle\langle A_3| + \ldots$$

which contributes to the commutator with : $\mathrm{e}^{\mathrm{i}(\phi_y-\phi_x)}\ |A_1\rangle\langle A_3| + \ldots$ **Bingo!**

Optimal choice $\phi_y - \phi_x = \pi/2$ (circular modulation) leading to:

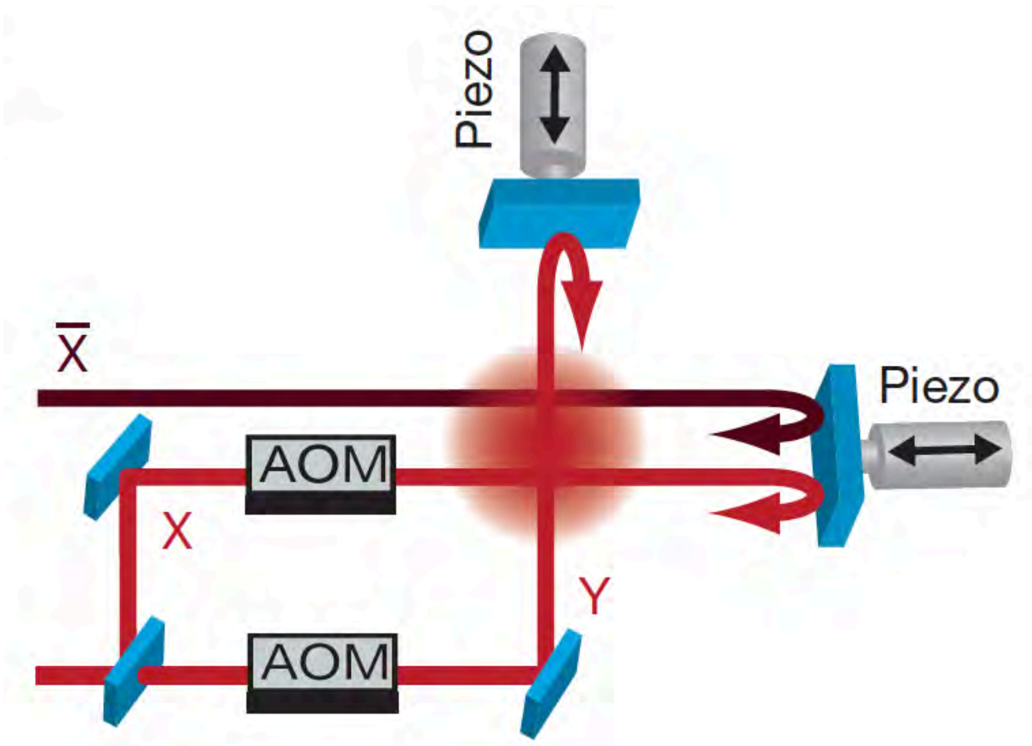
$$-\ \bar{J}'\left(\mathrm{e}^{+\mathrm{i}\pi/2}|A_2\rangle\langle A_1| + \mathrm{e}^{+\mathrm{i}\pi/2}|A_1\rangle\langle A_3|\right)\ +\ \mathrm{H.c.}$$

$$\bar{J}' = 2\frac{J^2}{\hbar\omega}\,\mathcal{J}_1^2(\kappa)$$

same for B sites

$$\kappa = ma\dot{\rho}_0/\hbar$$

The Zurich experiment

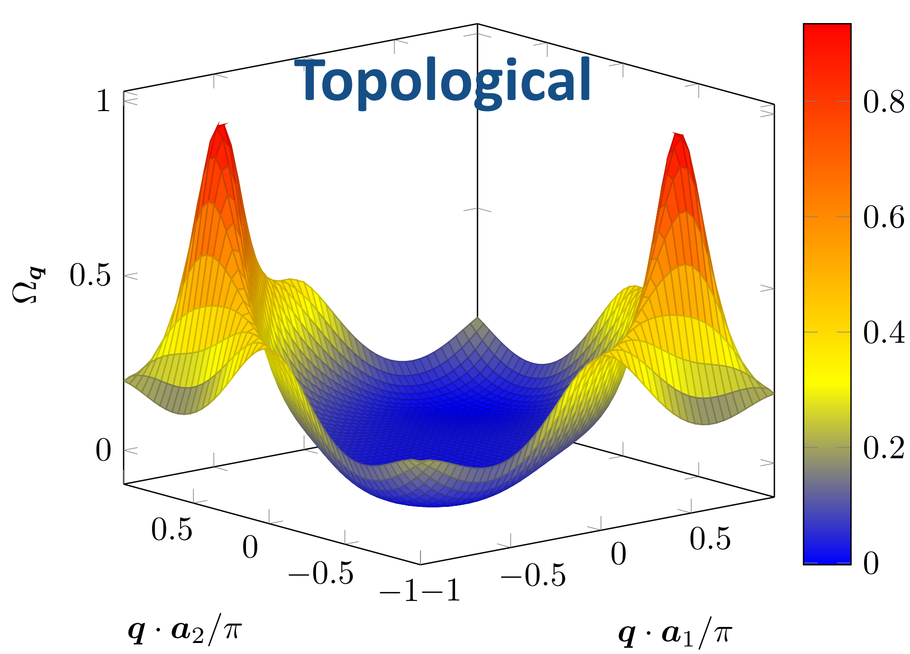


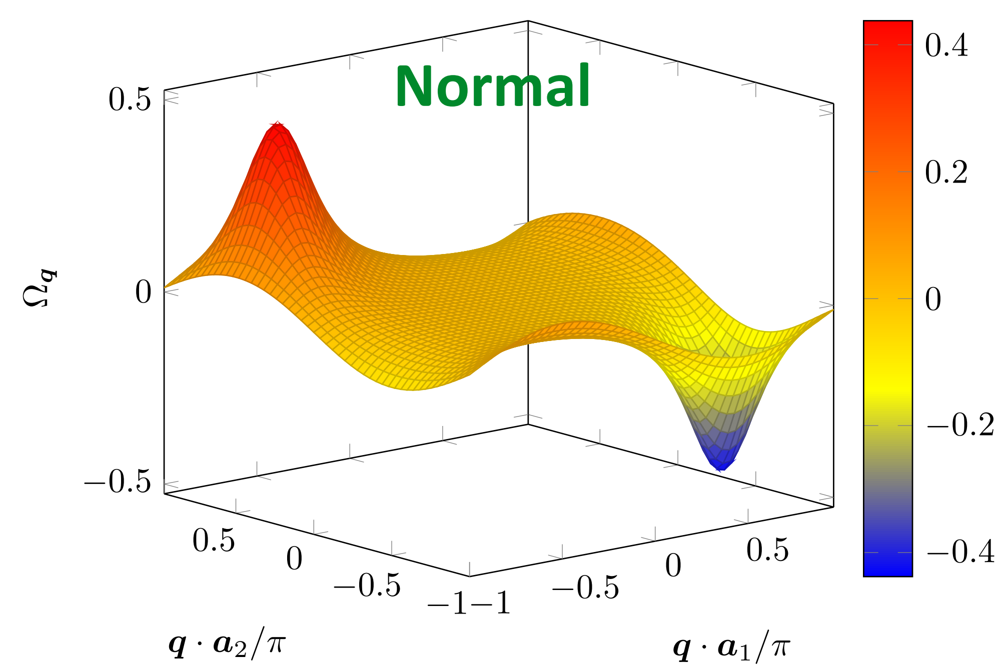
G. Jotzu et al., Nature 515, 237 (2014)

Brick-wall lattice where one controls the on-site energy $\pm\Delta$

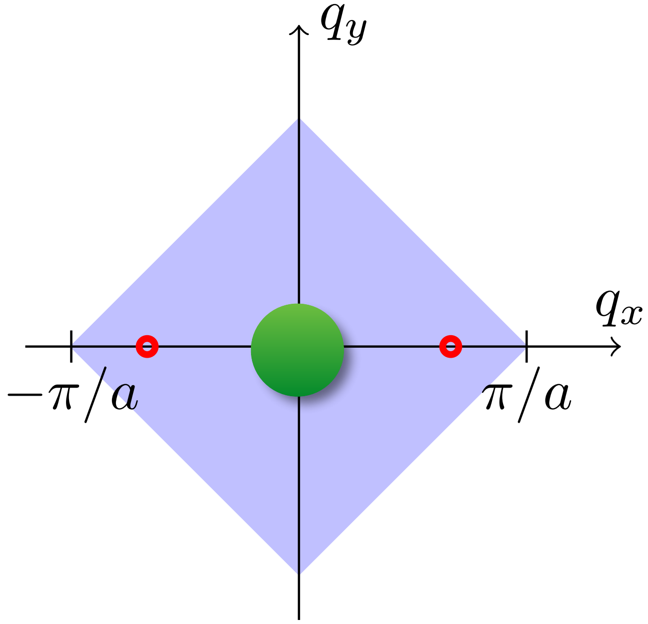
^{40}K atoms (fermions) prepared in a wave packet at the center of the Brillouin zone of the lowest band

Idea: probe the topology using the response to an external force





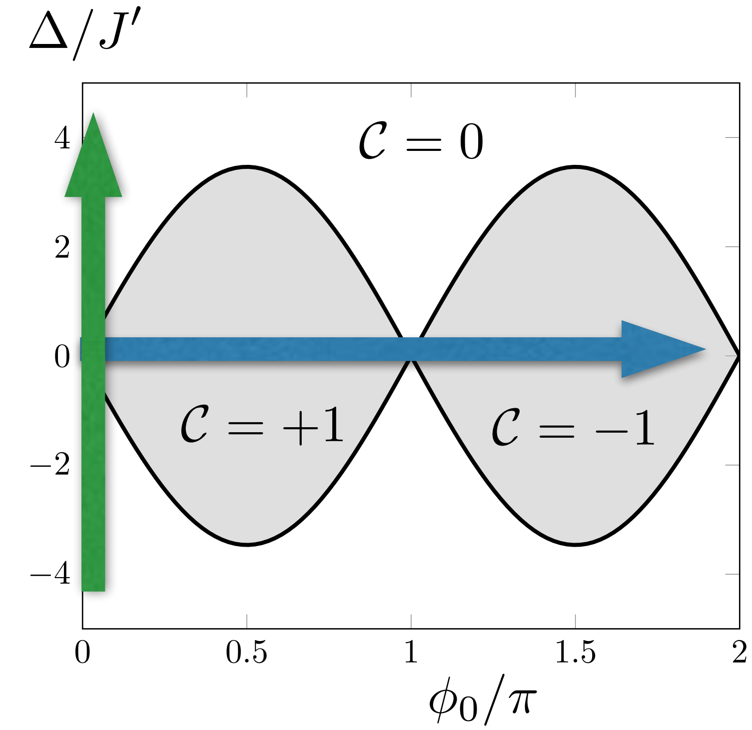
The Zurich experiment (2)



$$\hbar\,\frac{\mathrm{d}\boldsymbol{q}}{\mathrm{d}t} = \boldsymbol{F}$$

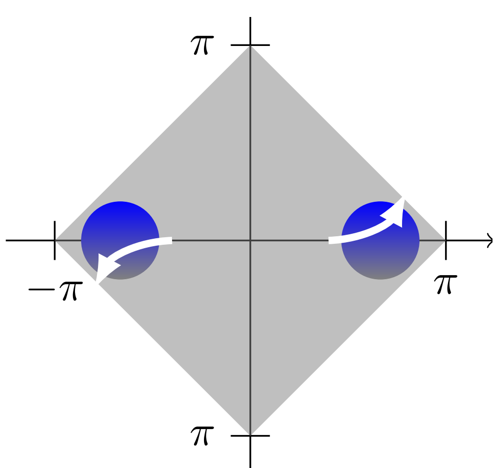
$$\hbar\boldsymbol{v} \;=\; \boldsymbol{\nabla}_{\boldsymbol{q}} E^{(0)}_{\boldsymbol{q}} + \boldsymbol{\Omega}_{\boldsymbol{q}} \times \boldsymbol{F}$$

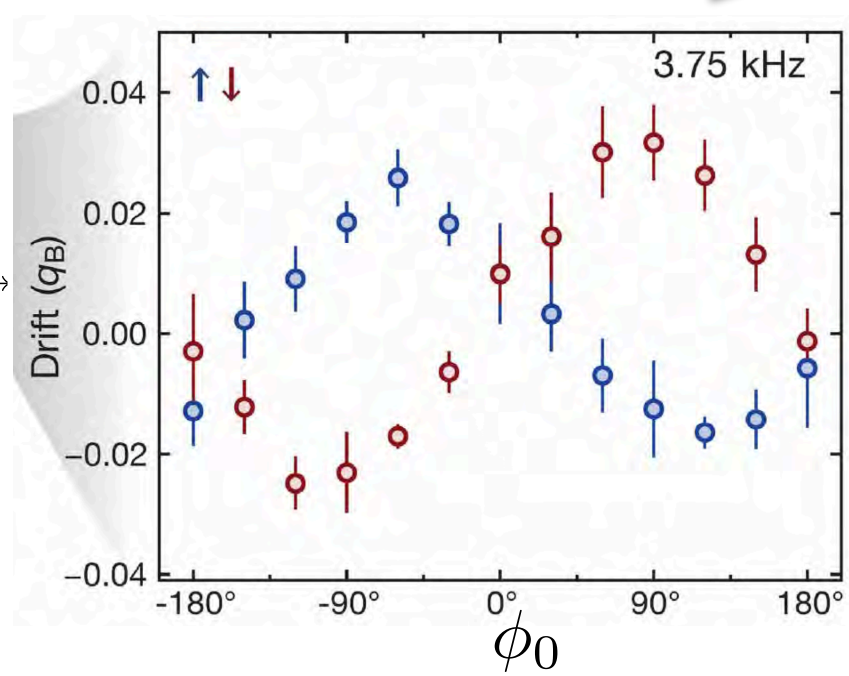
Group velocity   *Anomalous velocity*



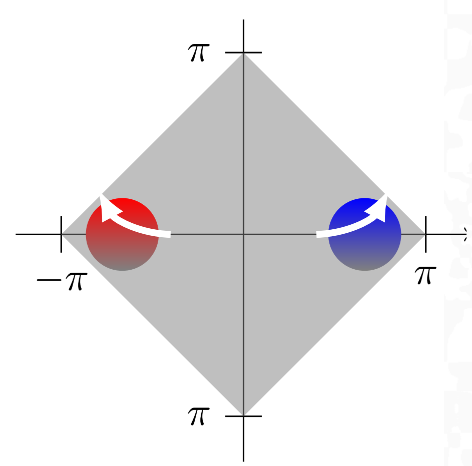
Magnetic gradient creating a force $\pm F_x$; what is the drift along y?

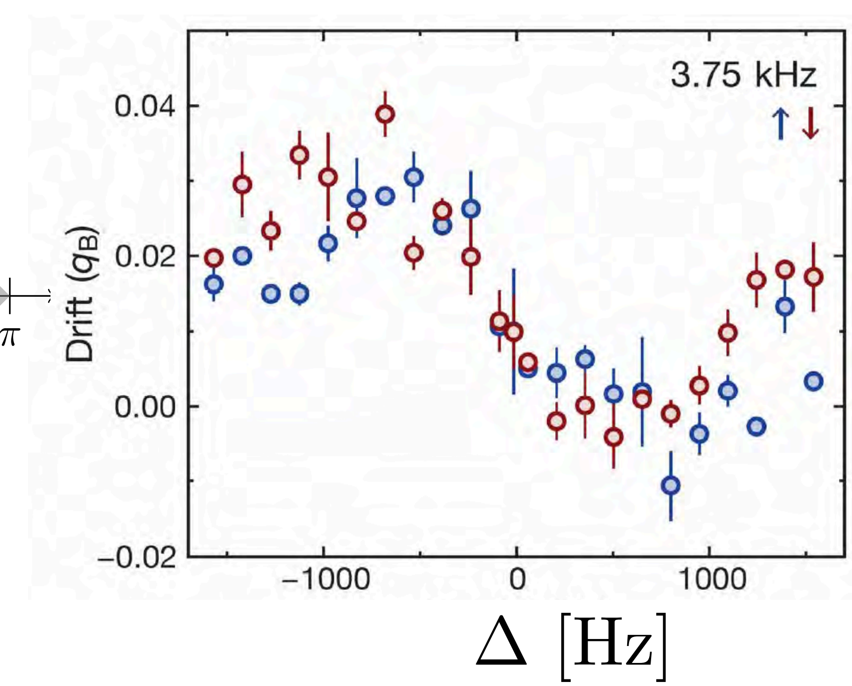
Topological





Normal



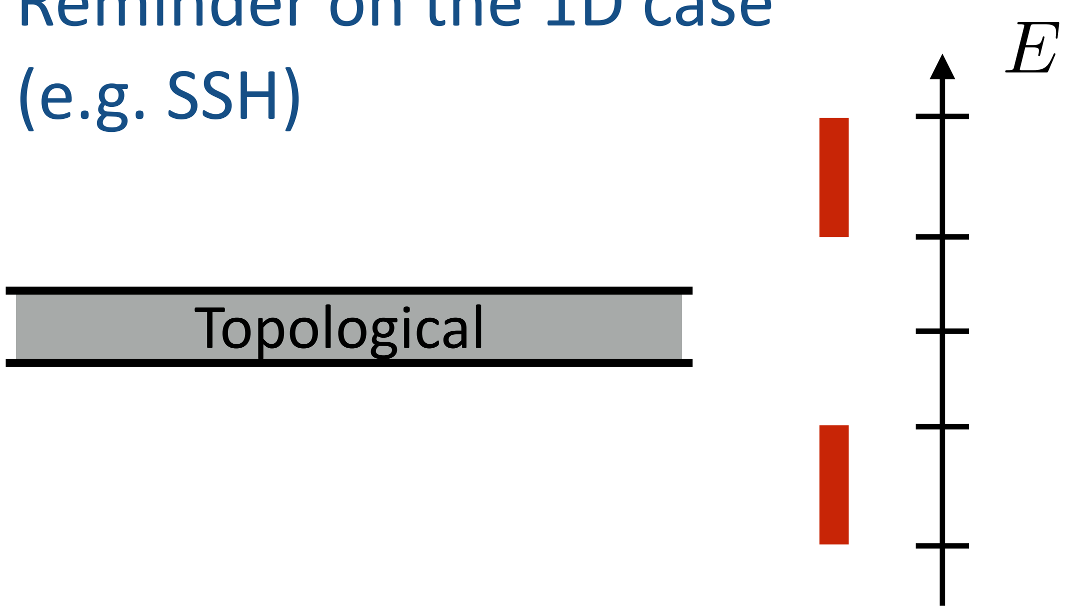


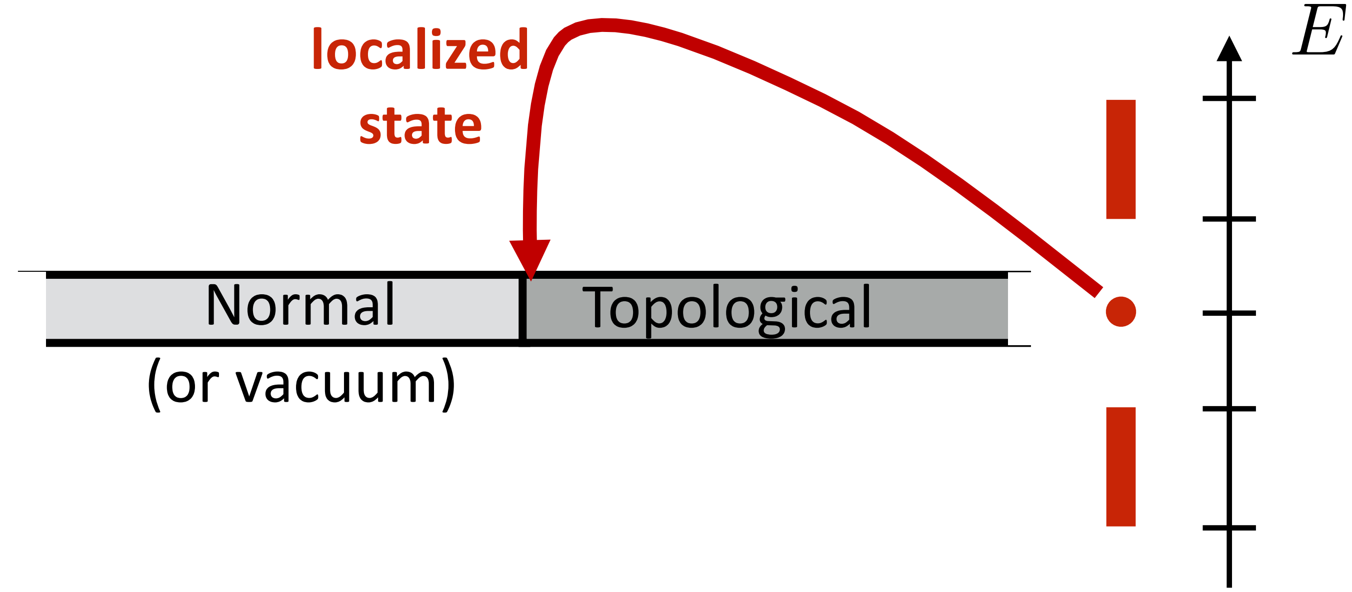
3.

Edge channels

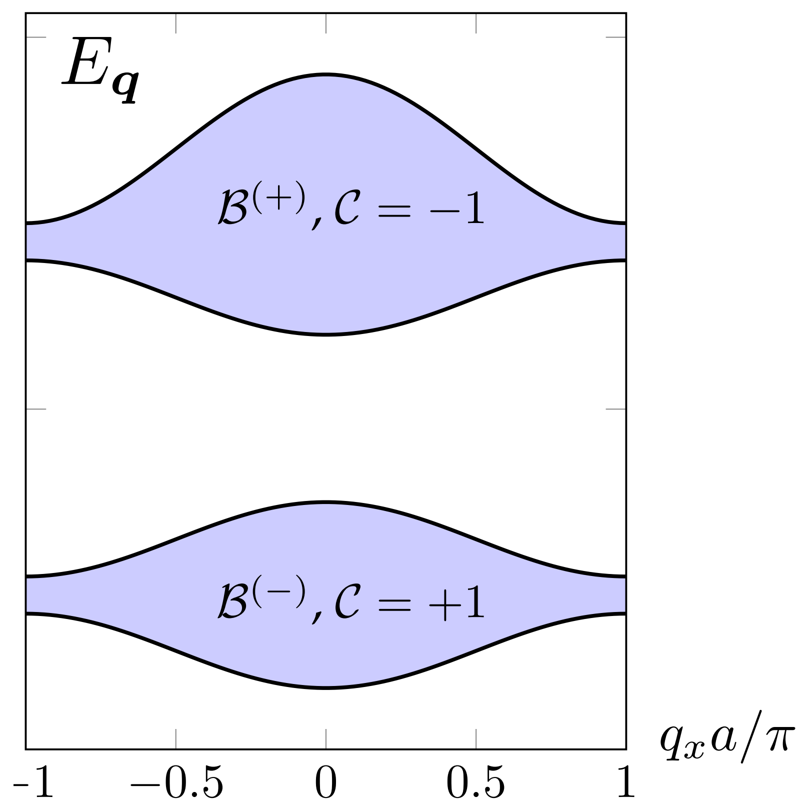
Edges states: 1D vs. 2D

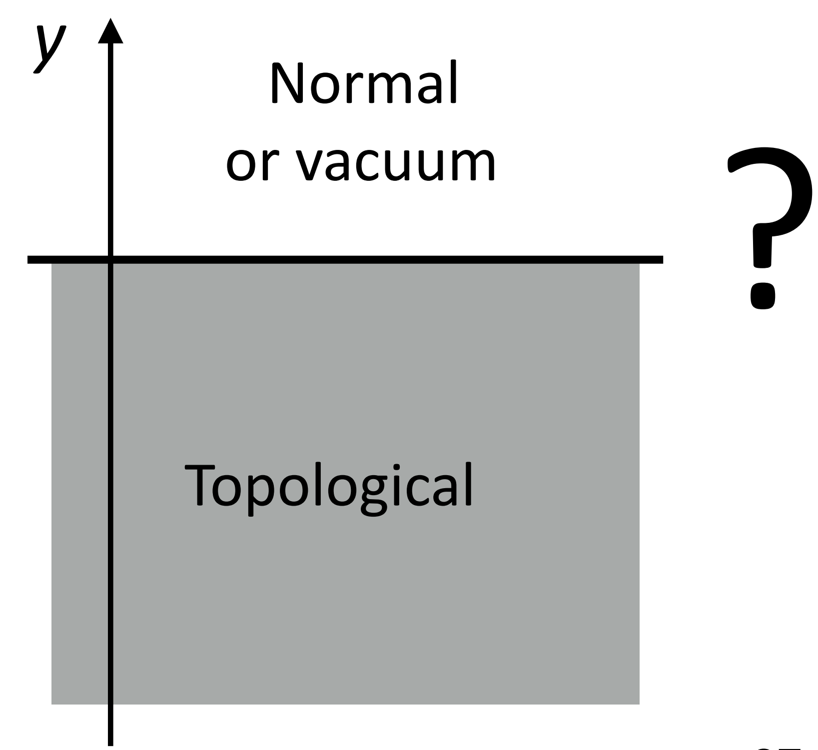
Reminder on the 1D case
(e.g. SSH)



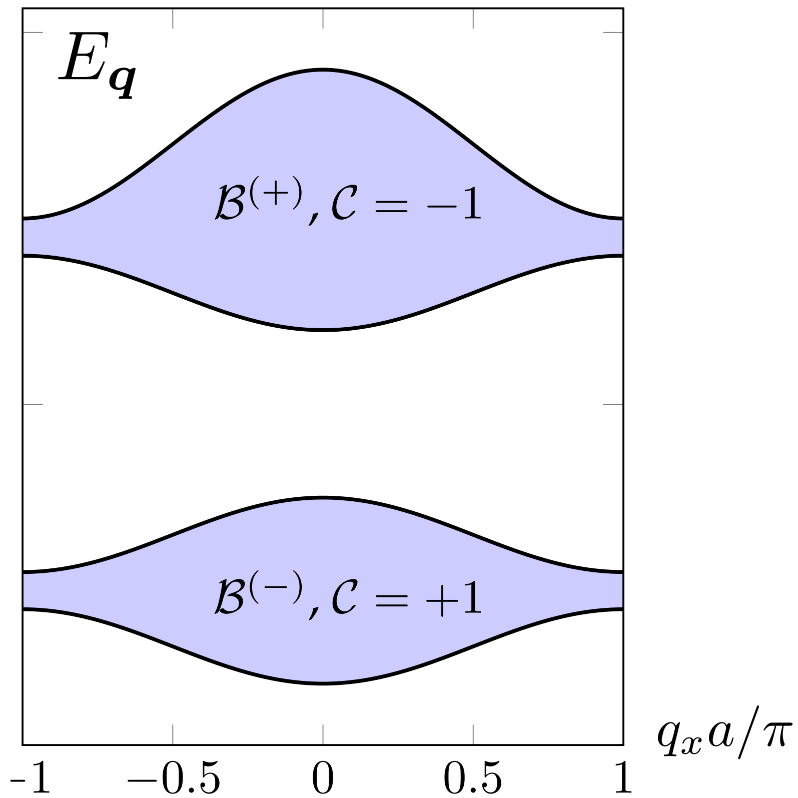


Switch to 2D and
assume that the lowest
band is topological
(for example Haldane)

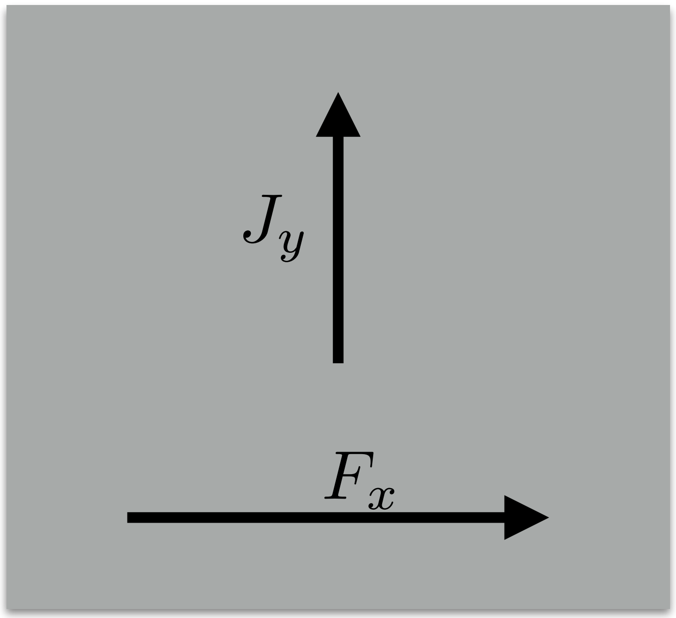




Why an edge state must exist

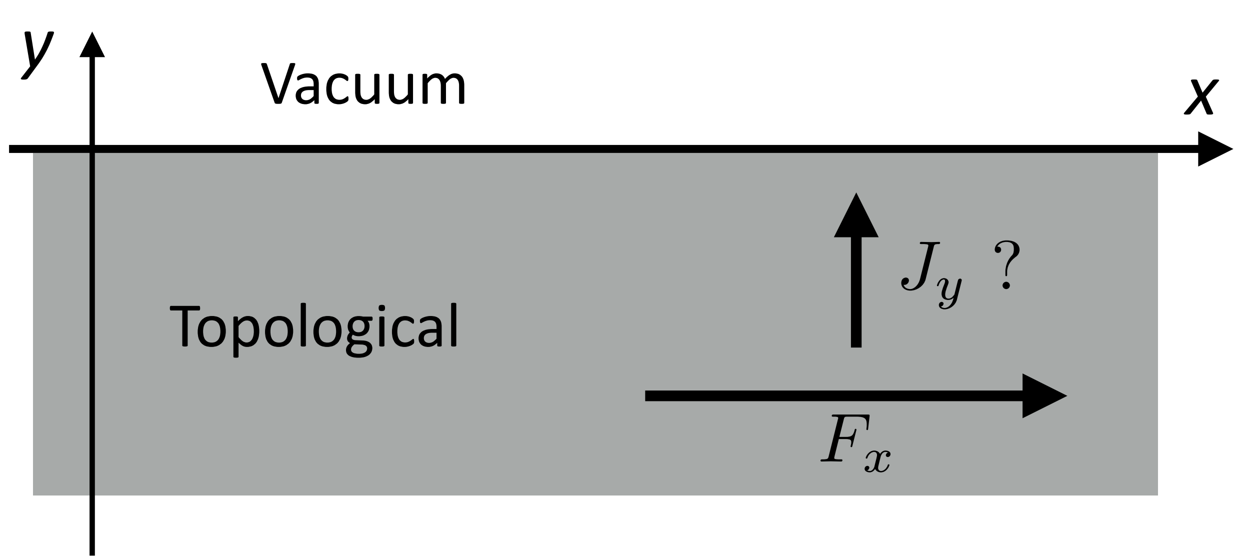


For an infinite plan and a filled band



$$\dot{q}_x = \frac{1}{\hbar} F_x$$

For a half-plane, we keep translation invariance along the x axis:
Bloch theorem is still valid along this direction

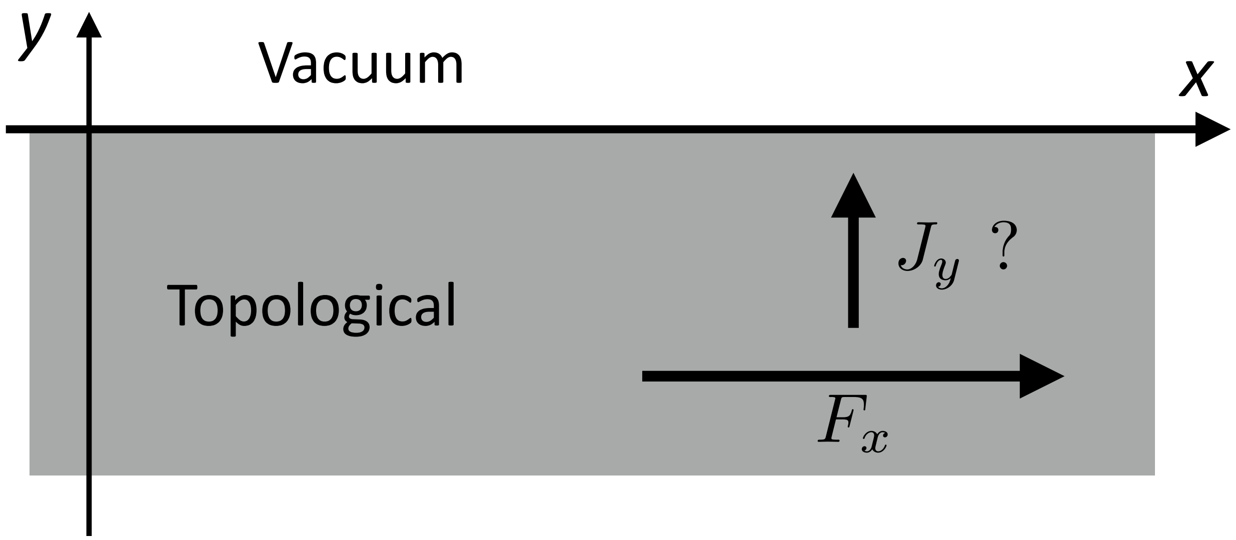


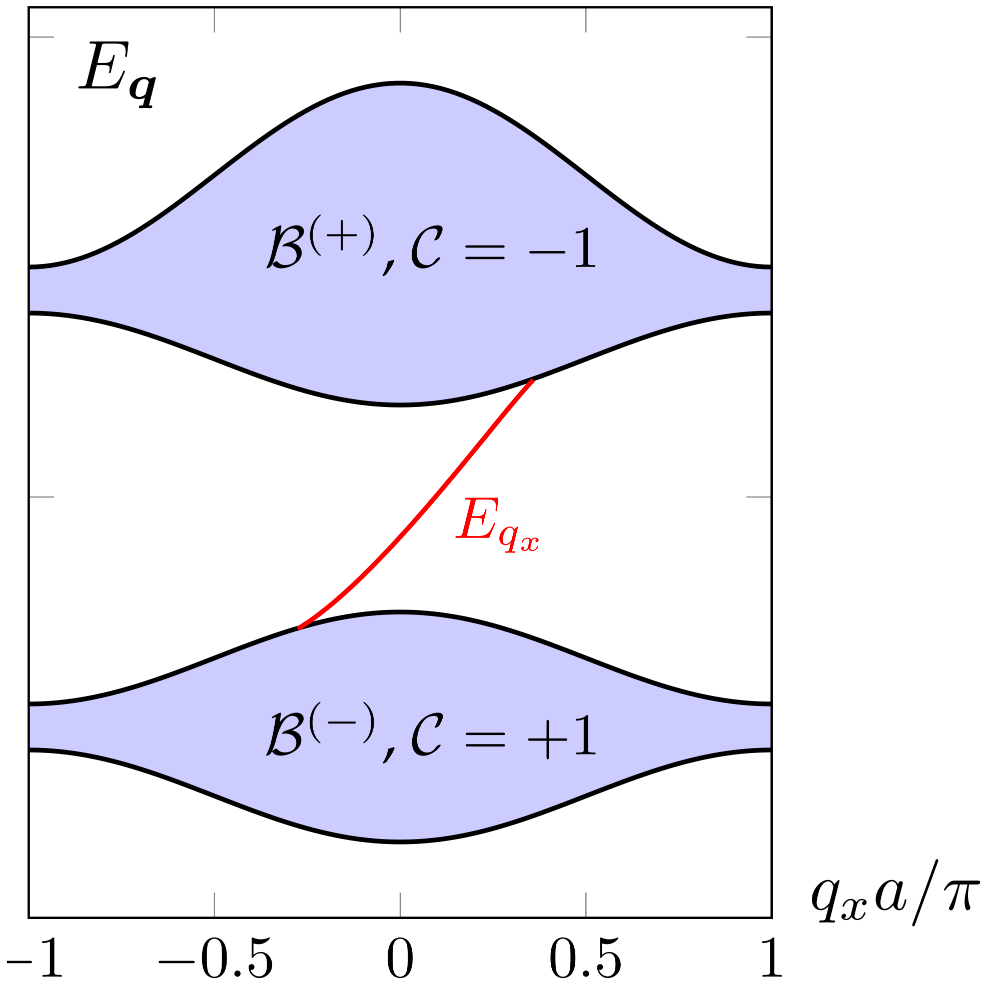
Is there a current along *y* ?

If this is the case, how can one avoid the accumulation of particles on the line *y*=0 ?

Why an edge state must exist (2)

$$\dot{q}_x = \frac{1}{\hbar} F_x$$





The edge channel creates a detour to send particles in the upper band. Once in the upper band, the particles travel back since

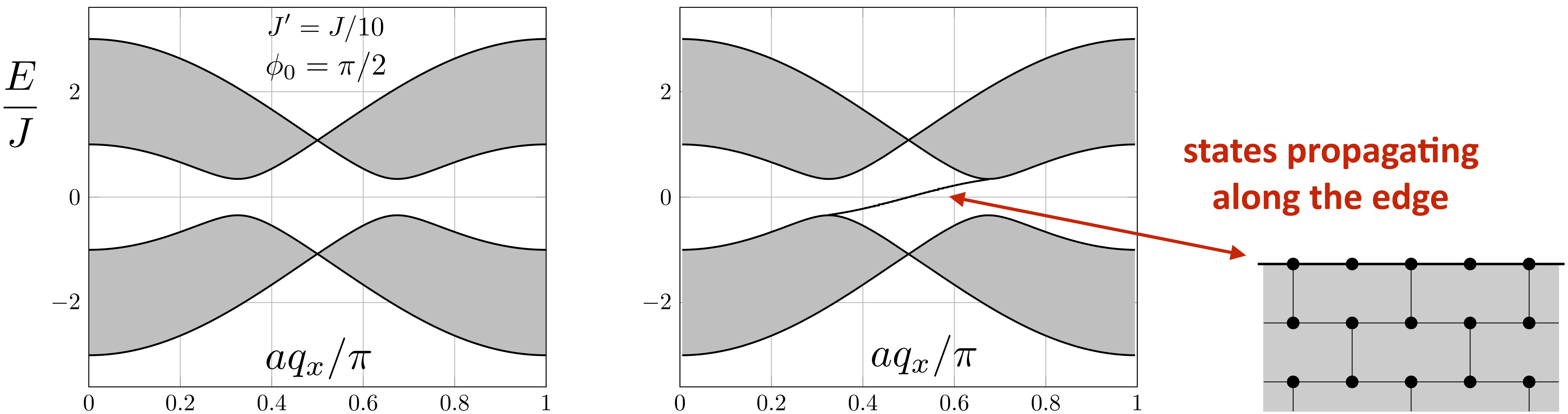
$$\mathcal{C} = -1 \;\Rightarrow\; J_y < 0$$

Robust channel as long as the two bands do not touch

Simple example of the bulk-edge correspondence

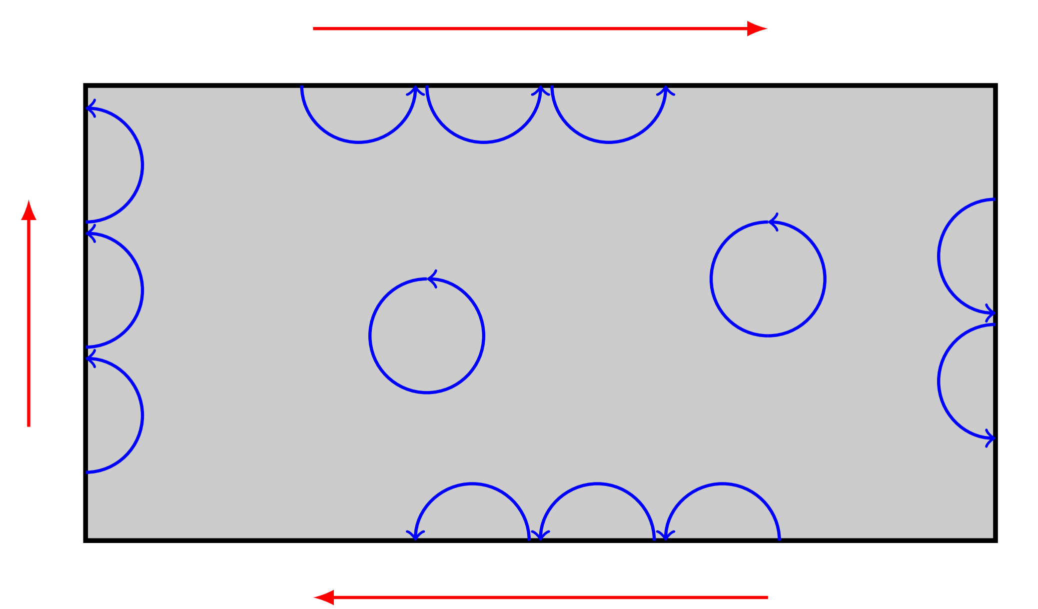
Edge channels and skipping orbits

Plane vs. **half-plane** for the brick-wall lattice (topological case)



Single mode edge channel with the group velocity : $\dfrac{1}{\hbar}\,\dfrac{\mathrm{d}E^{(\mathrm{edge})}_{q_x}}{\mathrm{d}q_x}$

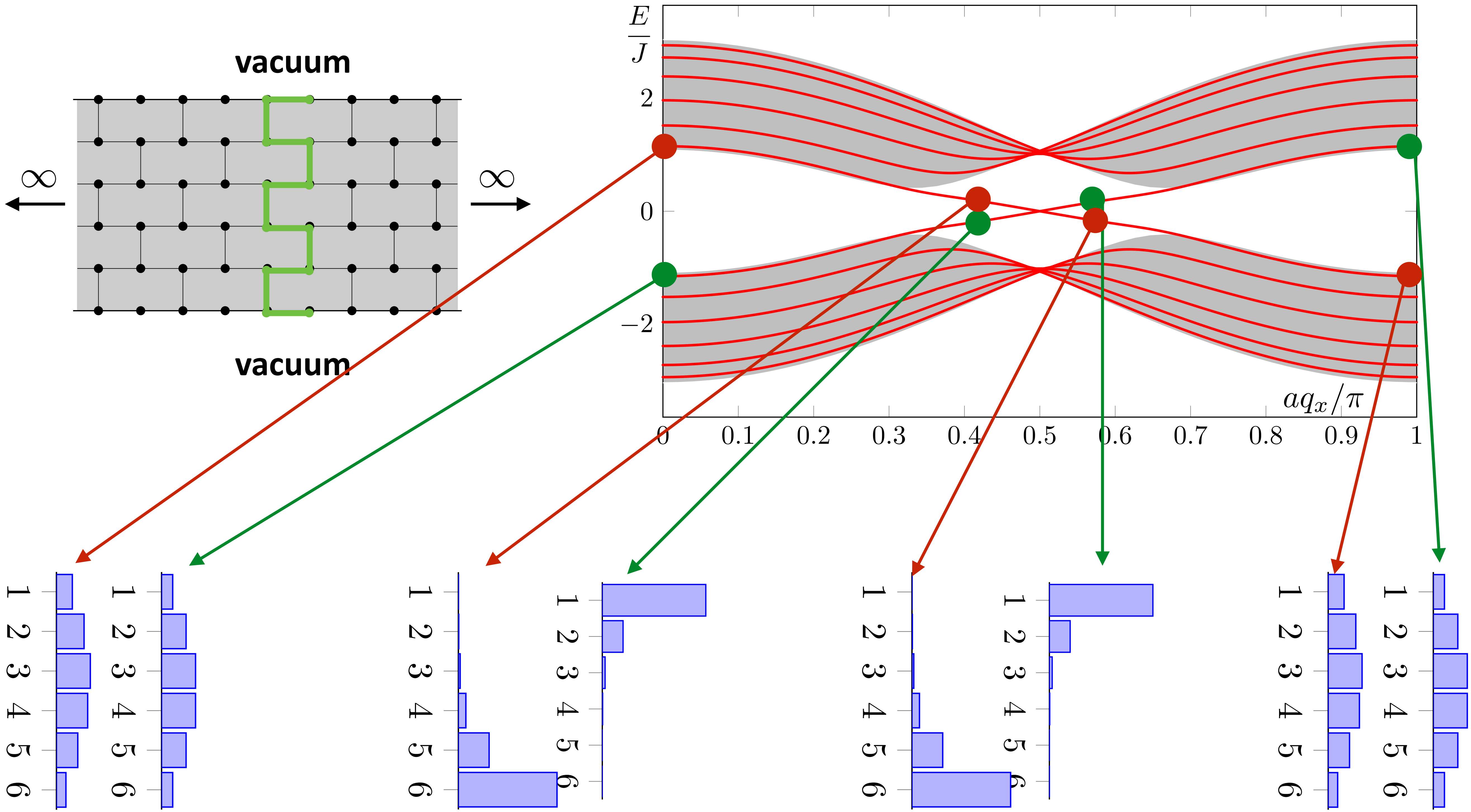
Equivalent of the skipping orbits for the quantum Hall effect



Edge states for a ribbon

Infinite ribbon along x with six lines along y

Diagonalisation of a 12x12 matrix for each value of q_x

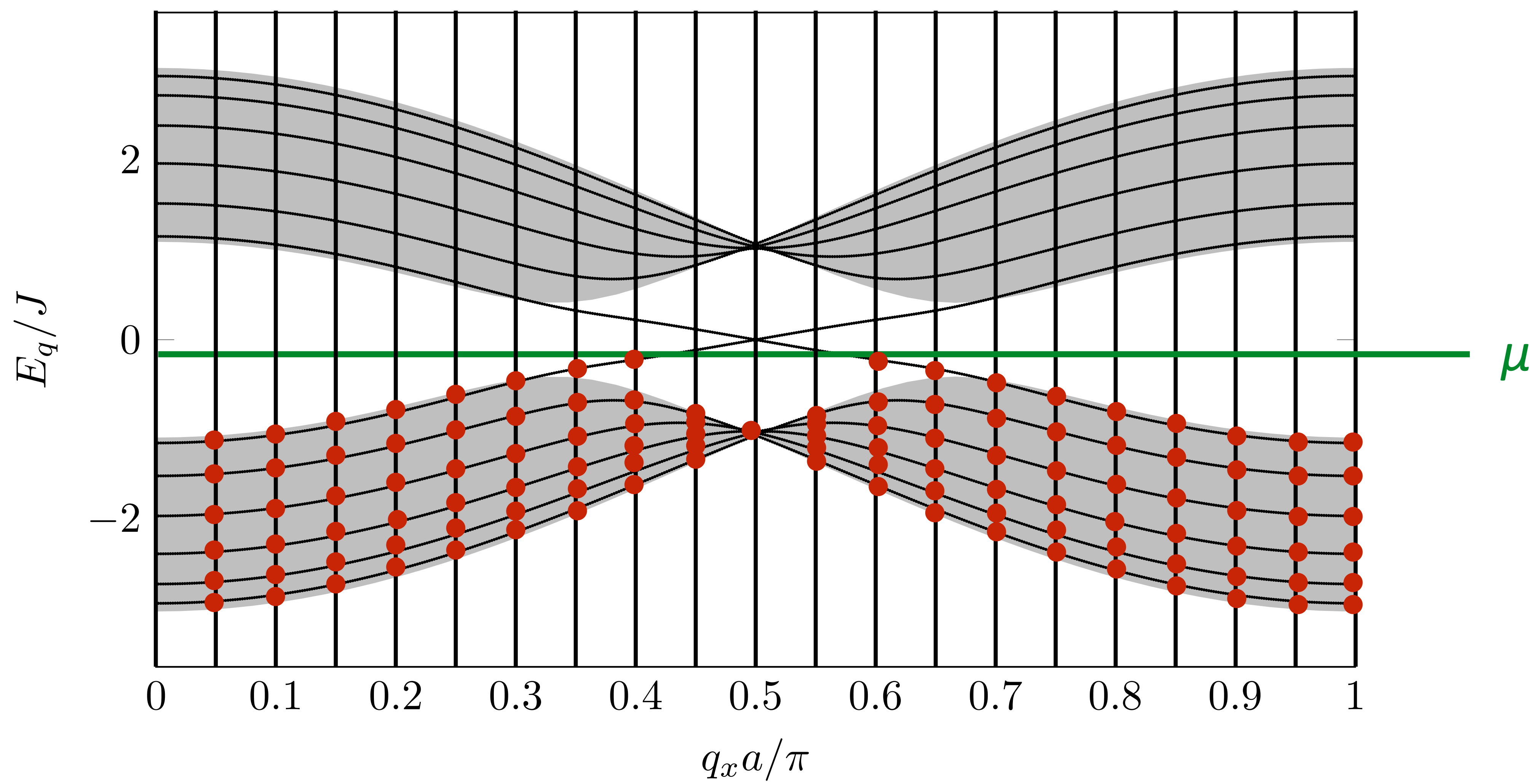


Reservoir and quantized transport

A reservoir imposes its chemical potential μ to the ribbon (still 6 lines along y)

Periodic boundary conditions along x (here n_x=20 sites) : q_x is quantized

$$\delta q_x = \frac{2\pi}{n_x a}$$



Opposite currents along the x axis on the lower edge and the upper edge of the ribbon

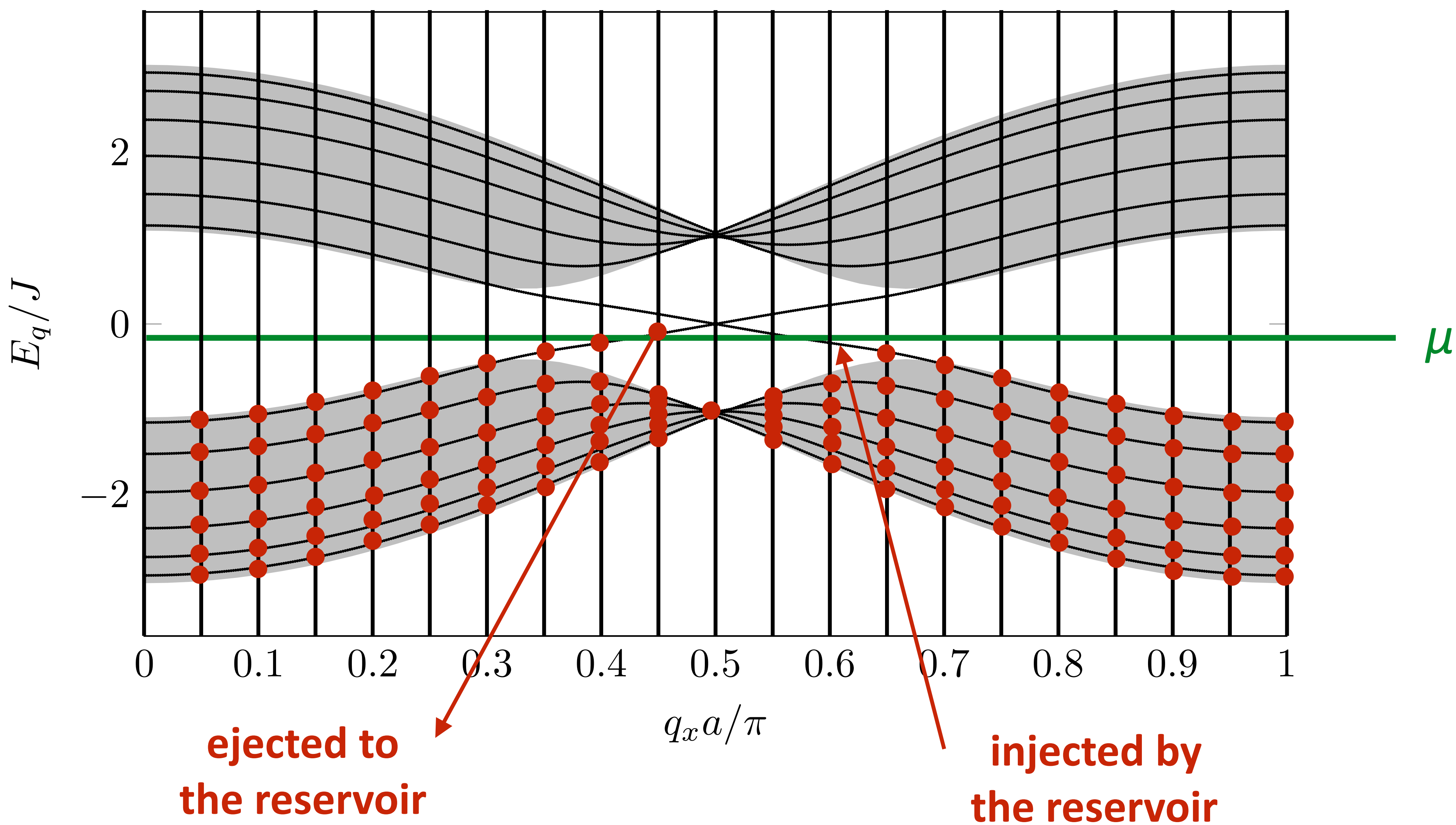
No current along y

Let us apply a force F_x along the x axis: what happens?

Reservoir and quantized transport

Force F_x : Bloch oscillations along x : $\hbar \dot{q}_x = F_x$

In a time step $\delta t = \dfrac{2\pi}{n_x a} \dfrac{\hbar}{F_x}$, the moments q_x increase by a quantum $\delta q_x = \dfrac{2\pi}{n_x a}$



Current density : $J_y = \dfrac{1/\delta t}{n_x a} = \dfrac{F_x}{2\pi\hbar}$ $\longrightarrow$ $\sigma_{yx} = \dfrac{\mathcal{C}}{2\pi\hbar}$ avec $\mathcal{C} = 1$

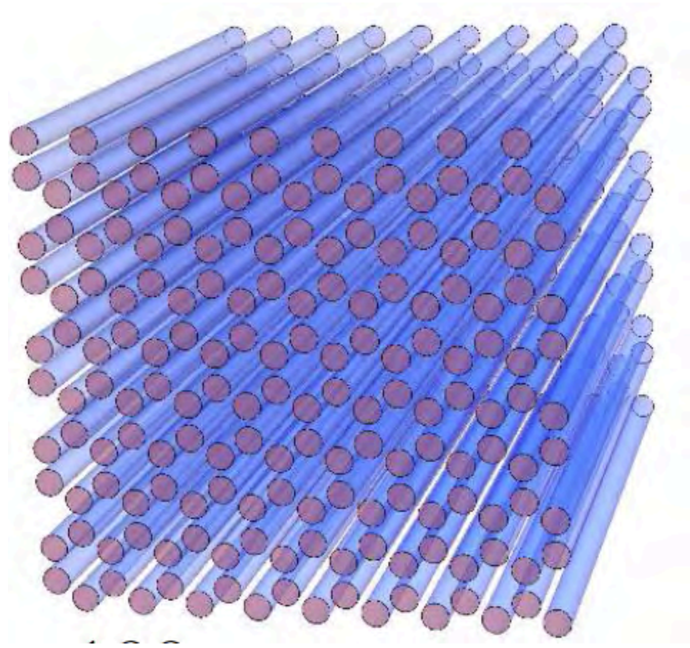
A photonic hexagonal lattice

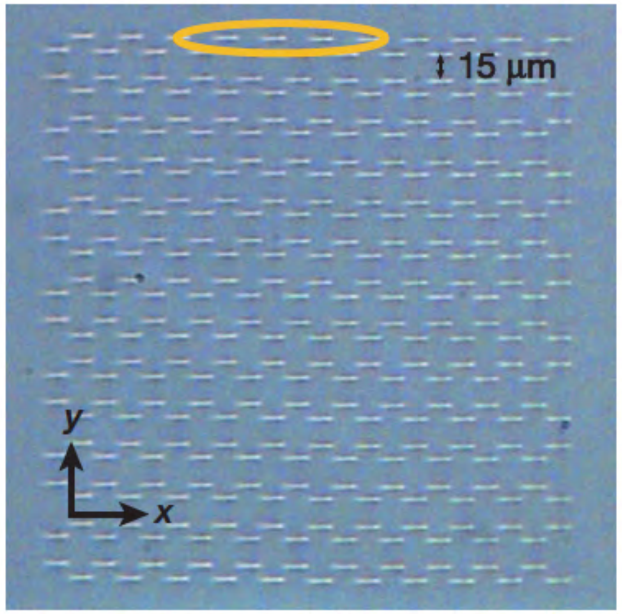
M. Rechtsman et al., Nature 496, 196 (2013) ; Technion & Jena

Wave guides engraved by a laser in in a piece of glass (refraction index 1.45), generated by a small index variation(10^{-3}): hexagonal lattice in the xy plane

Evanescent wave coupling between neighboring guides by (distance 15 µm)

Propagation over a distance of 10 cm along the z axis





$$\mathrm{i}\frac{\partial \mathcal{E}}{\partial z} = -\frac{1}{2k_0}\left(\frac{\partial^2 \mathcal{E}}{\partial x^2} + \frac{\partial^2 \mathcal{E}}{\partial y^2}\right) - \frac{k_0\,\Delta n(\boldsymbol{r})}{n_0}\mathcal{E}$$

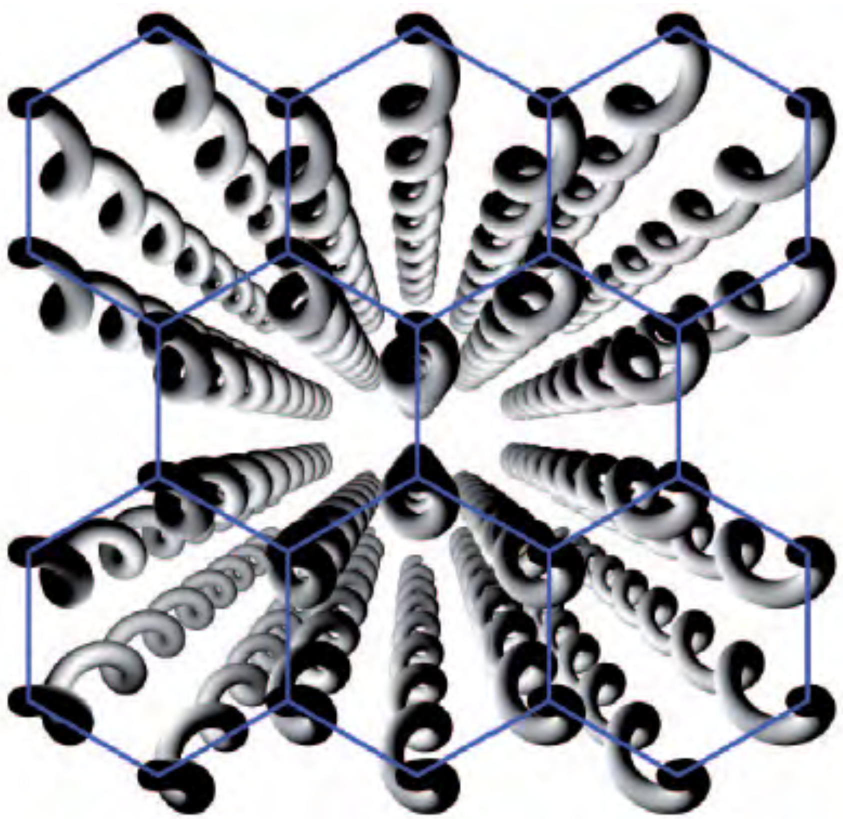
Identical to the 2D Schrödinger equation for a massive particle: graphene

Openeing a topological gap

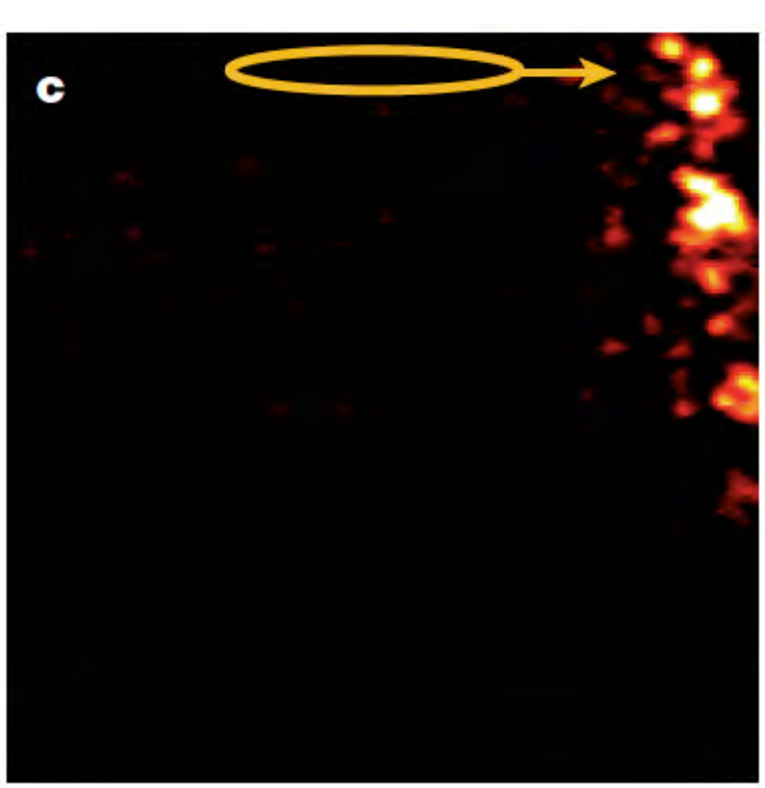
Wave guides with helicoidal shape: equivalent to a modulated lattice

Helix pitch: 5mm (20 cycles over the 10 cm propagation length)

Helix radius: 8 µm



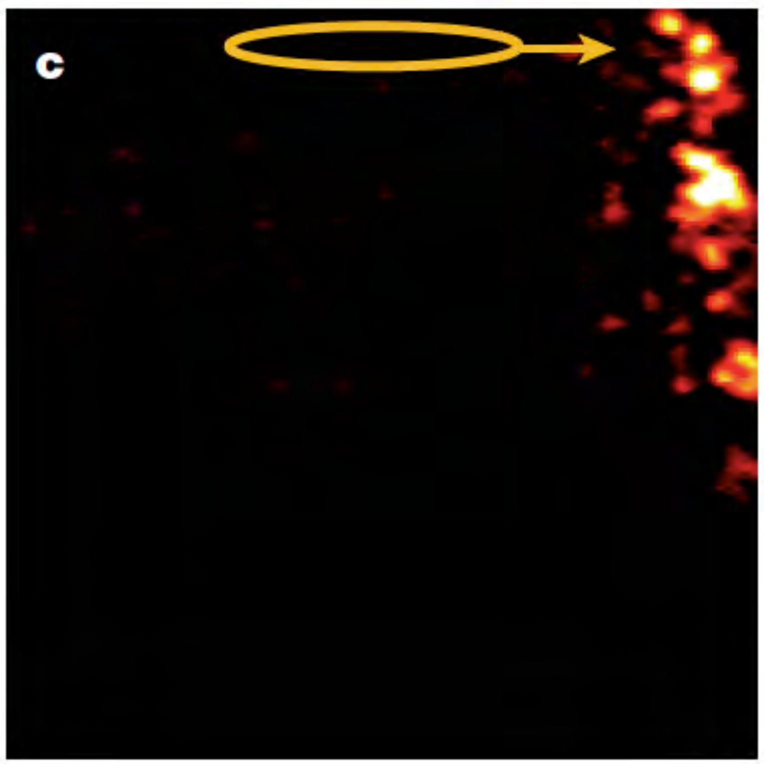
Illuminated zone in the entrance plane of the waveguides



Intensity distribution in the exit plane

Chiral edge states

Illuminated zone in the entrance plane



Intensity distribution in the exit plane

- No diffusion towards the bulk of the sample

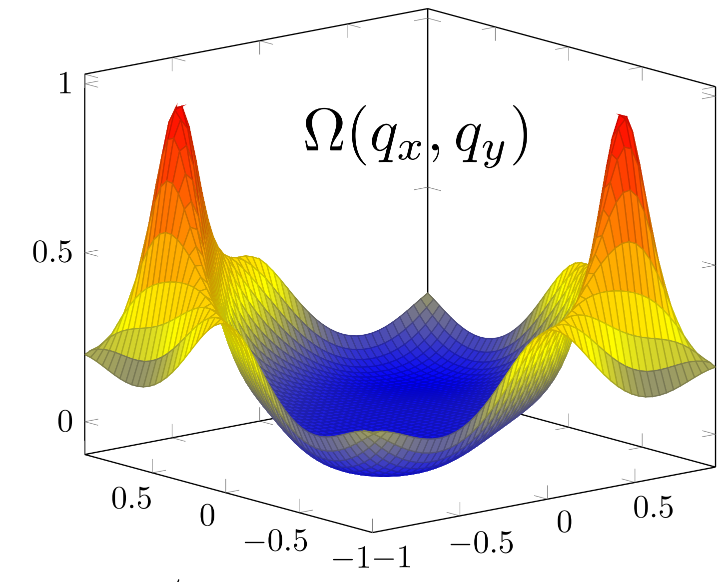
  Only the edge channel is significantly populated

- Light travelled in a chiral way, going only towards the right of the excitation zone

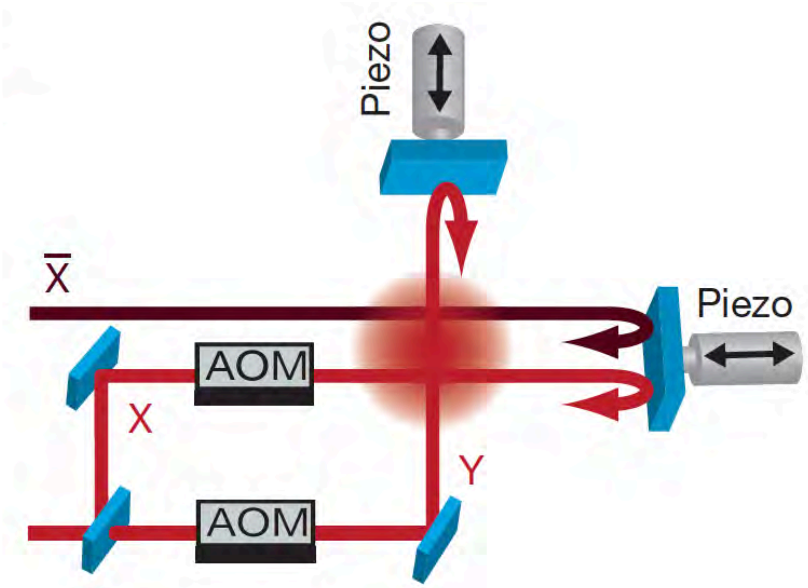
- The edge channel is robust: no back-reflexion at the top right corner

The non-sensitivity to added defects was also checked

Summary for this part

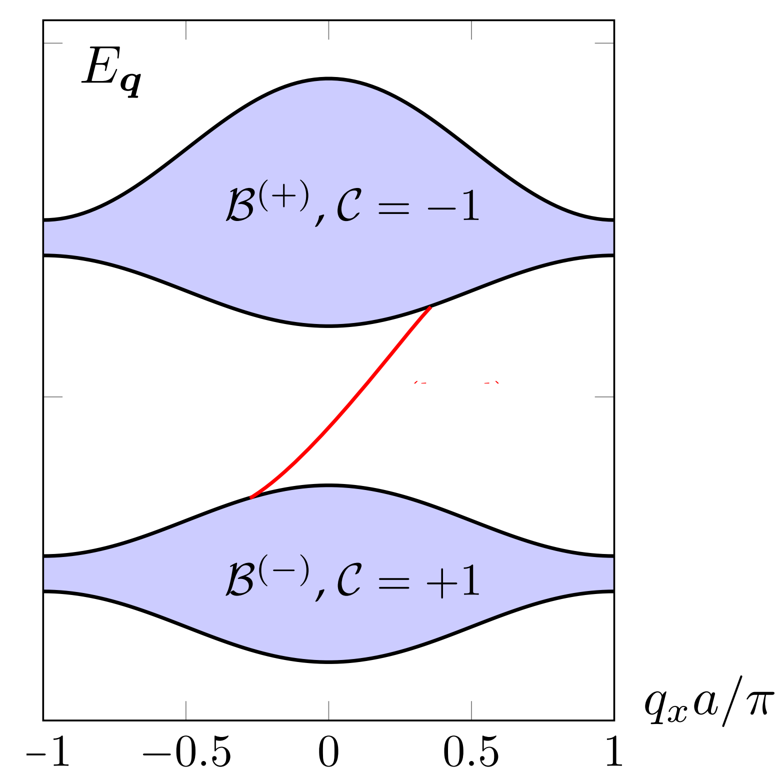


Construction and study of a 2-site unit cell model (Haldane or brick-wall) leading to topological bands



Model that can be implemented using a temporal variation of some parameters of the Hamiltonian, both for atoms and for guided light

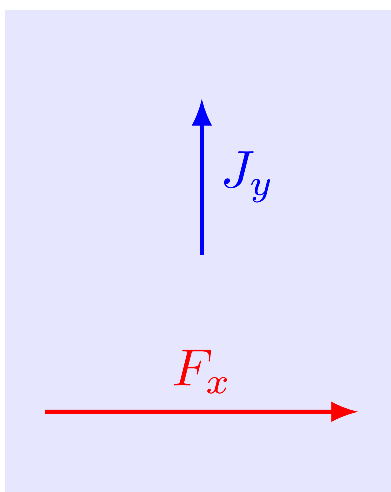
New physical criterion to characterize a topological band: edge channels that correspond currents travelling in a chiral way on the border of the sample



The Harper - Hofstadter model: Recovering the Hall effect

Summary of what we have seen so far

Characterization of topological bands in 1 and 2 dimensions



- ***Quantized conductivity for a filled band***

$$J_y = \sigma_{yx} F_x \qquad \text{avec} \qquad \sigma_{yx} = \frac{\mathcal{C}}{2\pi\hbar}$$

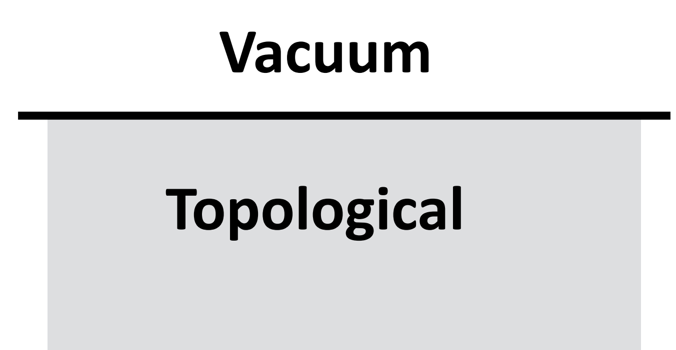
External force along the x axis

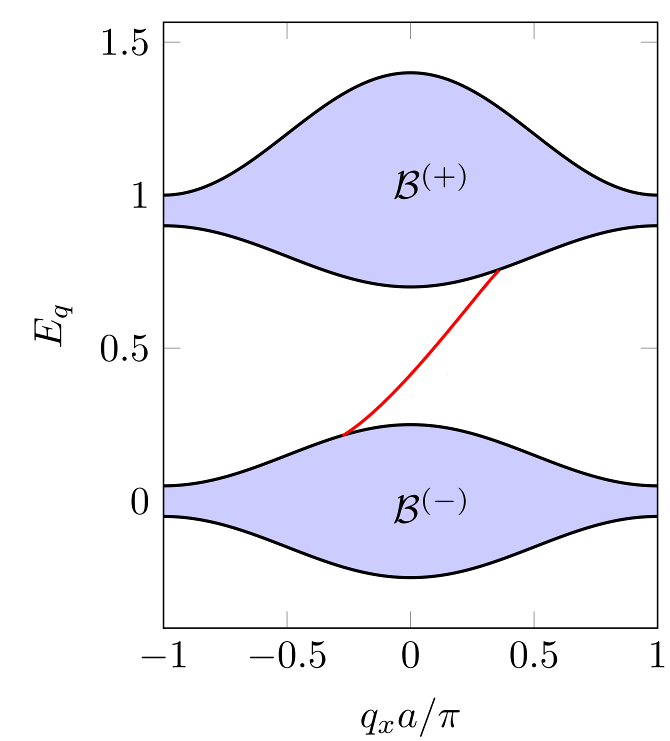
Particle current along the y axis

$\mathcal{C}$: Chern number (integer) associated to the band

- ***Existence of edge states/channels***

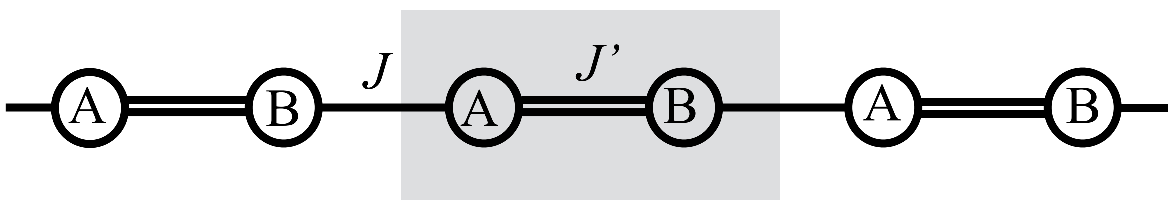
Single mode propagation channel at the interface between the two regions



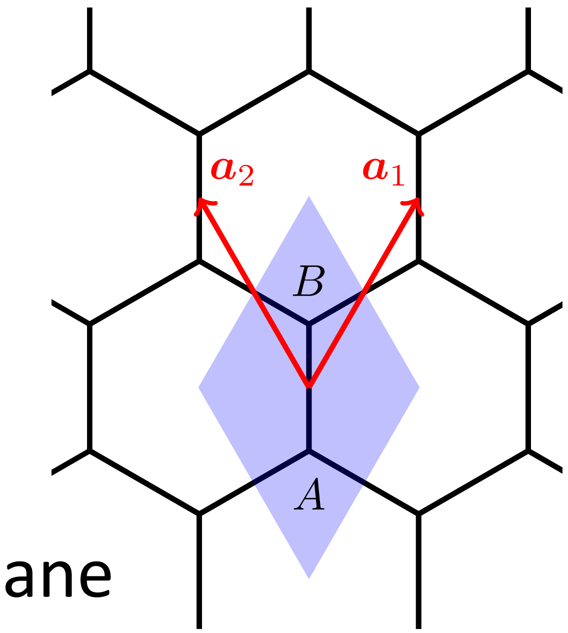


Summary of what we have seen so far (2)

Construction of models leading to topological bands



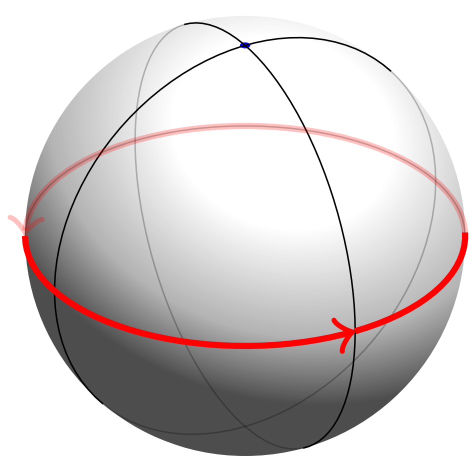
SSH



Haldane

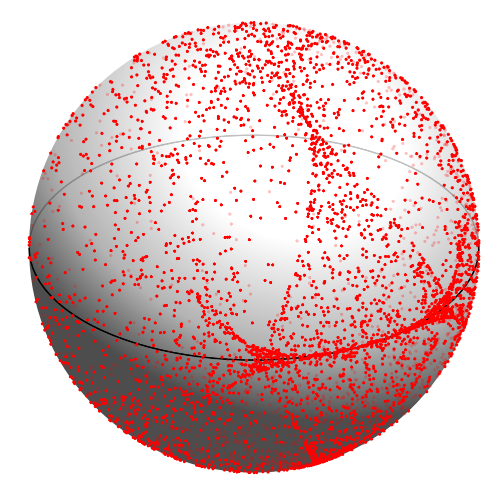
Two-site unit cell

$\longrightarrow$ The periodic Hamiltonian $\hat{H}_{\boldsymbol{q}}$ is a 2x2 matrix



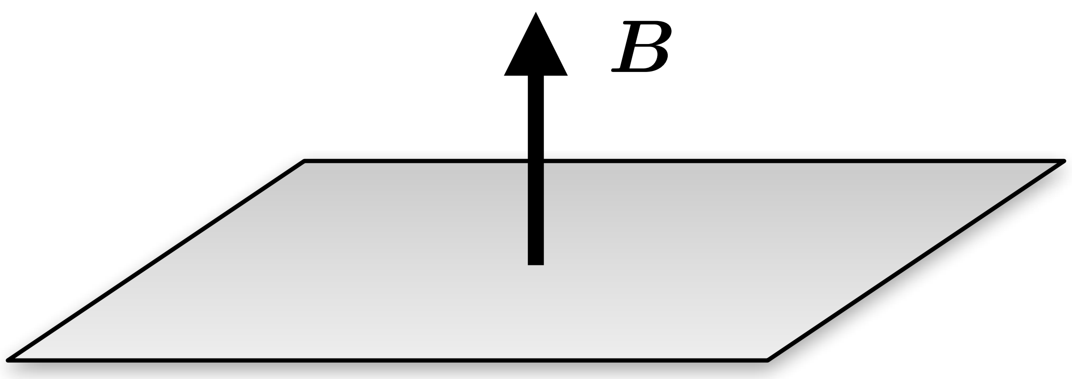
- Simple calculation
- Pseudo-spin 1/2

  Geometrical analysis using the Bloch sphere



Goal of this lecture

Historically, the notion of a topological band first appeared with a spatially continuous problem



2D electron gas (quantum well)

Here also the Hall conductance σ_{yx} is quantized:
What is the link with the discretized models we have studied so far?

Outline

1. The Harper-Hofstadter model

Going from a continuous to a discrete model

Tight-binding regime, but with a “large” unit cell

Only nearest-neighbor couplings

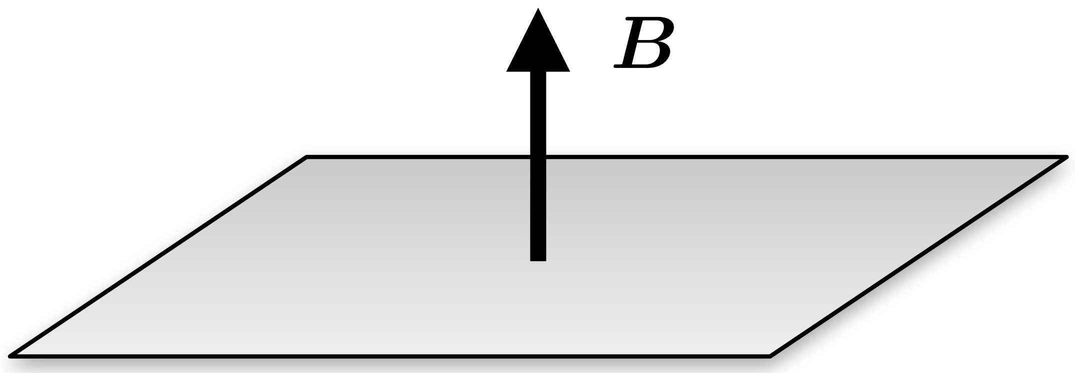
2. Implementation with cold atoms

Measurement of a Chern number

3. Implementation in photonics

Edge states and topological laser

The Landau levels



Particle with charge ***e*** freely moving in the *xy* plane in the presence of a magnetic field

$$\hat{H} = \frac{\left(\hat{\boldsymbol{p}} - e\boldsymbol{A}(\hat{\boldsymbol{r}})\right)^2}{2m} \qquad \begin{cases} \hat{\boldsymbol{p}} = -\mathrm{i}\hbar\boldsymbol{\nabla}_{\boldsymbol{r}} & \textit{canonical momentum} \\ \boldsymbol{\nabla}\times\boldsymbol{A} = \boldsymbol{B} & \text{for example: } \boldsymbol{A}(\boldsymbol{r}) = -By\,\boldsymbol{u}_x \end{cases}$$

Possible approach: use of the *kinetic momentum* $\hat{\boldsymbol{\Pi}} = \hat{\boldsymbol{p}} - e\boldsymbol{A}(\hat{\boldsymbol{r}})$

The Hamiltonian reads $\hat{H} = \dfrac{1}{2m}\left(\hat{\Pi}_x^2 + \hat{\Pi}_y^2\right)$ with $[\hat{\Pi}_x, \hat{\Pi}_y] = \mathrm{i}\,\hbar e B$

Formal analogy with a 1D Harmonic oscillator

$$\hat{H} = \frac{\hbar\omega}{2}\left(\hat{X}^2 + \hat{P}^2\right) \qquad [\hat{X}, \hat{P}] = \mathrm{i}$$

Landau levels (2)

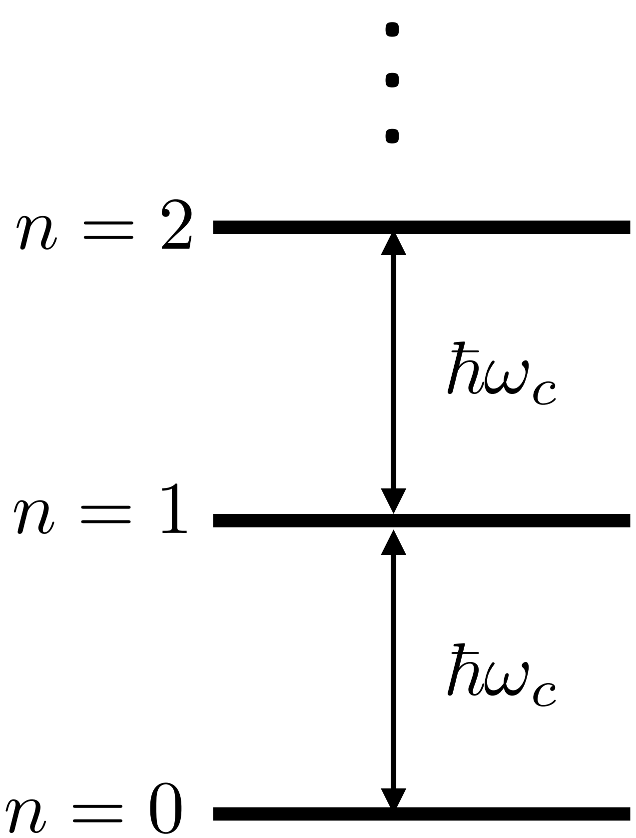
The analogy with the formalism of a 1D Harmonic oscillator indicates that the energy spectrum consists in equally spaced energy levels

$$\hat{H} = \frac{1}{2m}\left(\hat{\Pi}_x^2 + \hat{\Pi}_y^2\right)$$

$$[\hat{\Pi}_x, \hat{\Pi}_y] = \mathrm{i}\,\hbar e B$$

$$E_n = \hbar\omega_c\,(n + 1/2)$$

$\omega_c = eB/m$: cyclotron frequency



Each energy level has a macroscopic degeneracy:

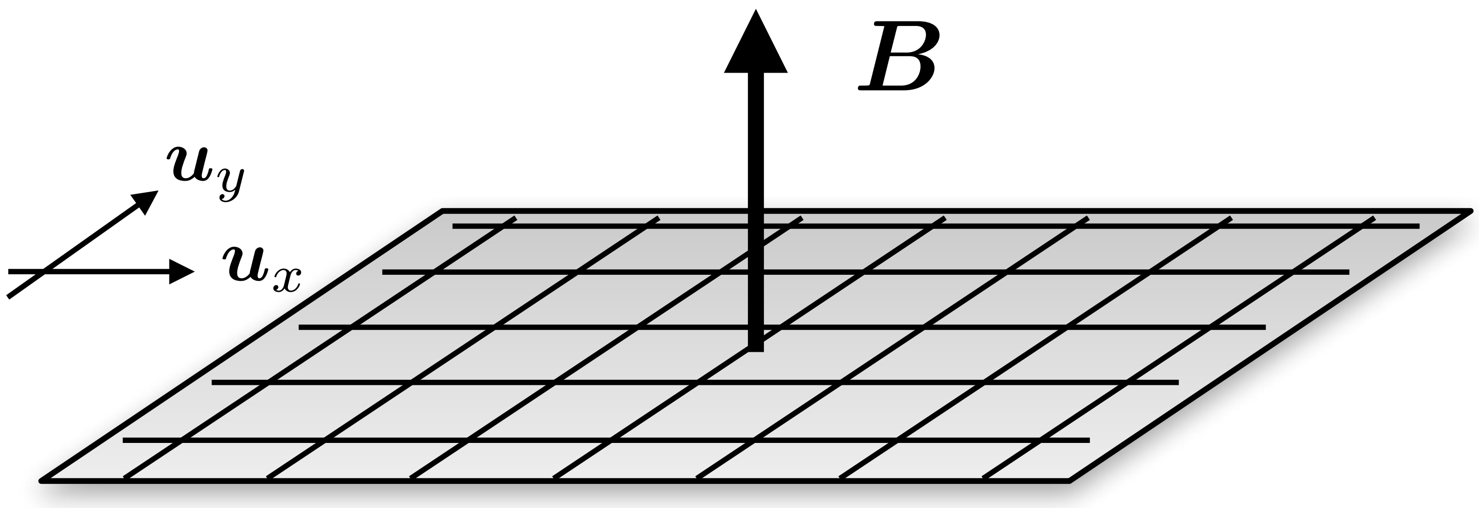
$$\text{degeneracy} = \frac{\mathcal{S}}{2\pi\ell^2}$$

$\mathcal{S}$: area of the sample

$\ell = (\hbar/eB)^{1/2}$
magnetic length

Flat energy bands with $\mathcal{C} = 1$

Square lattice and magnetic field



Discretize the *xy* plane:

$$A_{\boldsymbol{j}} \quad : \qquad \boldsymbol{r}_j = a\,(j_x \boldsymbol{u}_x + j_y \boldsymbol{u}_y)$$

Nearest neighbor coupling:

$$\hat{H}_0 = -J \sum_{\langle \boldsymbol{j},\boldsymbol{j}'\rangle} |A_{\boldsymbol{j}'}\rangle\langle A_{\boldsymbol{j}}| \quad \textit{in the absence of a magnetic field}$$

Energy band: $E_{\boldsymbol{q}} = -2J\,[\cos(q_x a) + \cos(q_y a)]$ (2D Hubbard model)

With the magnetic field: $J \quad \text{remplaced by} \quad J\ \exp[\,\mathrm{i}\,\gamma(\boldsymbol{j} \to \boldsymbol{j}')]$

Role of the phase factor: ensure the non-zero Aharonov-Bohm phase on a contour

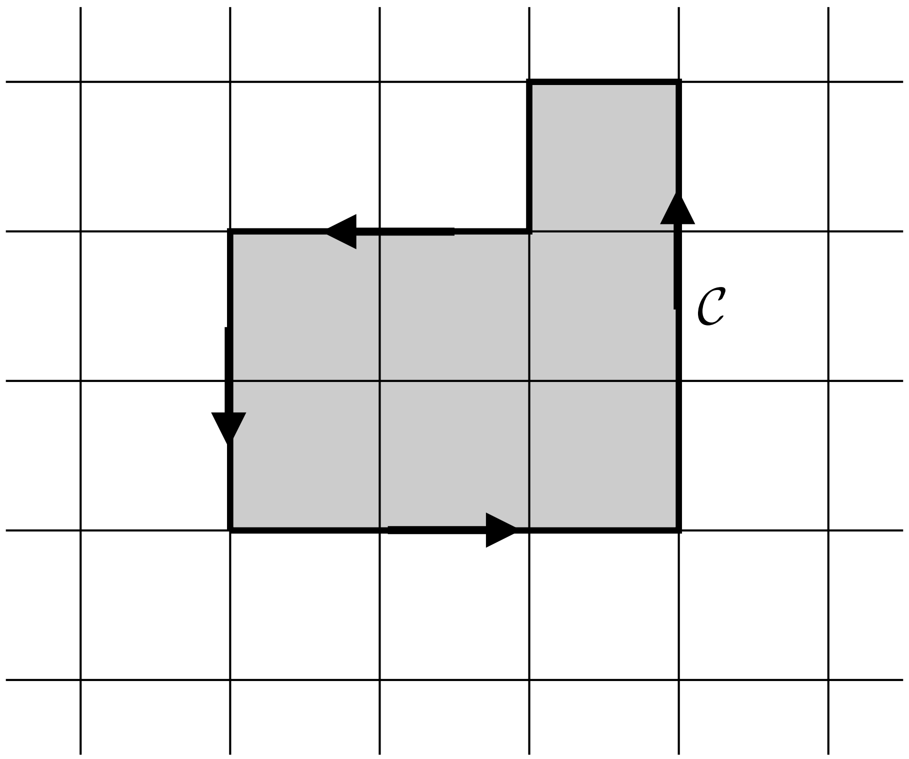
Choice of the phase factor $J \; \exp[\mathrm{i}\,\gamma(\boldsymbol{j} \to \boldsymbol{j}')]$

Arbitrary contour on the square lattice

$$\Phi(\mathcal{C}) \equiv \sum_{\text{contour}} \gamma(\boldsymbol{j} \to \boldsymbol{j}')$$

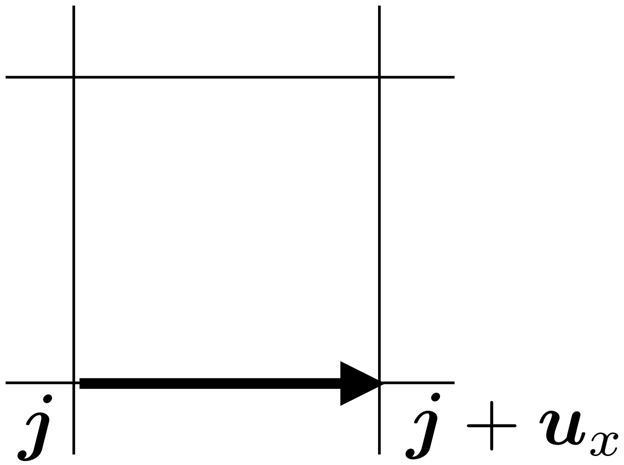
must be proportional to the flux ϕ of the magnetic field across $\mathcal{C}$:

$$\Phi(\mathcal{C}) = \frac{e}{\hbar}\phi \qquad \mathrm{mod}\ (2\pi)$$



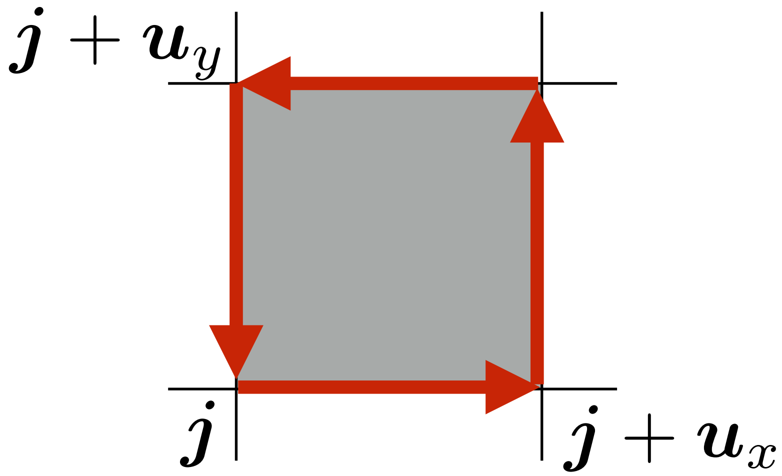
A possible solution (Peierls) : Take the vector potential $\boldsymbol{A}(\boldsymbol{r})$ for the continuous case and set for each link of the lattice

$$\gamma(\boldsymbol{j} \to \boldsymbol{j}') = \frac{e}{\hbar}\int_{a\boldsymbol{j}}^{a\boldsymbol{j}'} \boldsymbol{A}(\boldsymbol{r}) \cdot \mathrm{d}\boldsymbol{r}$$



$\gamma(\boldsymbol{j}' \to \boldsymbol{j}) = -\gamma(\boldsymbol{j} \to \boldsymbol{j}')$ ($\hat{H}$ hermitian)

Magnetic flux per unit cell



With this discretization of space, the magnetic field is defined by its flux through a unit cell withe area *a*²

$$\Phi_{\text{cell}} \equiv \sum_{4 \text{ links}} \gamma(\boldsymbol{j} \to \boldsymbol{j}')$$

Two values Φ_{cell} and $\Phi_{\text{cell}} + 2\pi$ correspond to the same tunnel coefficients

$$J \;\exp[\mathrm{i}\,\gamma(\boldsymbol{j} \to \boldsymbol{j}')]$$

and thus to the same physical situation

One usually sets $\Phi_{\text{cell}} = 2\pi\,\alpha$ and choose α such that

$$0 \;\leq\; \alpha \;< 1$$

In what follows we will typically have $\alpha = 1/4$

The magnetic unit cell

When replacing the tunnel coefficient J by $J\ \exp[\mathrm{i}\,\gamma(\boldsymbol{j} \to \boldsymbol{j}')]$, we loose the spatial periodicity of the initial lattice

Complete loss?
Not always: example of the flux α = 1/4

$$\longleftrightarrow \qquad \Phi_{\text{cell}} = \frac{\pi}{2}$$

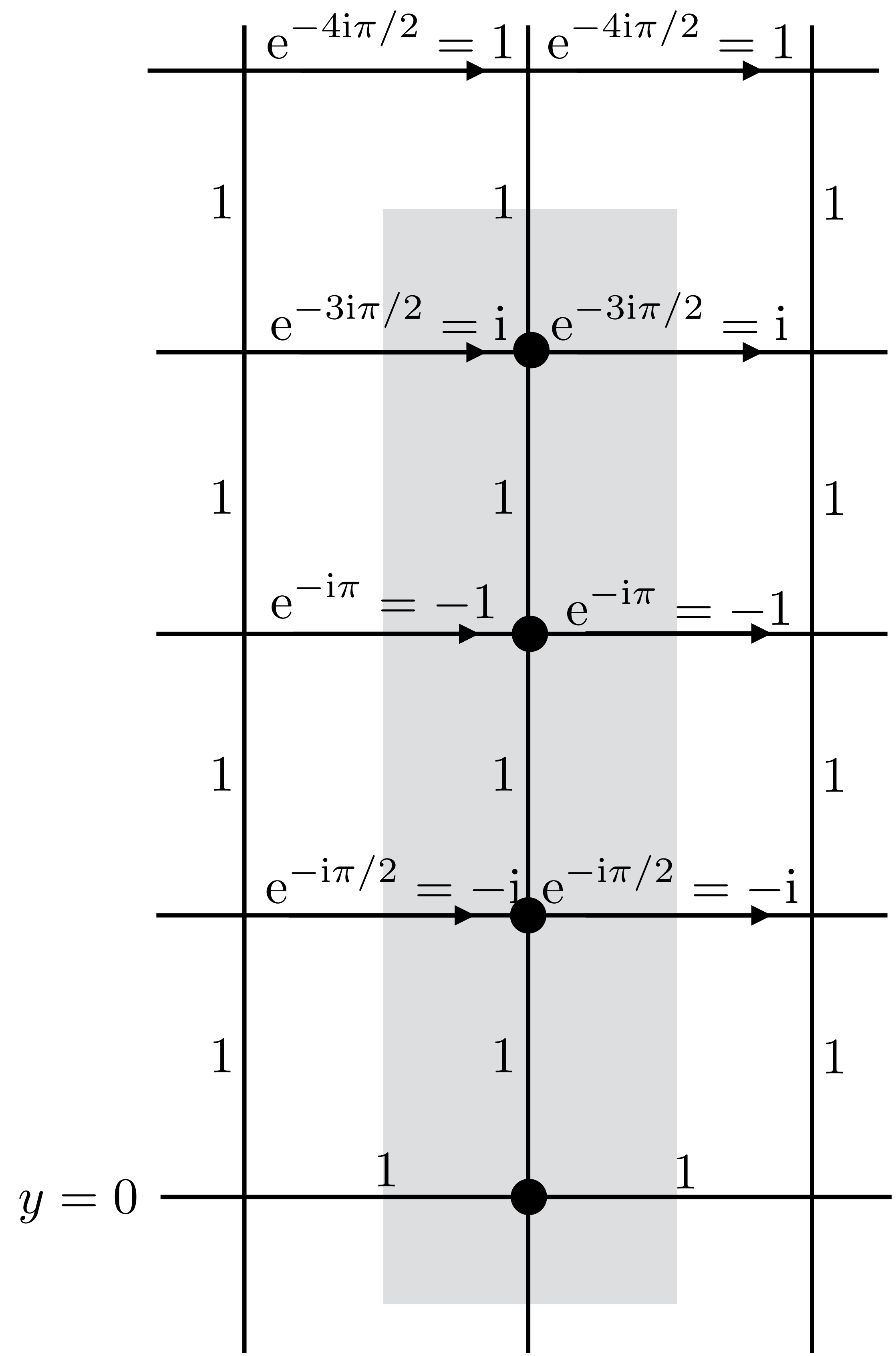
Gauge choice: $\quad \boldsymbol{A}(\boldsymbol{r}) = -By\,\boldsymbol{u}_x$

What are the tunnel coefficients

$$J\ \exp[\mathrm{i}\,\gamma(\boldsymbol{j} \to \boldsymbol{j}')]$$

according to the prescription

$$\gamma(\boldsymbol{j} \to \boldsymbol{j}') = \frac{e}{\hbar} \int_{a\boldsymbol{j}}^{a\boldsymbol{j}'} \boldsymbol{A}(\boldsymbol{r}) \cdot \mathrm{d}\boldsymbol{r} \ \text{?}$$

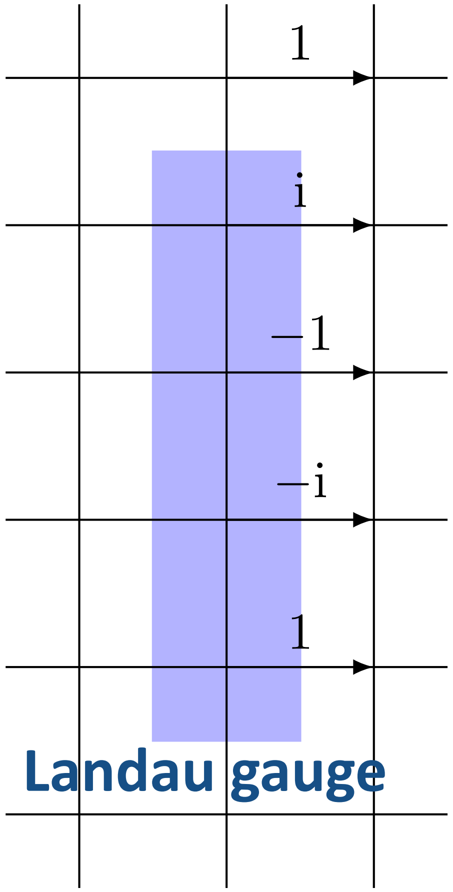


We now have a 4-site unit cell

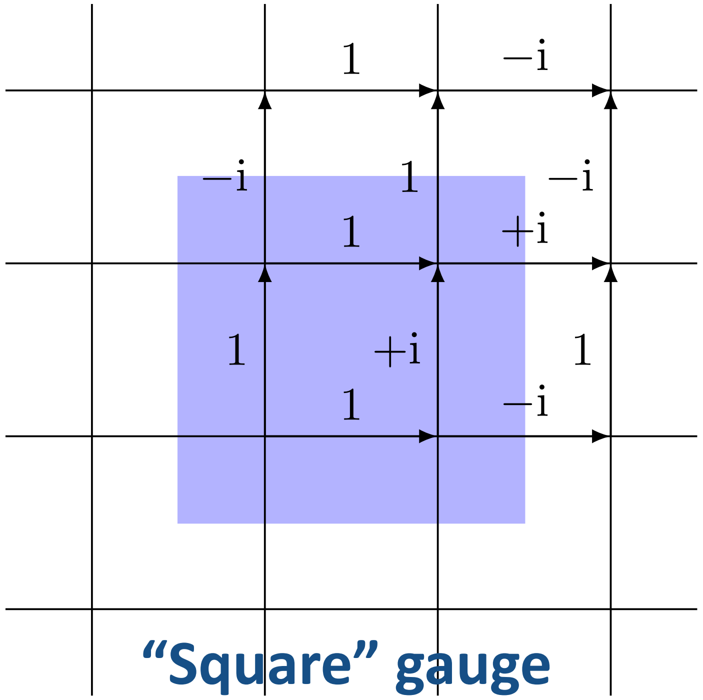
The choice of the magnetic unit cell

For a given flux $\Phi_{\text{cell}} = 2\pi\ \alpha$, several possible choices

Example for flux α = 1/4 :



Landau gauge



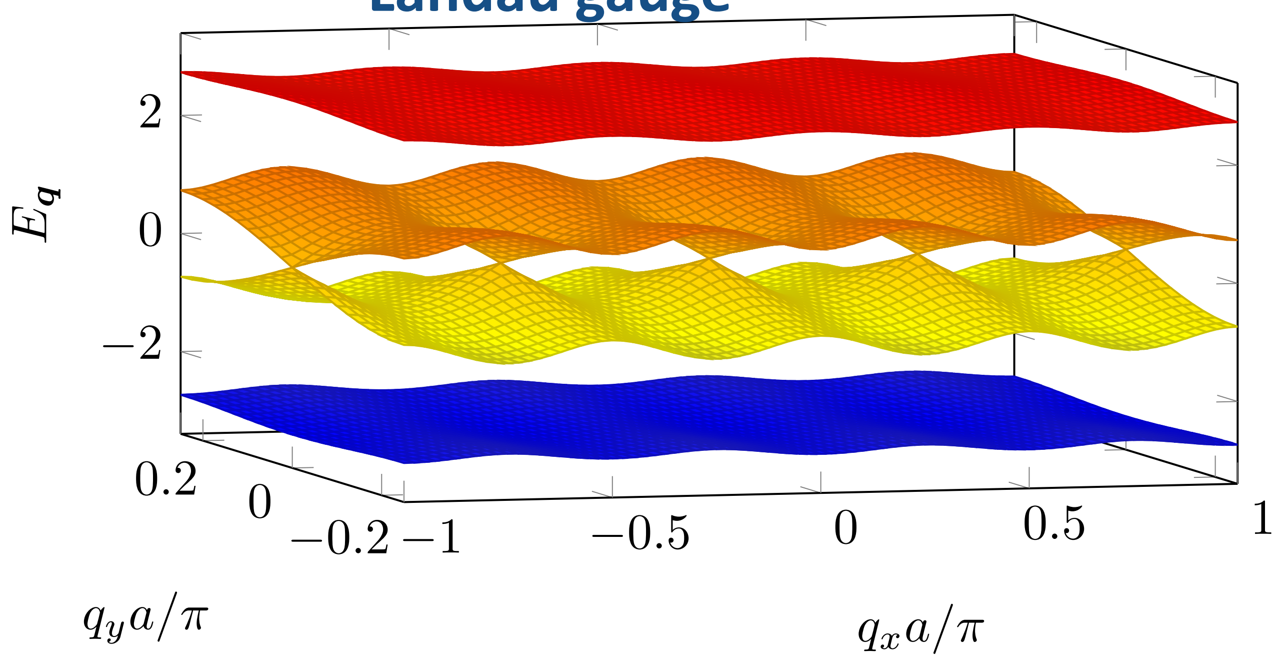
"Square" gauge

All choices lead to the same predictions for physical properties (spectrum, Chern number): gauge invariance

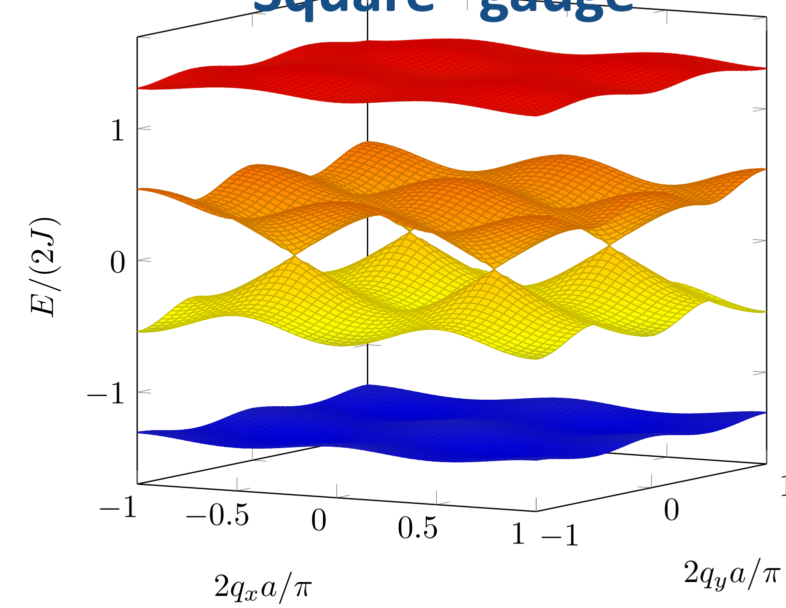
We recover the continuous limit for $\alpha = 1/p\ \rightarrow\ 0$

Energies and topology for flux α = 1/4

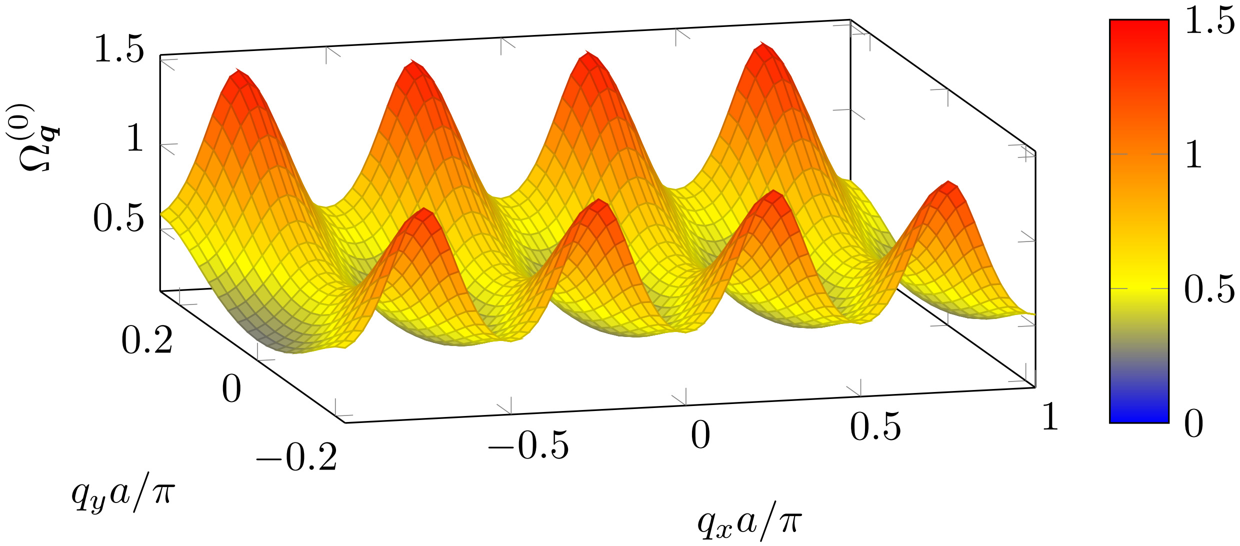
Landau gauge



"Square" gauge



Nearly flat lowest band: this is interesting when one looks for strongly correlated states in the presence of interactions



Berry curvature of the lowest band

positive everywhere

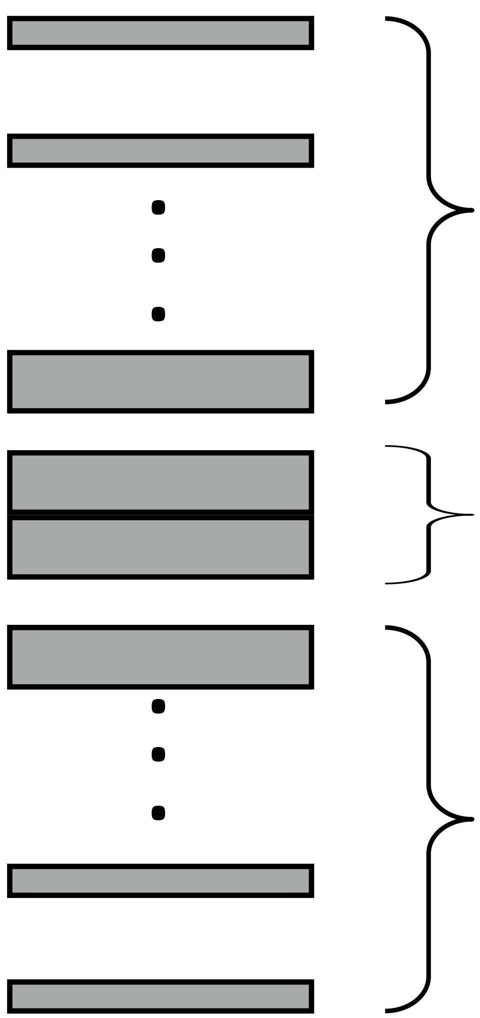
$$\mathcal{C} = \frac{1}{2\pi} \iint_{\mathrm{ZB}} \Omega_{\boldsymbol{q}} \, \mathrm{d}^2 q = 1$$

Berry curvature for the Harper-Hofstadter problem

Flux $\alpha = 1/p$, with p integer (assumed even here)

Thouless *et al* (TKNN), 1982

- Magnetic unit cell with p sites
- The lowest energy band for B=0 splits in p bands, symmetric / $E = 0$

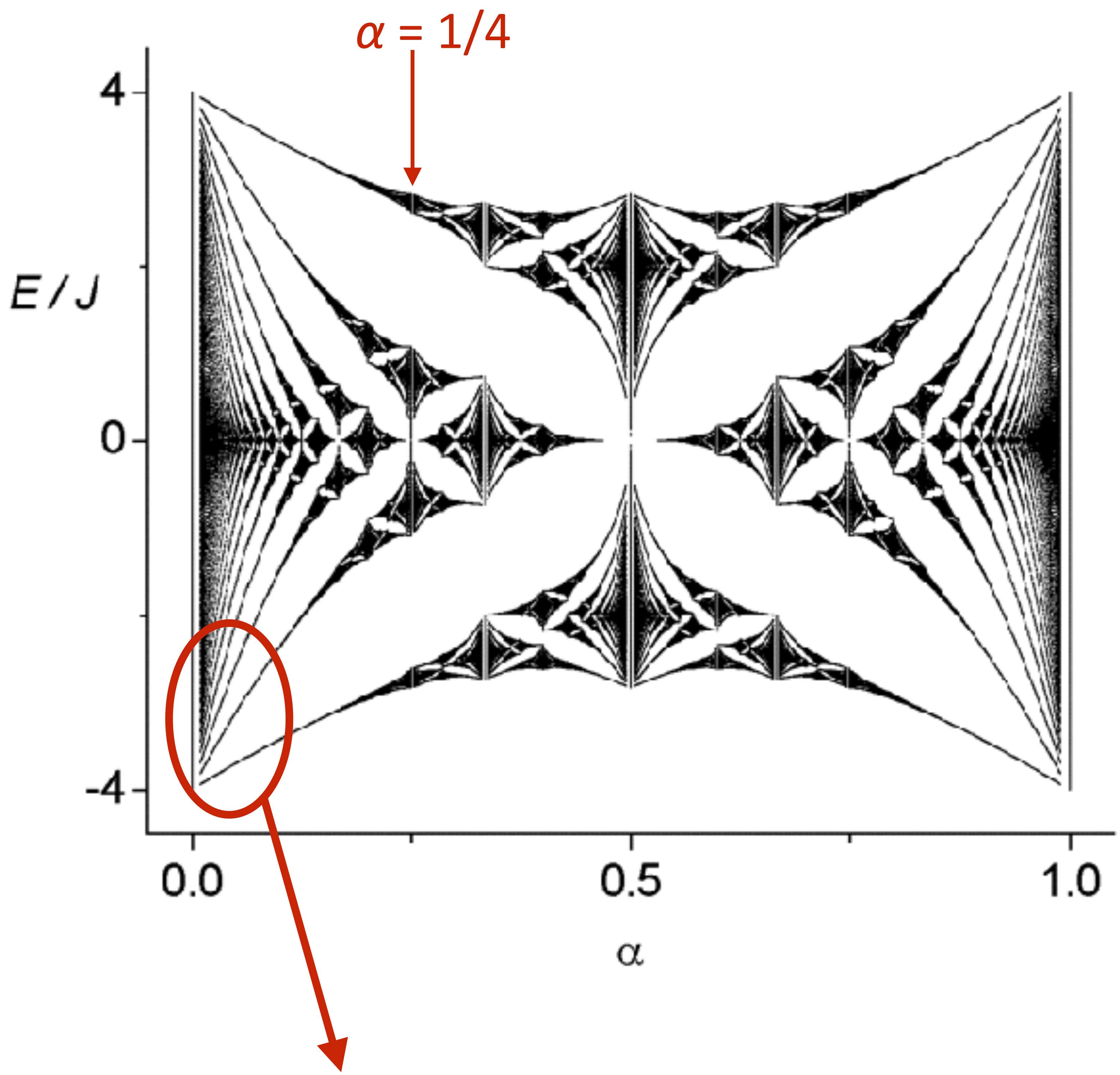


$\frac{p}{2} - 1$ bands, each with $\mathcal{C} = 1$

2 central bands in contact at Dirac points $\mathcal{C} = -p + 2$

$\frac{p}{2} - 1$ bands, each with $\mathcal{C} = 1$

The general spectrum: Hofstadter butterfly



Calculated here for rational fluxes

$$\alpha = p'/p$$

$$1 \ \leq \ p, p' \ \leq \ 100$$

Fractal

Very narrow and equally spaced levels: one recovers the Landau spectrum

2.

Implementation with cold atoms

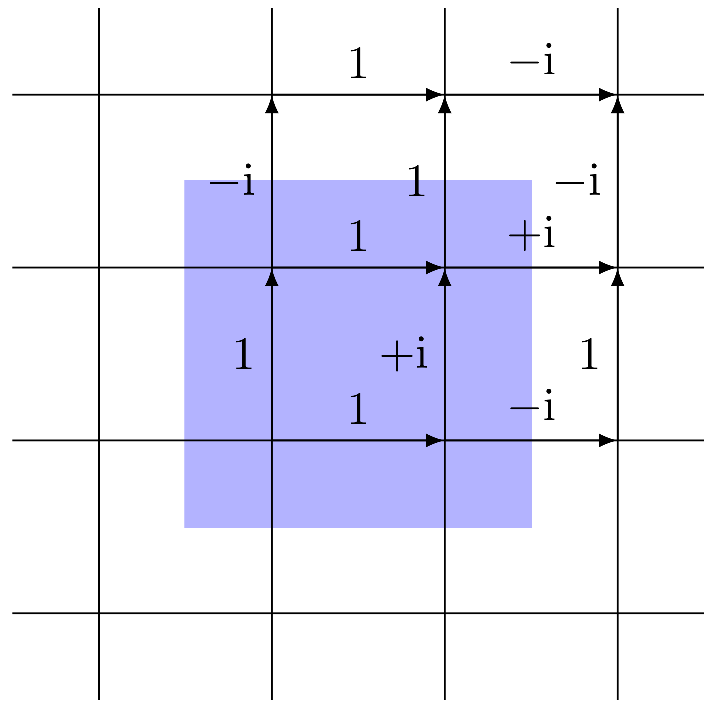
M. AIDELSBURGER, M. LOHSE, C. SCHWEIZER, M. ATALA, J. T. BARREIRO,
S. NASCIMBÈNE, N. R. COOPER, I. BLOCH & N. GOLDMAN

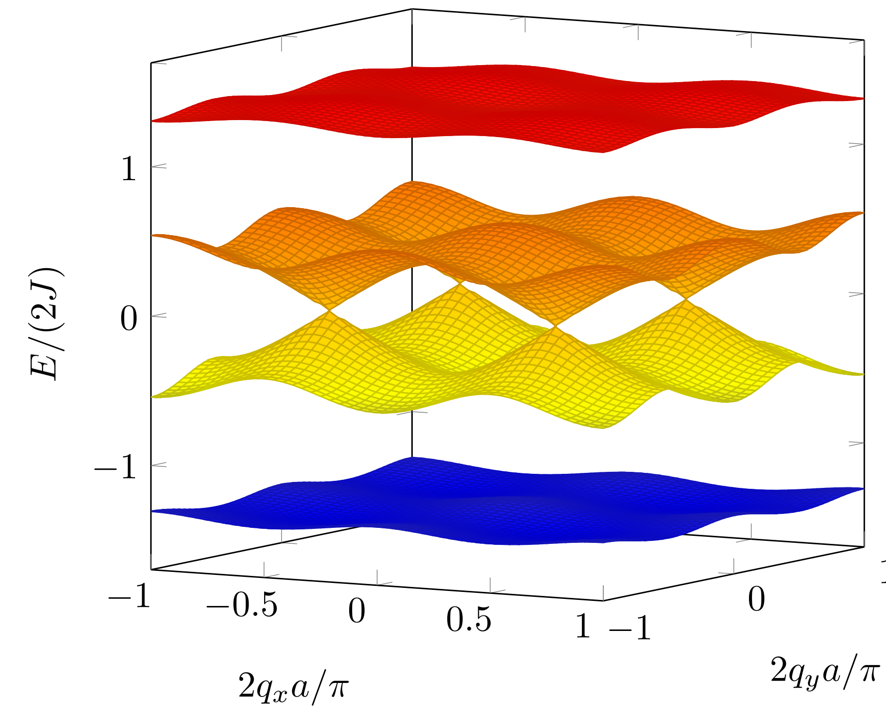
Measuring the Chern number of Hofstadter bands with ultracold bosonic atoms

Nature Physics **11**, p. 162 (2015)

Use of a superlattice

Target: flux $\alpha = 1/4$

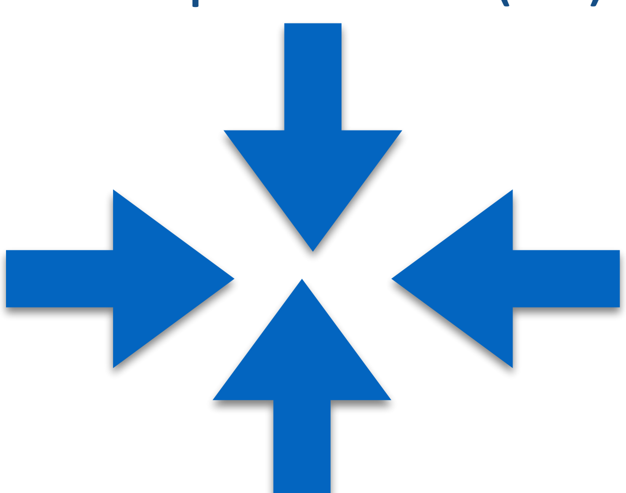


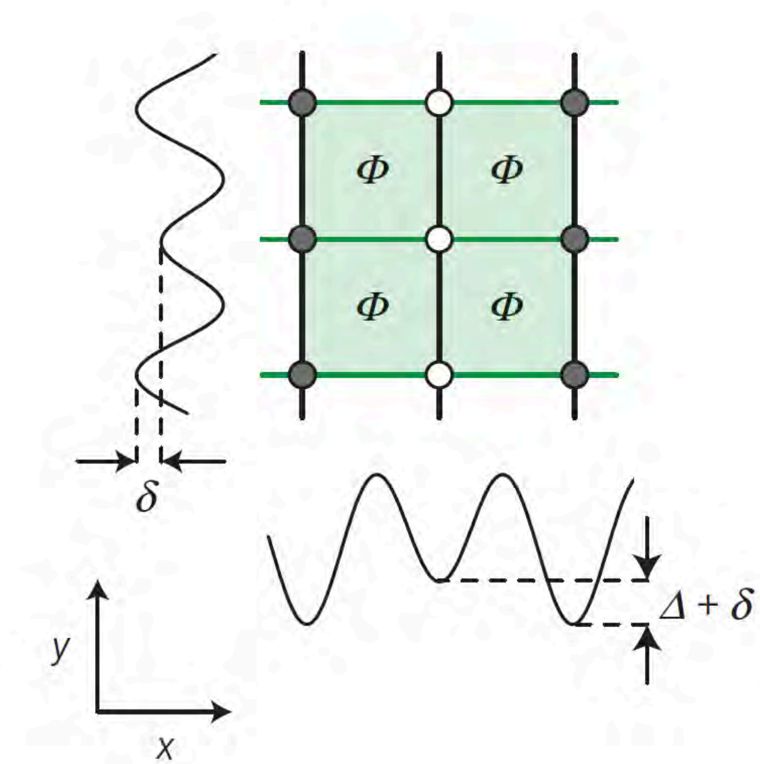


How to create complex tunnel matrix elements?

How to put atoms in the lowest band?

2D optical lattice (λ=0.77 μm)
and additional superlattice (2λ)

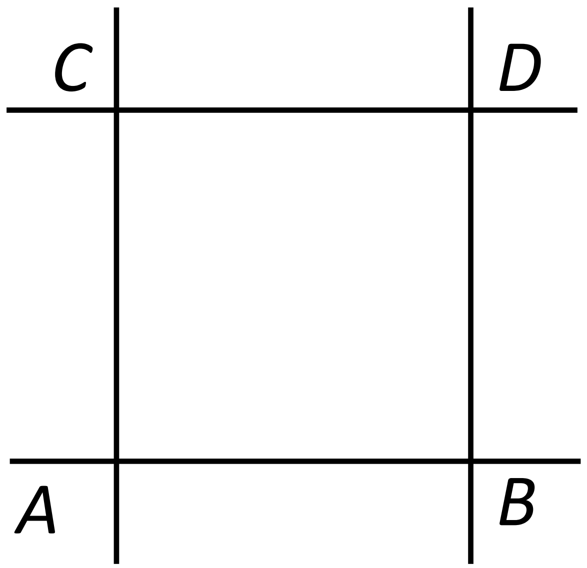




Use of a superlattice

- Start with a simple square lattice: All tunnel matrix elements equal to *J*

- Add a superlattice along the *x* axis:



$$E_A = E_C = 0, \quad E_B = E_D = \Delta$$

Tunneling is blocked along the *x* axis

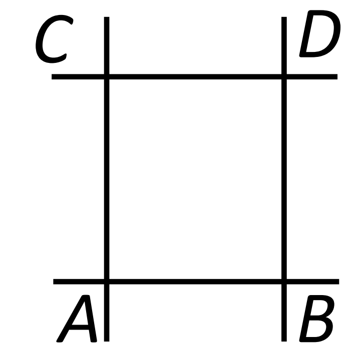
- Modulation at frequency $\Delta/(2\pi\hbar)$: recover tunneling along *x* with a phase controlled by the modulation

- Add another superlattice along *x* and *y*

$$E_A = -\delta, \quad E_B = E_C = 0, \quad E_D = +\delta$$

δ : control parameter

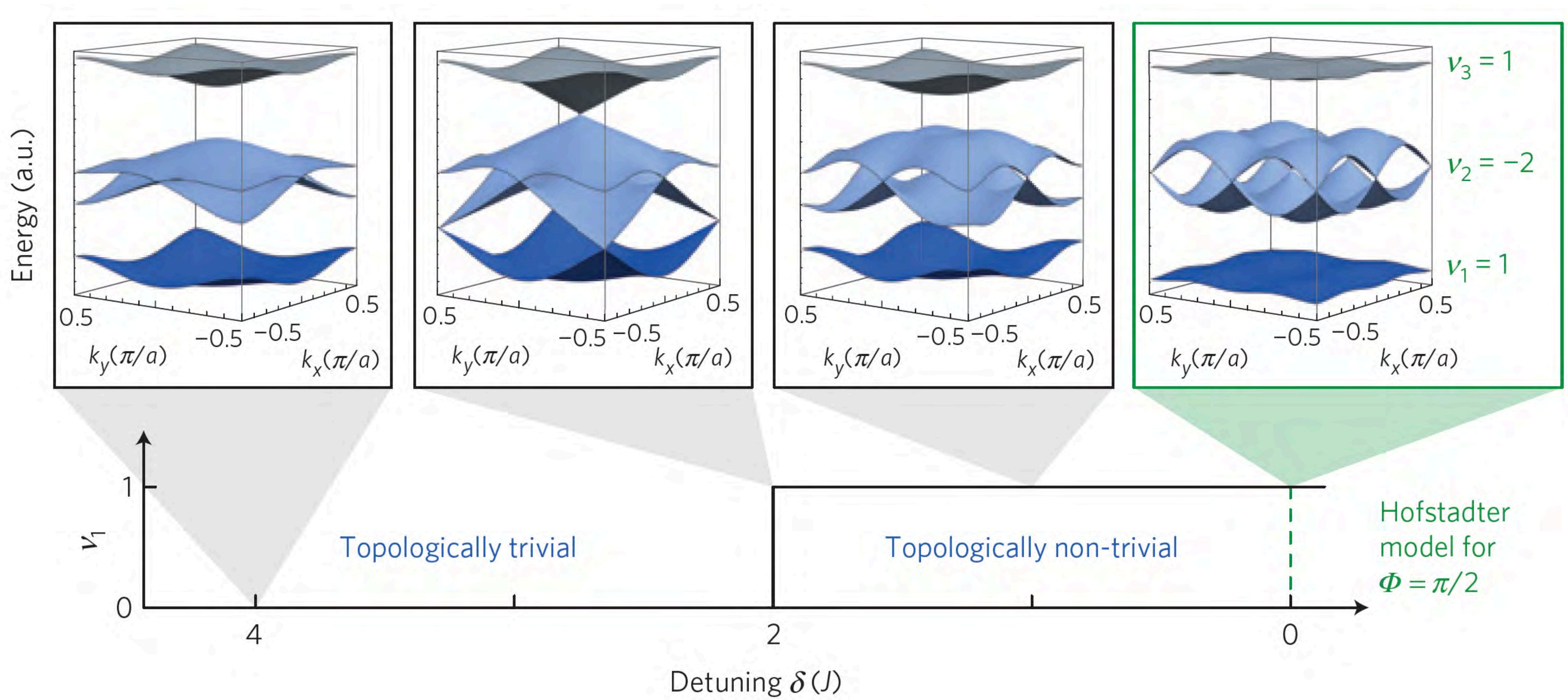
Preparation of atoms in the topological band



$$E_A = -\delta, \quad E_B = E_C = 0, \quad E_D = +\delta$$

δ large and positive: all atoms go on the A sites

Slowly decrease δ until δ=0 : Harper-Hofstadter Hamiltonian



For the detection: inverse path and "band-mapping" technique

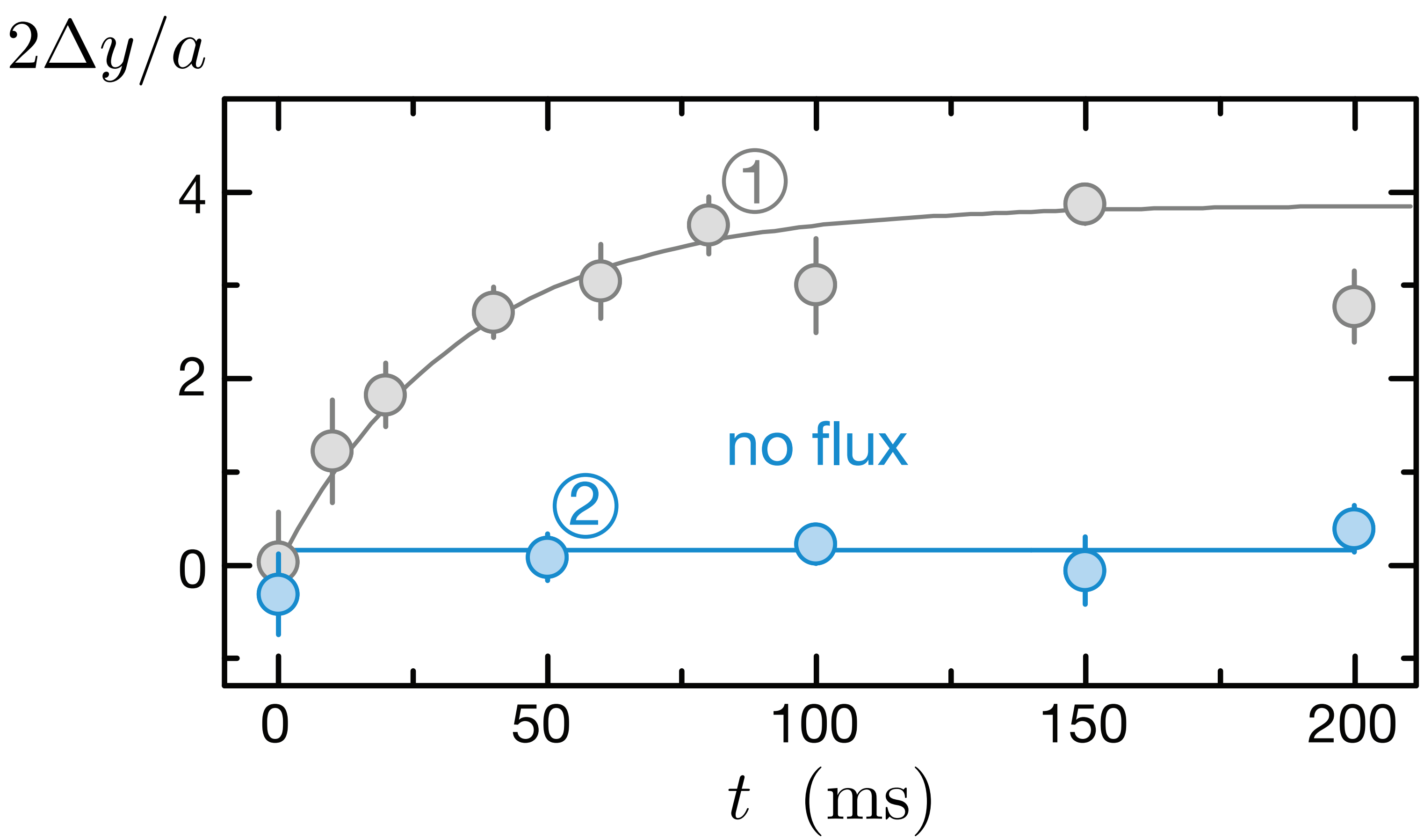
The Chern number measurement

Uniform population of the states $|\psi_{\boldsymbol{q}}^{(n)}\rangle$ of a given band

Apply a force F_x along the x axis (dipole force of an auxiliary beam)

A previously obtained result gives for the velocity of the center-of-mass along y:

$$v_y = \mathcal{C}\ \frac{2}{\pi}\ \frac{F_x a^2}{\hbar}$$

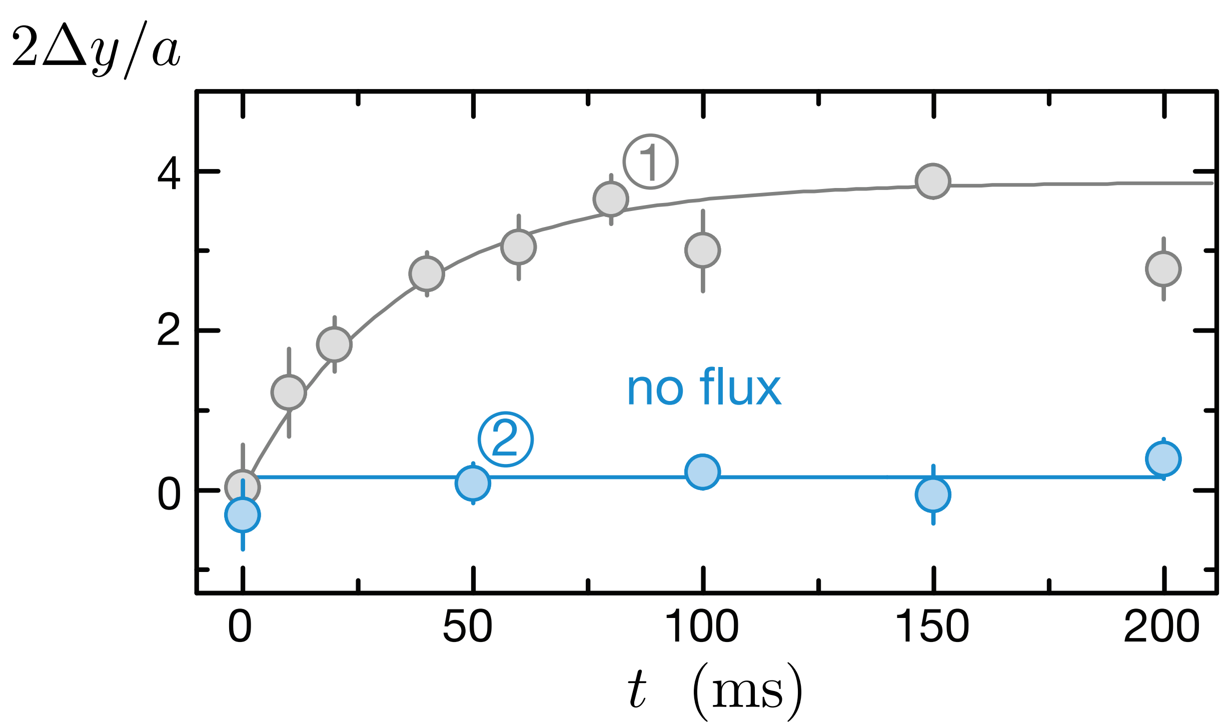


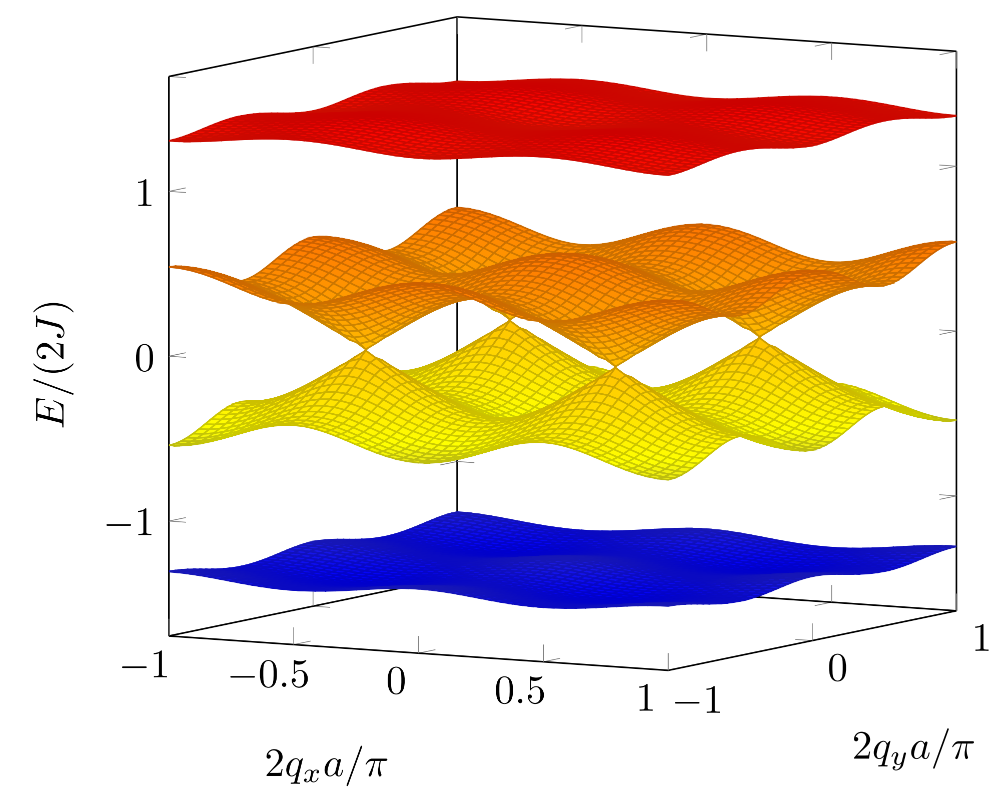
Linear variation at the origin

$$\mathcal{C} = 0.9\,(2)$$

AIDELSBURGER et al, 2015

A non-trivial problem: Heating!





Heating linked to the temporal modulation combined with two-body interaction: transfer of the micro-motion energy to the slow motion

At long time, all bands are equally populated:

$$\sum_{j=1}^{4} \mathcal{C}^{(j)} = 0 \longrightarrow \text{The displacement along } y \text{ stops}$$

3.

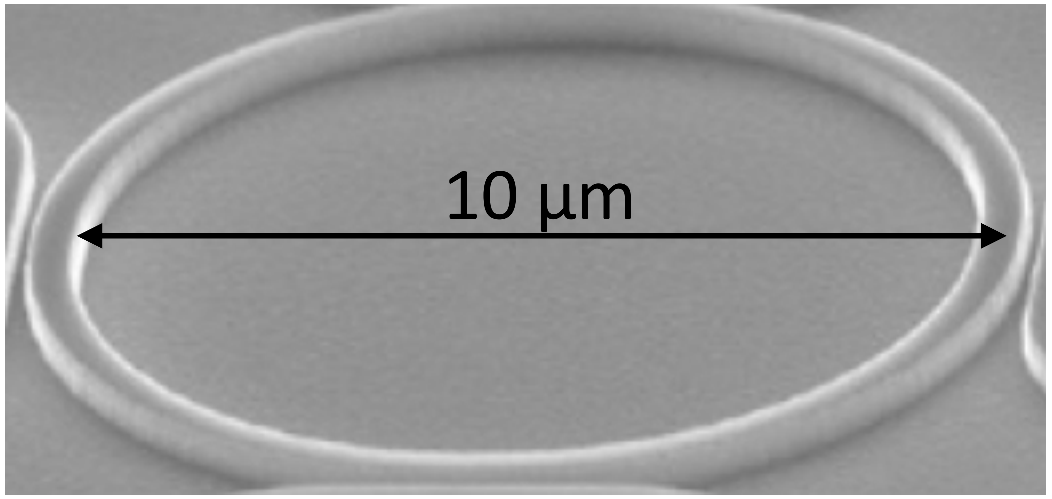
Hall effect in photonics

Mohammad HAFEZI, S MITTAL, J FAN, A MIGDALL & JM TAYLOR
Imaging topological edge states in silicon photonics
Nature Photonics 7, p. 1001 (2013)

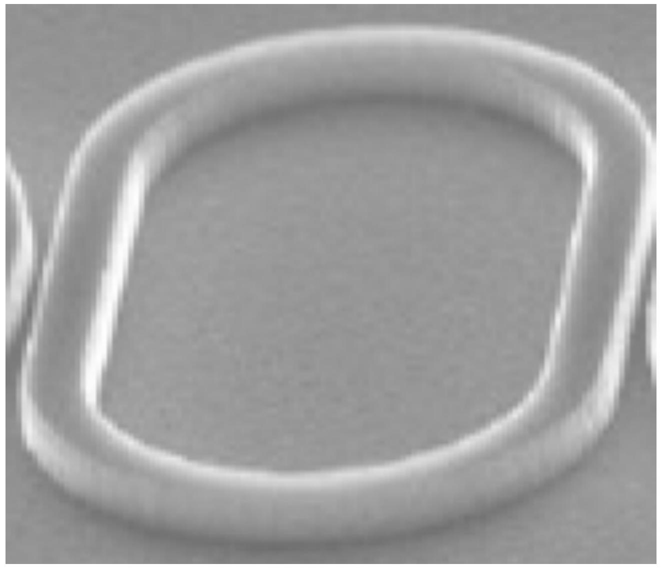
Miguel A. BANDRES, Steffen WITTEK, Gal HARARI, Midya PARTO, Jinhan REN, Mordechai SEGEV,
Demetrios N. CHRISTODOULIDES & Mercedeh KHAJAVIKHAN
Topological insulator laser : Experiments
Science 359, p. 4005 (2018)

Elementary bricks

Goal: Harper-Hofstadter lattice with an ajustable flux α



BANDRES et al, Science 2018



Ring-shaped waveguide,
resonant with light ($Q>10^4$),
in silica for the passive version

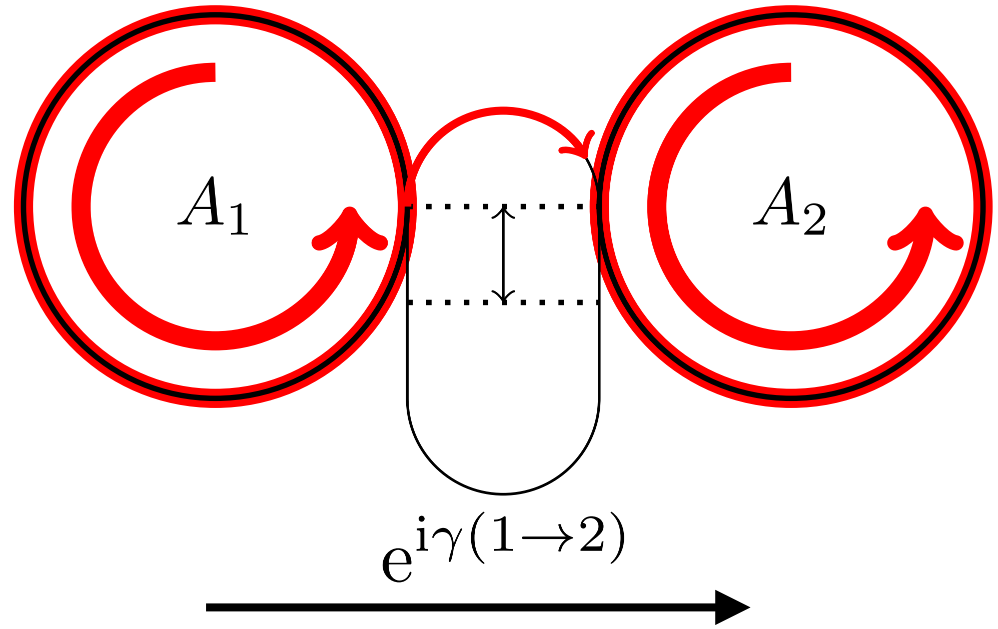
Link between two sites,
non resonant

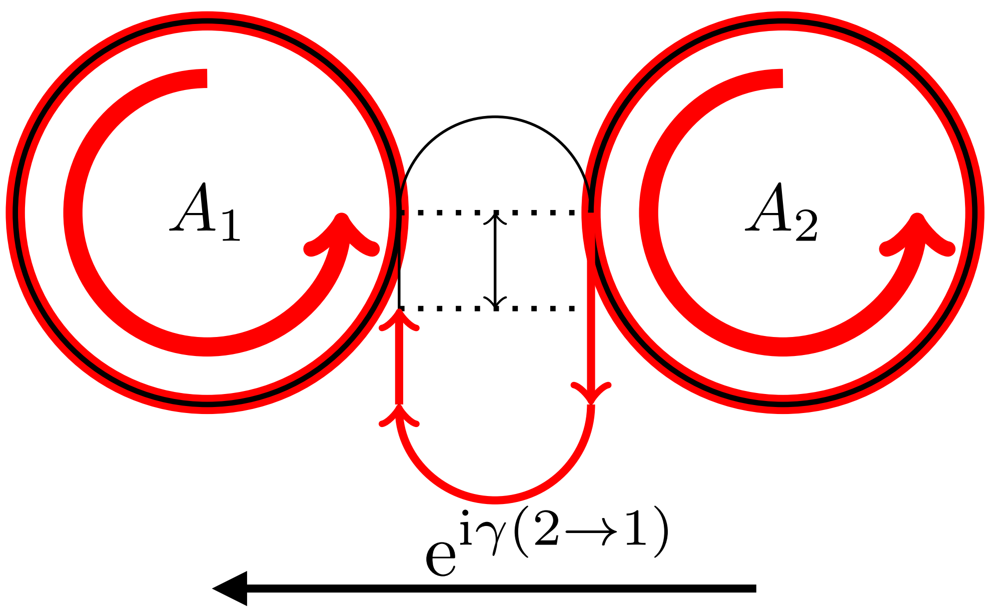
Plays the role of a lattice site

Coupling between two rings

Suppose (for the moment) that light rotates in the positive sense on the site resonators

Transfer from ring A_1 to ring A_2 along the upper part of the link

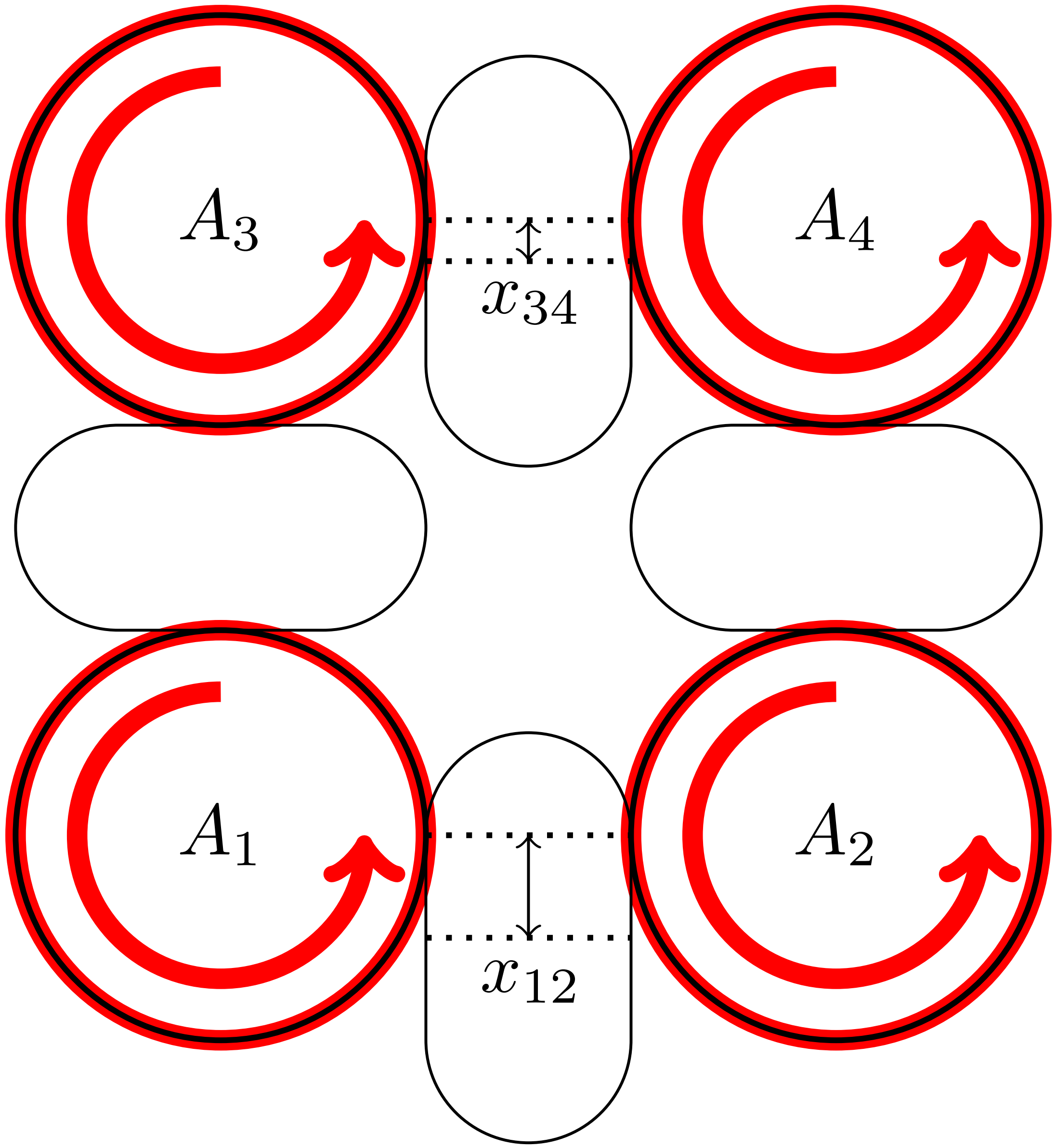




Transfer from ring A_2 to ring A_1 along the lower part of the link

One can thus achieve : $\gamma(2 \to 1) = -\gamma(1 \to 2)$

A square cell of the Harper-Hofstadter lattice



$x_{34} \neq x_{12} \longrightarrow \gamma(3 \to 4) \neq \gamma(1 \to 2)$

Direct implementation of Landau gauge

The accumulated phase on a unit cell is a function of the difference $x_{34} - x_{12}$

Allows one to adjust the flux $\Phi_{\text{cell}} = 2\pi\ \alpha$

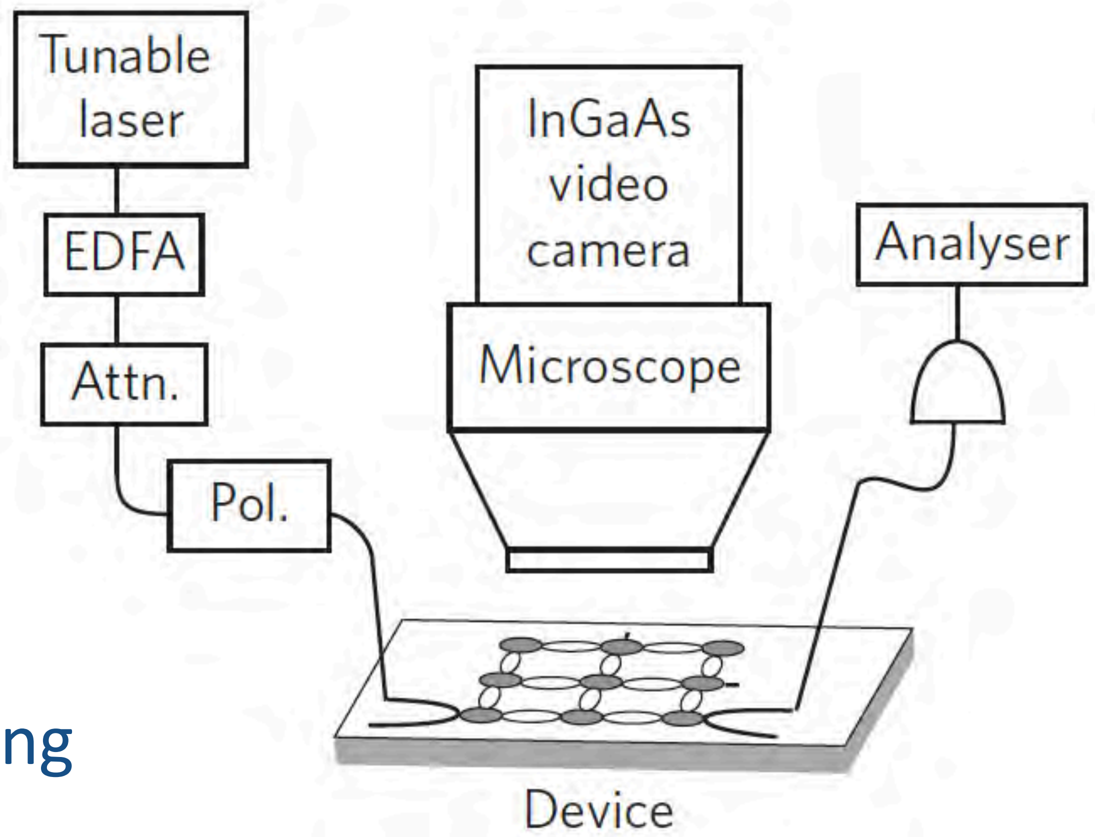
Observation of edge channels

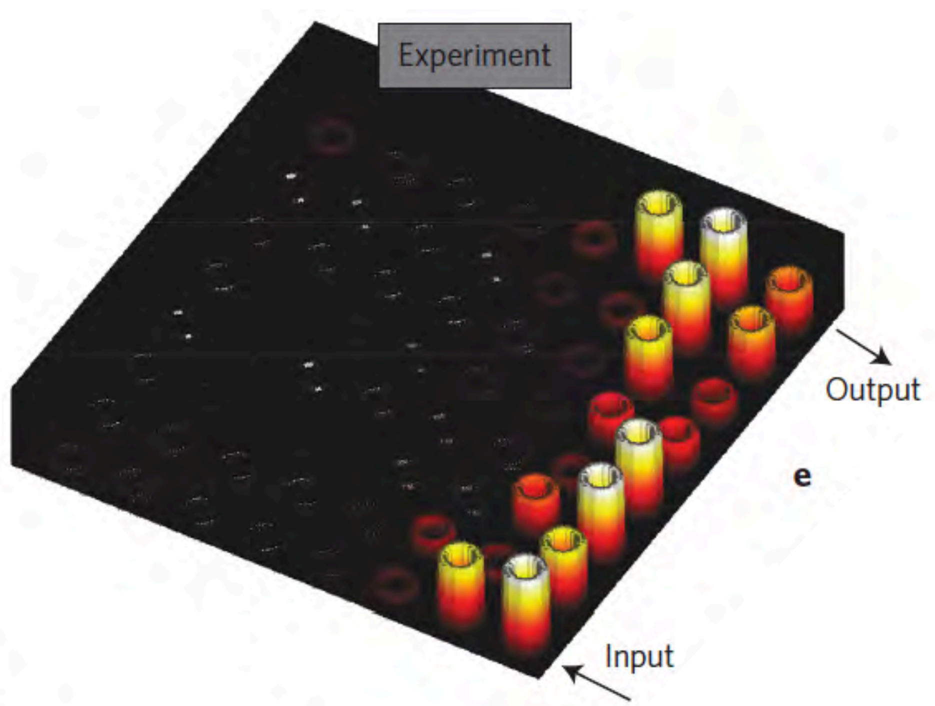
HAFEZI et al. (2013)

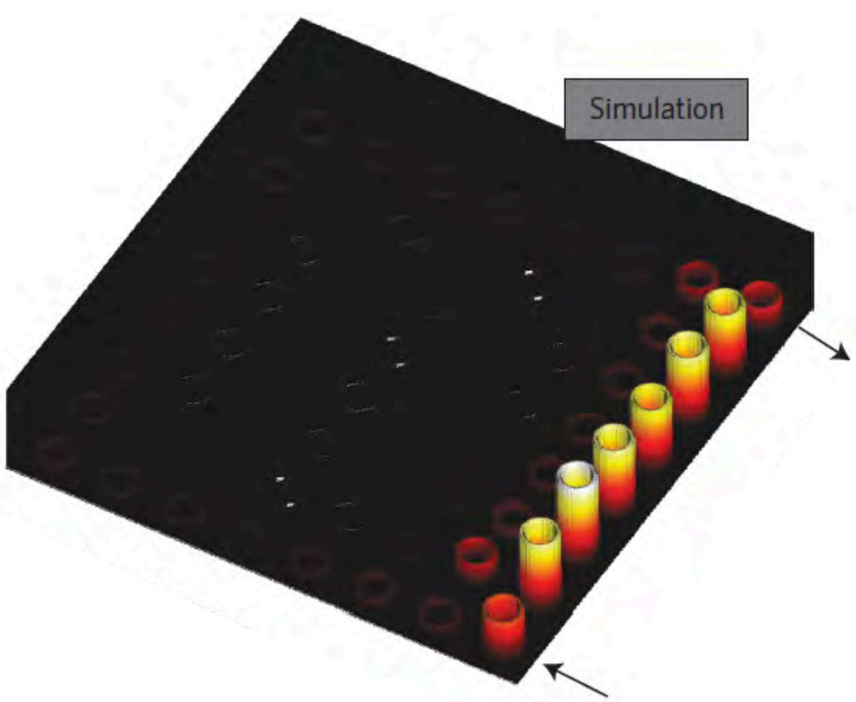
Square lattice with 8x8 rings linked together with $\alpha = 0.15$

The light is injected on one of the outer rings

Measurement of the light scattered by each ring



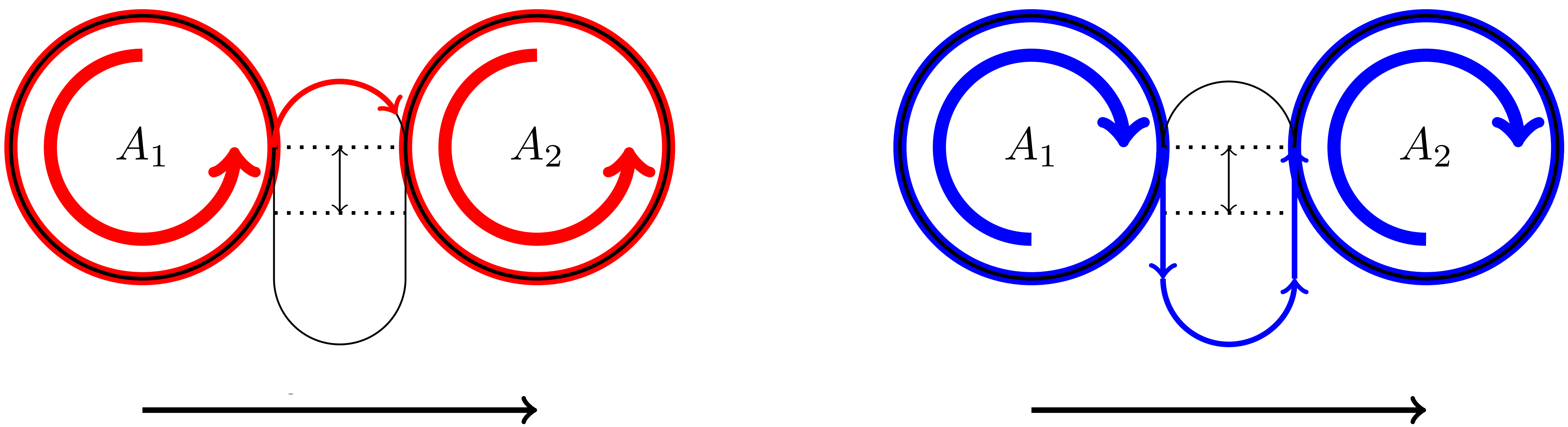




What about time-reversal symmetry?

Here no optical component breaks time-reversal symmetry (one would need some Faraday effect)

By imposing an orientation of the propagation in the rings, we took into account only half of the possible configurations



One passes from flux α to flux $-\alpha$ by exchanging the rotation sign

Spin Hall effect : $|\pm\rangle$ each have a non-trivial topology with opposite effects

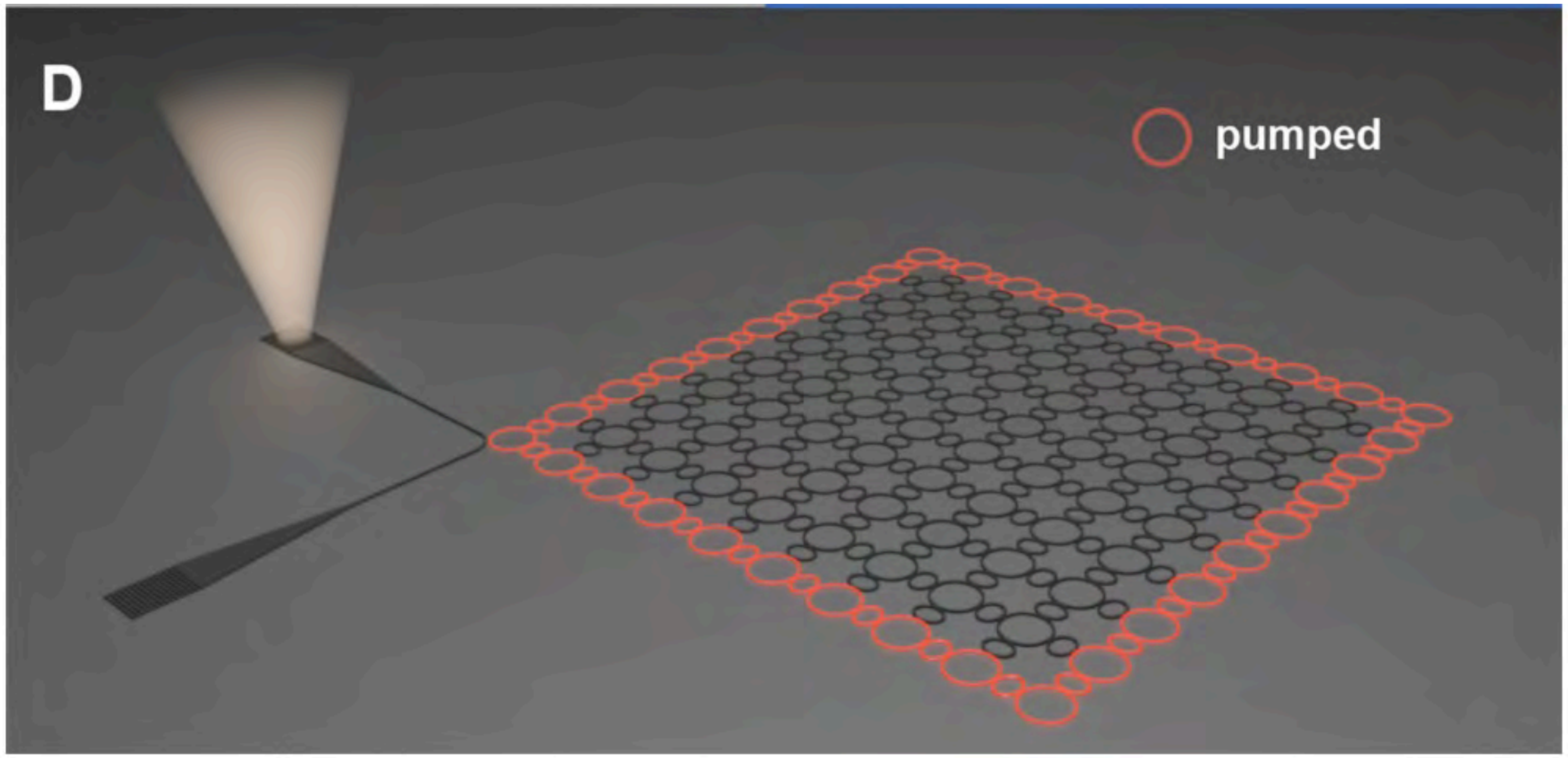
Robust as long as retro-diffusion is negligible

Implementation in active photonics: topological laser

BANDRES et al., Science, 2018, Technion + Orlando

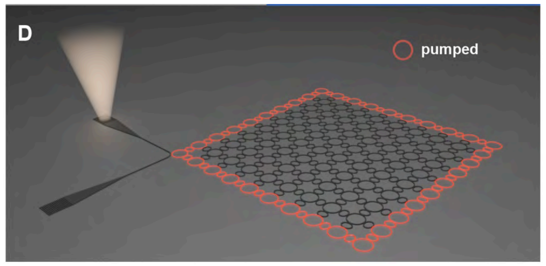
Platform with a InGaAsP quantum well pumped with 1.06 μm light

Lattice of 10x10 resonators, coupled by links achieving a flux $\alpha = 1/4$

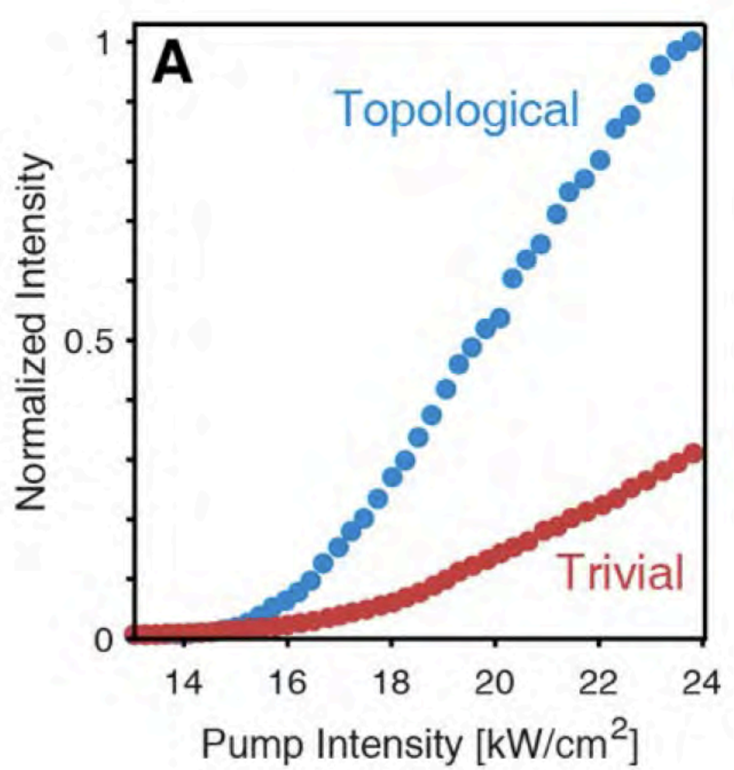


Mesurement of the light directly emitted by the resonators, as well as in input-output couplers at the corner of the sample

Characteristics of the topological laser

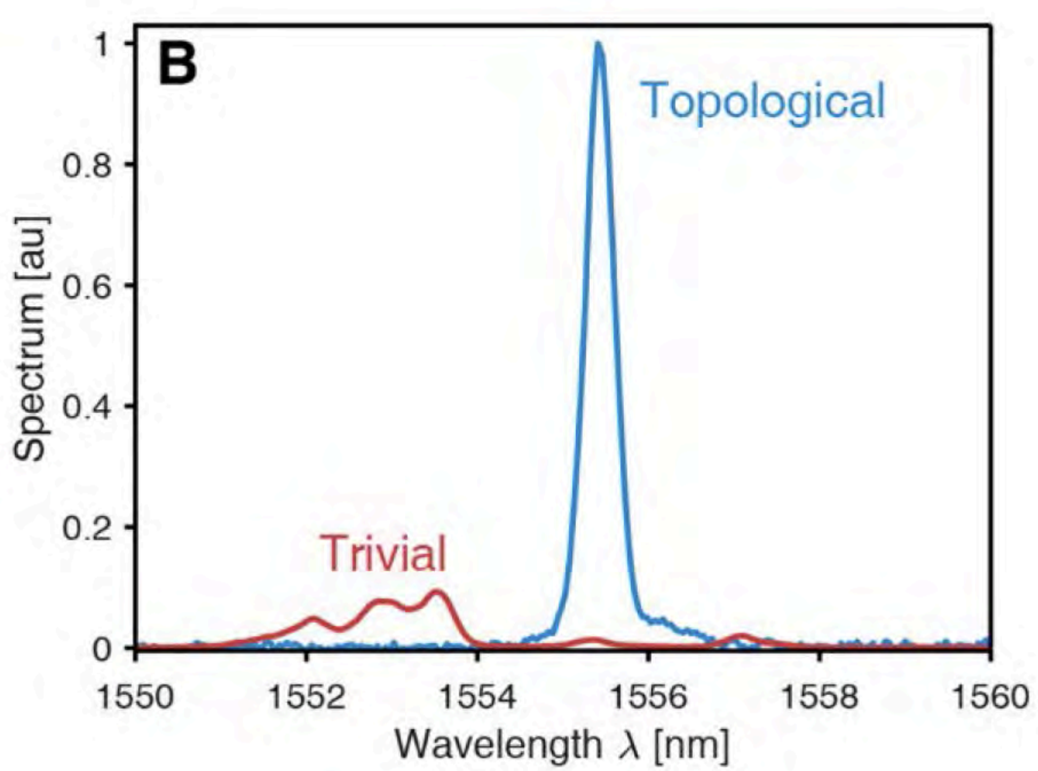


BANDRES et al.
Science 2018



Better efficiency than the same device with $\alpha = 0$

No losses linked to the diffusion towards the bulk of the sample



Narrower spectrum for the topological laser

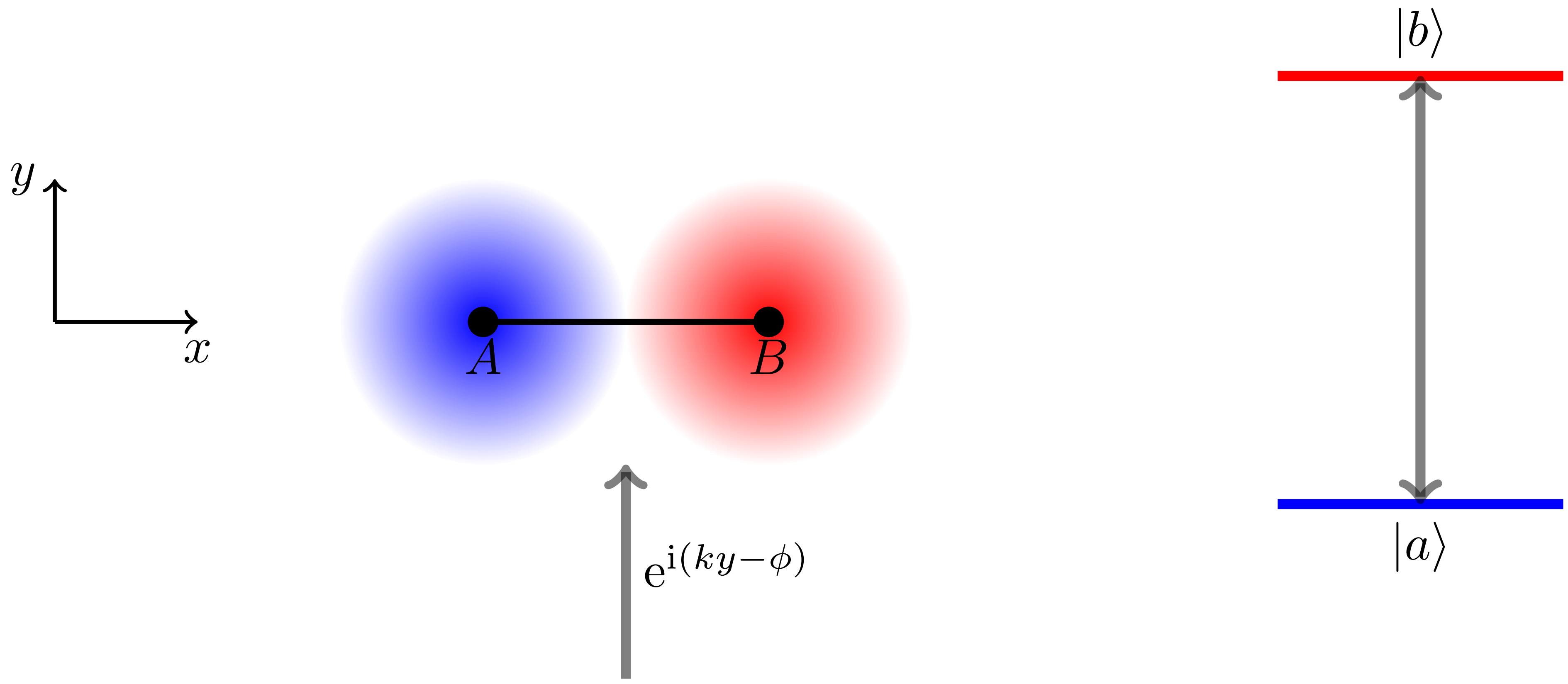
Avoid the non-homogenous broadening due to disorder in the sample

A few perspectives (among many)…

- Use of atomic internal structure
- Synthetic dimensions
- Quench dynamics
- Dynamical gauge fields

Optical transition and laser assisted tunneling

Two-site problem in real space {A, B}, and two internal states $|a\rangle, |b\rangle$



An atom in the internal state $|a\rangle$ (resp. $|b\rangle$) is trapped on site A (resp. B)

In the process $|a\rangle + \text{photon} \longrightarrow |b\rangle$, the phase ϕ gets “printed” on the atomic wave function: equivalent to a complex tunnel matrix element

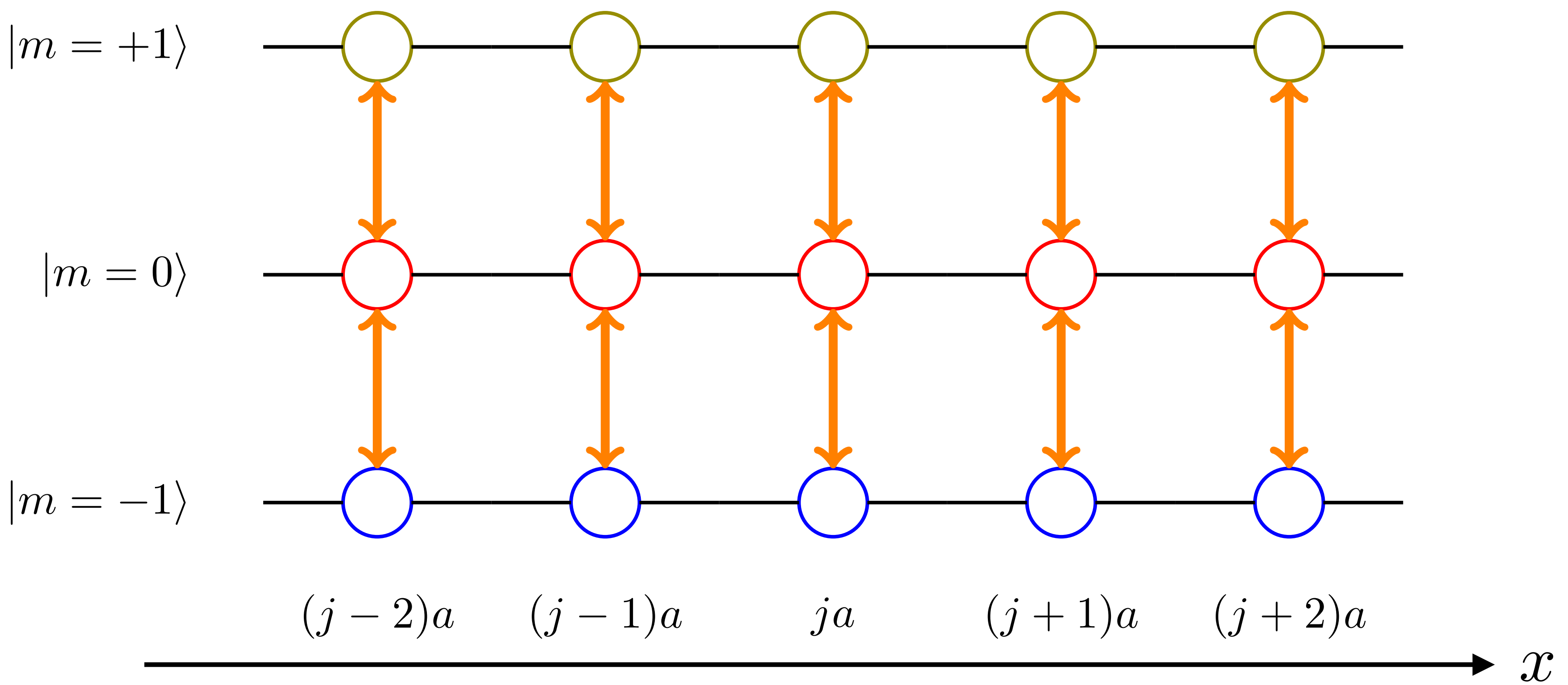
Can be generalized to a whole optical lattice

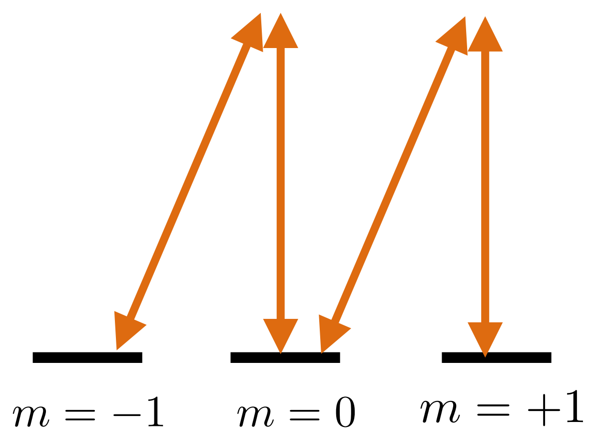
Proposed by Celi et al, 2014

Synthetic dimensions

The internal states of an atom can play the role of one of the two dimensions of the xy plane

Example of a 3-line ribbon with an atom of angular momentum *J* = 1



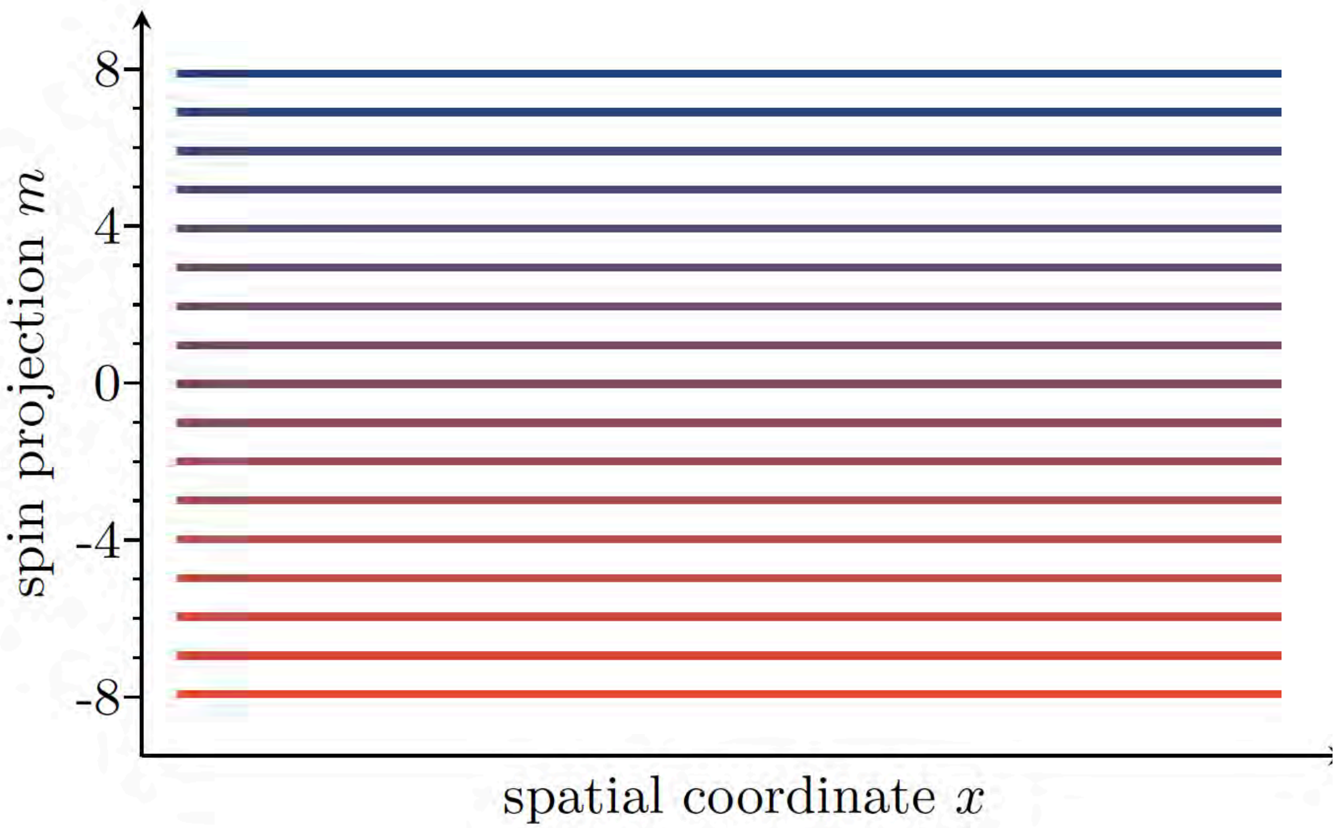


Normal lattice along *x* and synthetic along *y*

Observation of edge states (Florence, Maryland, Boulder), but no real bulk

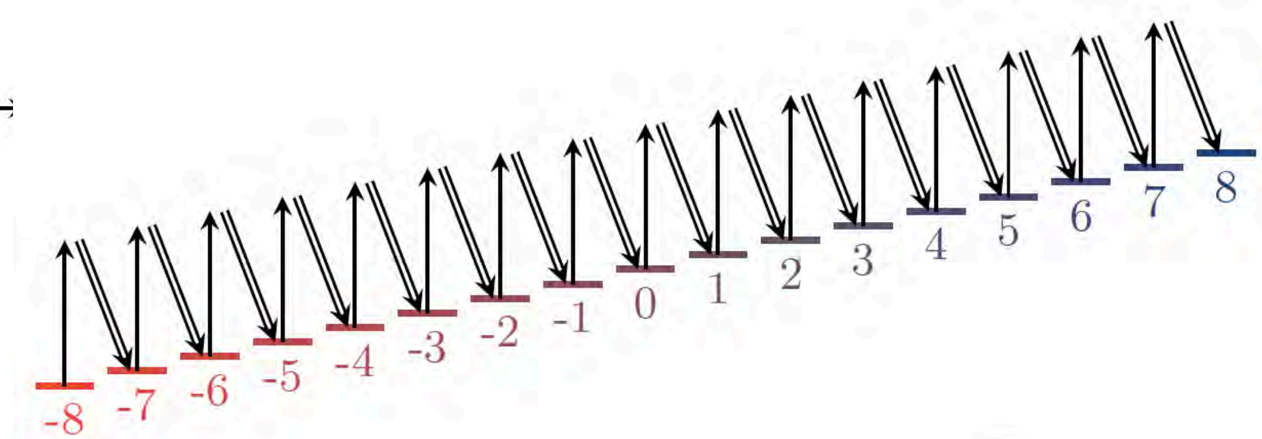
A new playground for synthetic dimensions: Atomic species with large spin

Dysprosium atoms (^{162}Dy) have a J=8 ground state: 17 Zeeman sublevels



Effective stripe geometry:

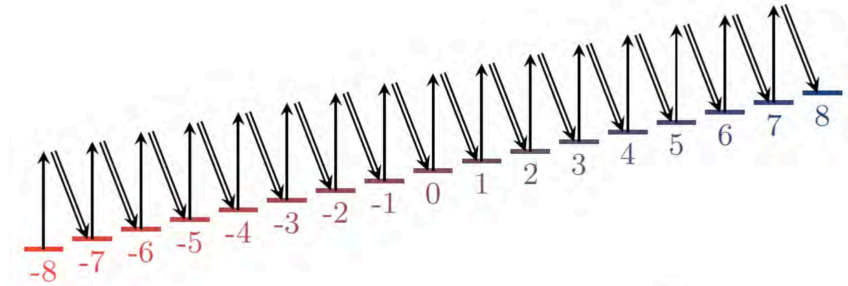
- Continuous motion along real x
- Nearest neighbor hopping along synthetic y, via stimulated Raman transitions



PhD: T. Chalopin, A. Evrard, A. Fabre, T. Satoor
Permanent: R. Lopes, S. Nascimbene, J.Dalibard

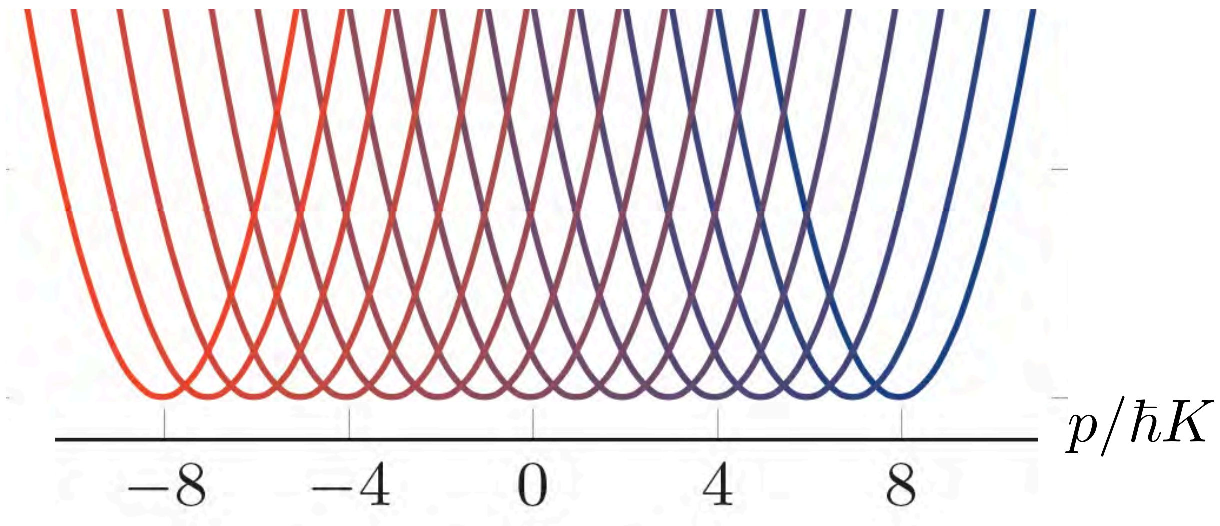
Postdoc: V. Makhalov

Reaching a flat lowest band

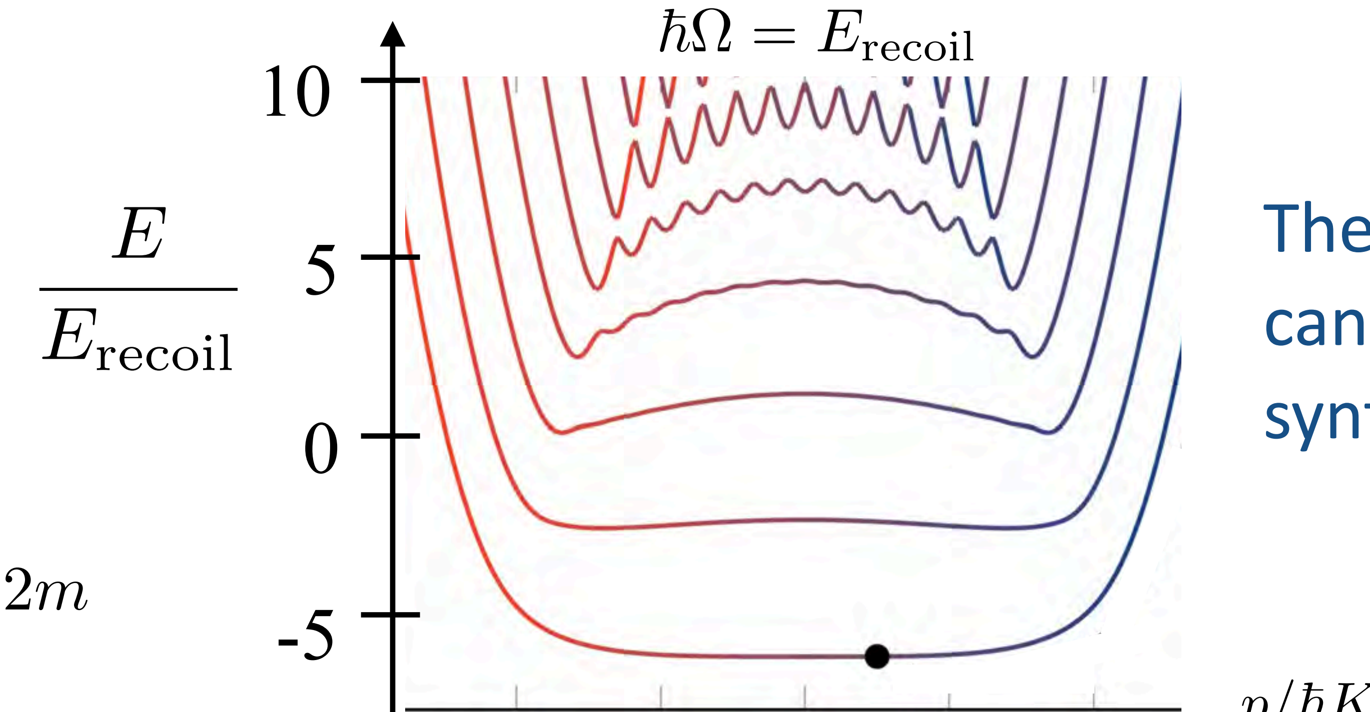


Families of coupled states : $\mathcal{F}(p) = \{|g_m, p - m\hbar K\rangle,\ m = -8, \ldots, , +8\}$

Energy diagram for a vanishing Raman coupling: 17 parabolas



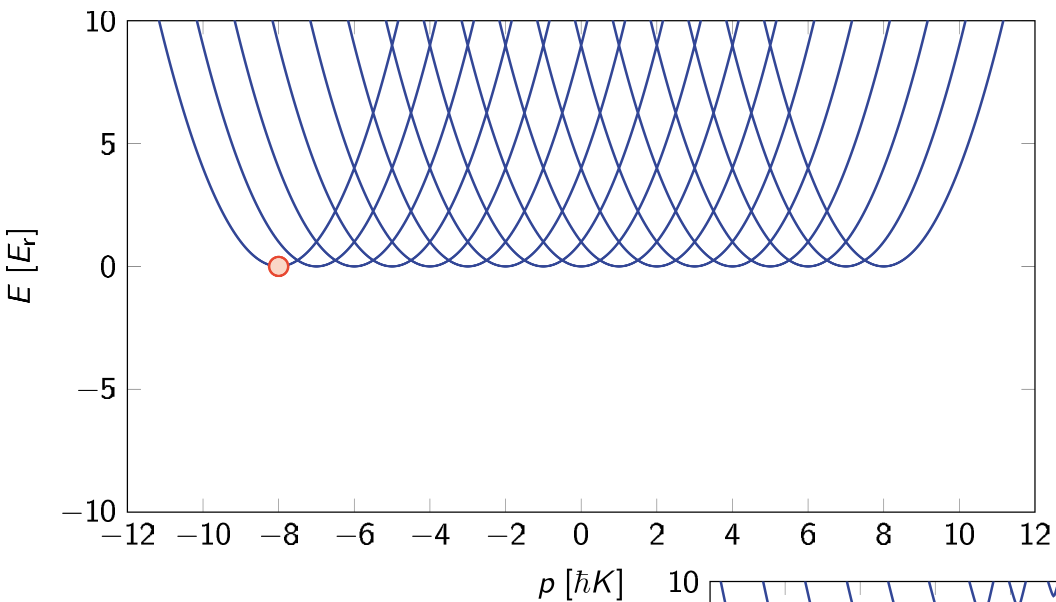
$$\hat{H} = \frac{\hat{p}^2}{2m} - \frac{\hbar\Omega}{2}\left(\hat{J}_+ \mathrm{e}^{\mathrm{i}Kx} + \hat{J}_- \mathrm{e}^{-\mathrm{i}Kx}\right)$$



These quasi-flat bands can be viewed as synthetic Landau levels

$E_{\mathrm{recoil}} \equiv \hbar^2 K^2/2m$

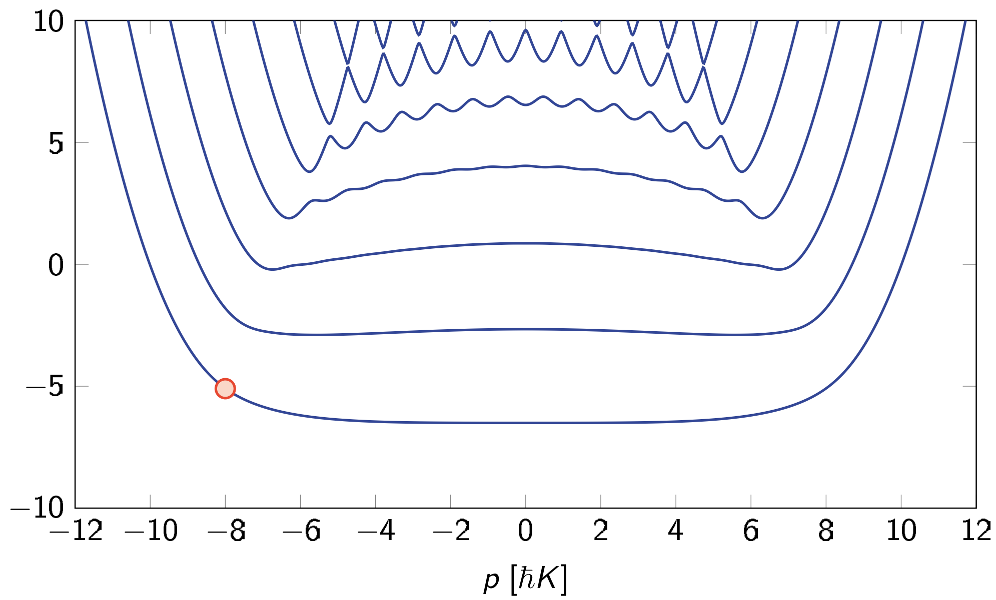
Preparation at a given quasi-momentum



Start with a polarized [162]Dy BEC in $|m_z = -8\rangle$

20000 atoms, $k_{\mathrm{B}}T \sim 0.2\,E_{\mathrm{recoil}}$

Raman coupling $\Omega = 0$

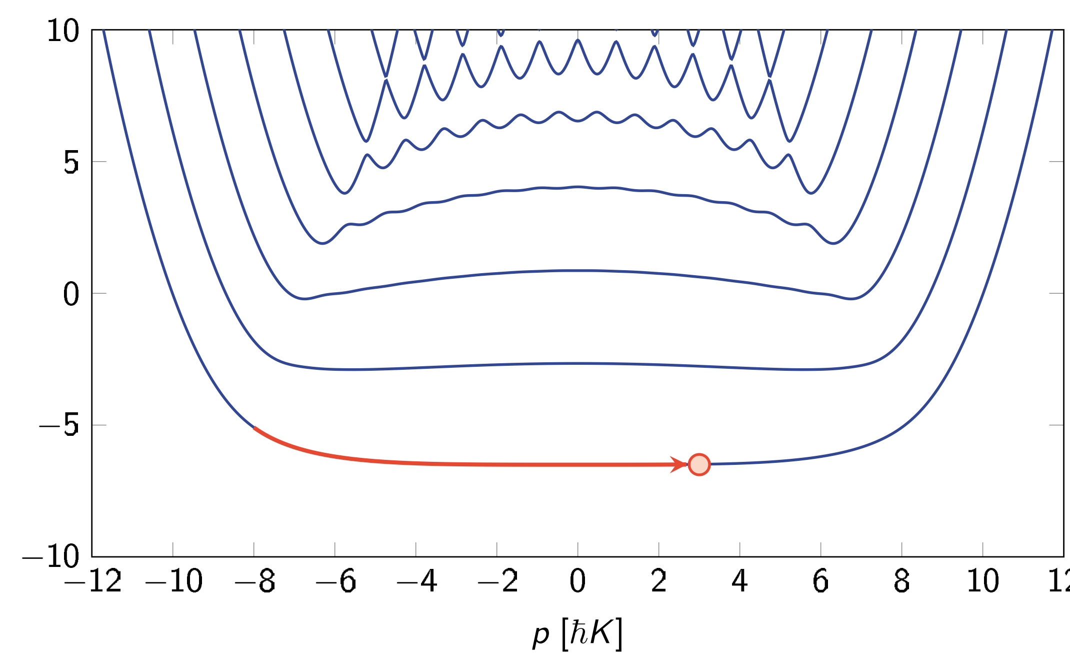


Adiabatically raise the coupling Ω

$\Omega/2\pi \sim 12\,\mathrm{kHz}$

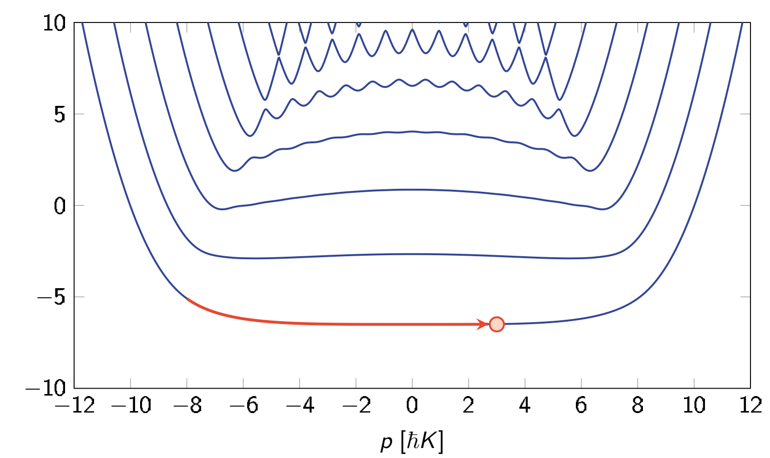
Apply a force F along the real dimension:

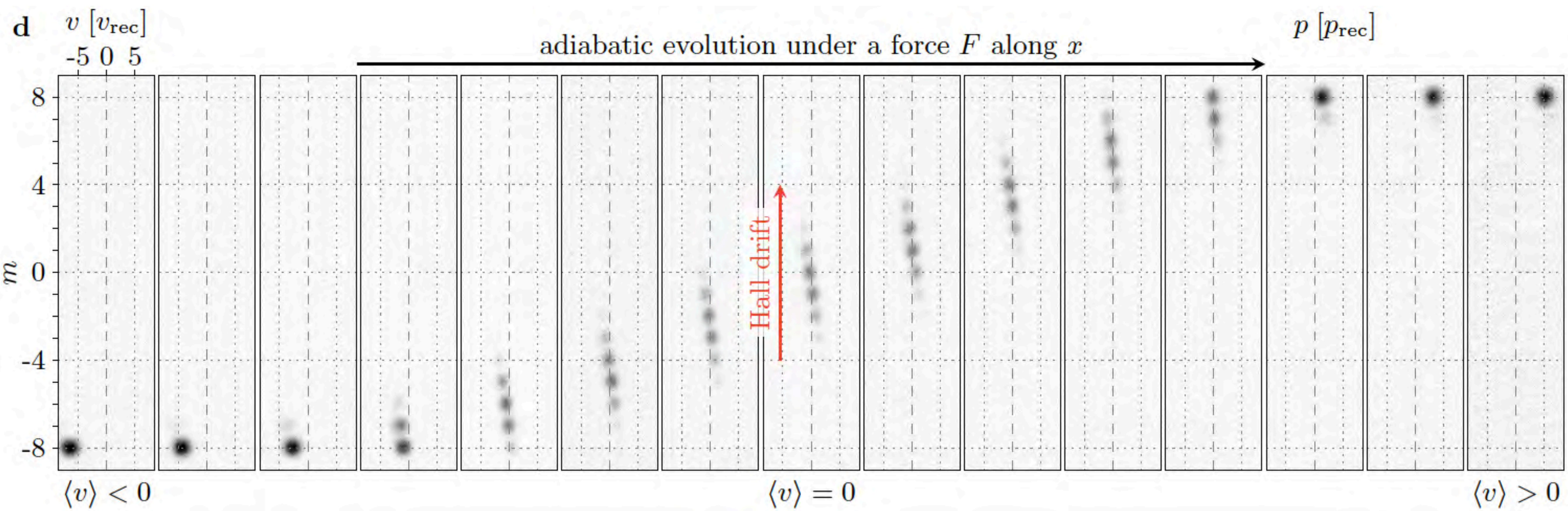
$$\dot{p} = F \longrightarrow p(t) = p_0 + Ft$$



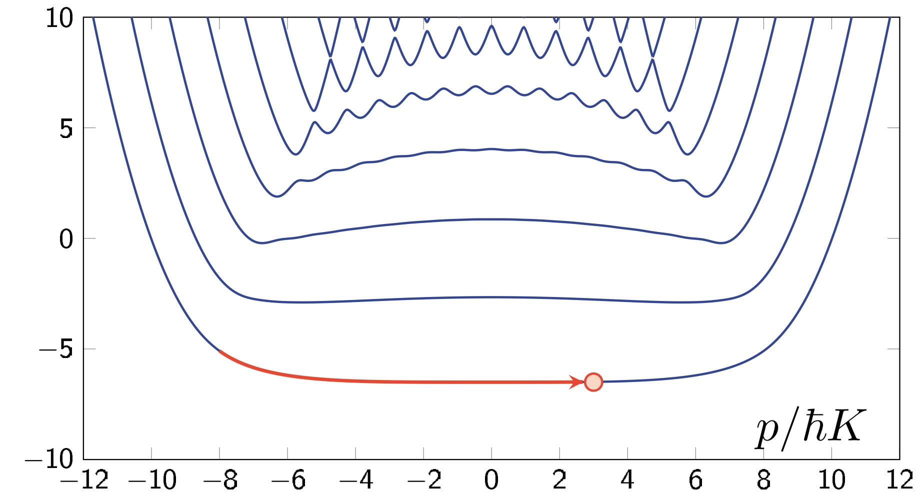
Diagnosis along the real (x) and synthetic (y) directions

Flat band : $\langle v \rangle = \dfrac{\mathrm{d}E}{\mathrm{d}p} \approx 0$





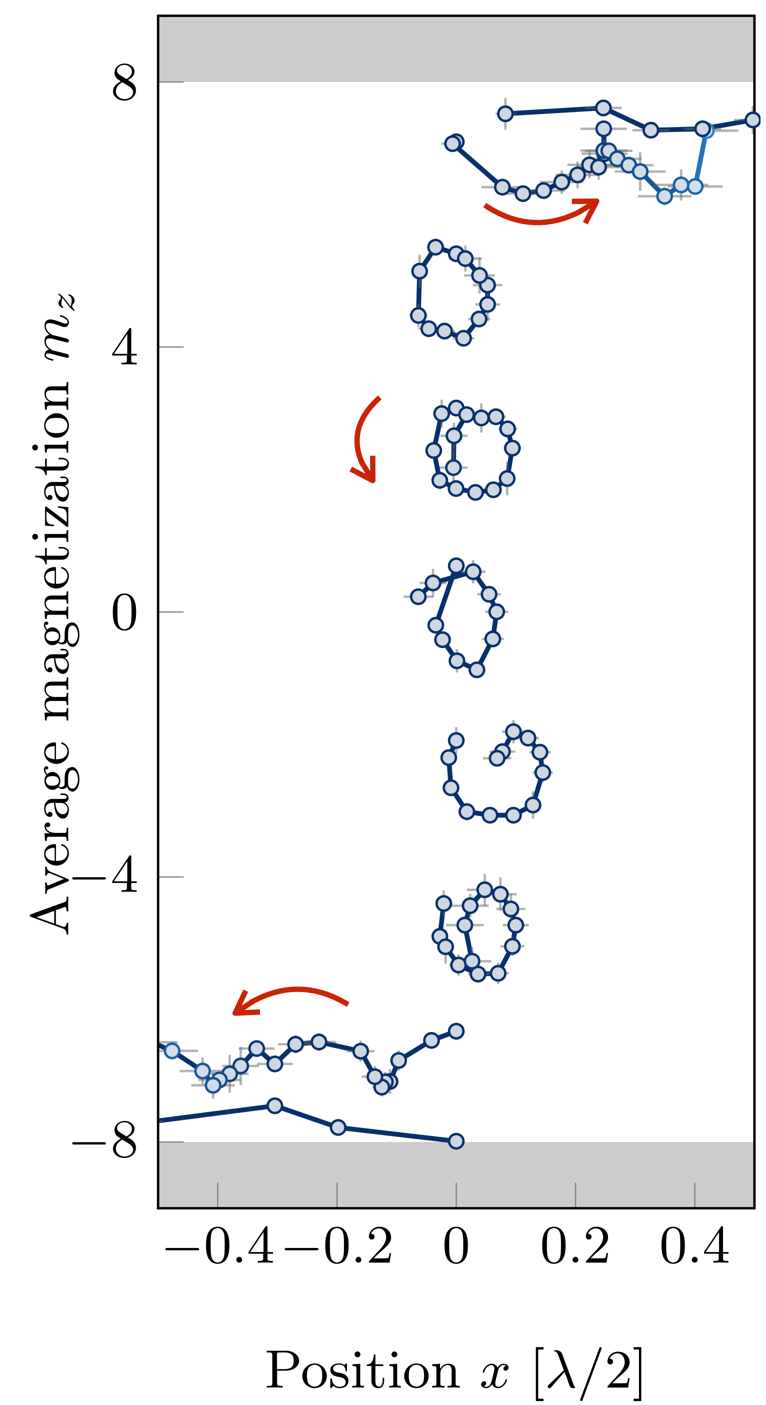
Observing cyclotron orbits



- Prepare a LLL state at a given momentum p
- Sudden (small) velocity kick populating slightly the second LL
- Monitor the evolution of $m_z(t)$ and $v(t)$ [hence $x(t)$]

Trajectories with one point every 2 microseconds

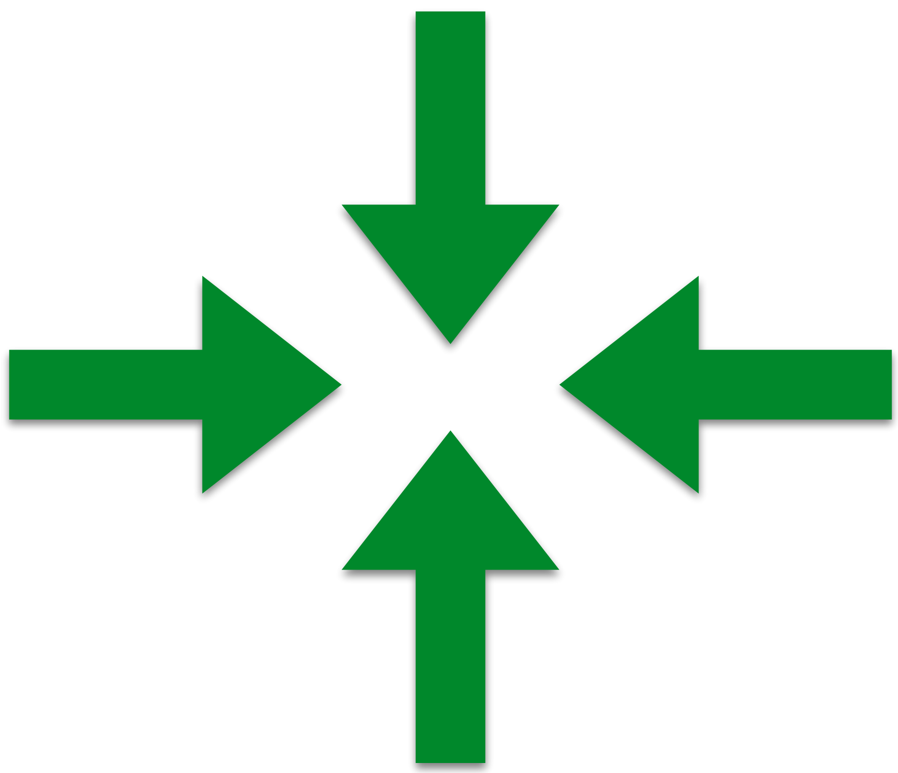
Cyclotron period $\sim$ 20 microseconds

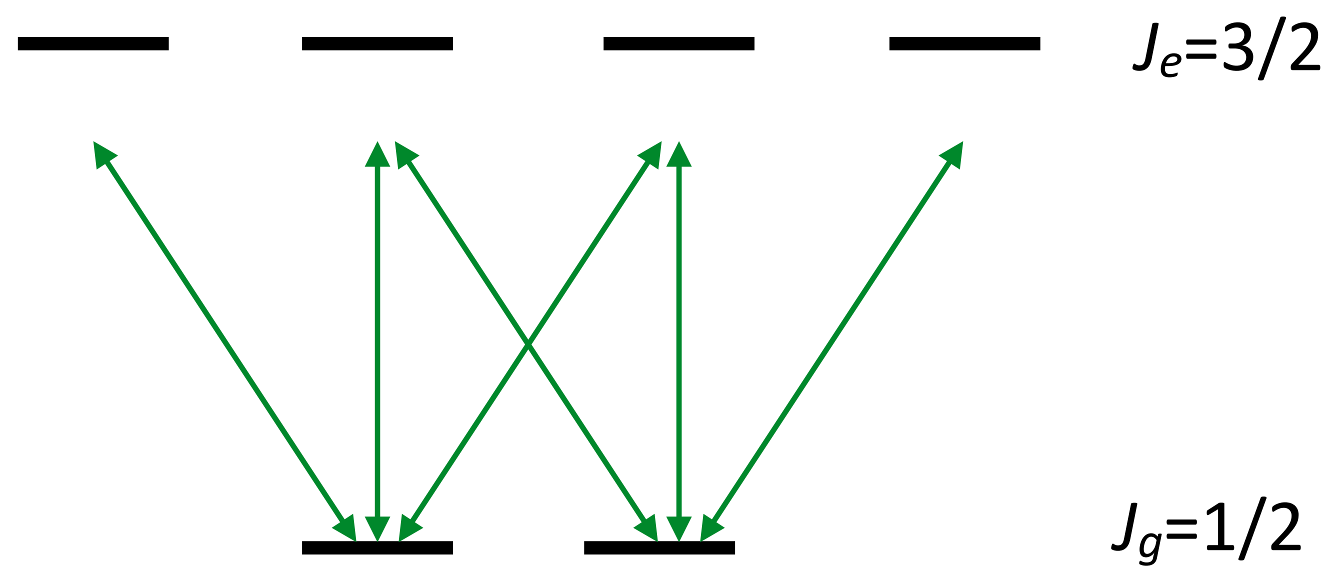


Band with the same topology as the lowest Landau level

Dressed states and adiabatic following

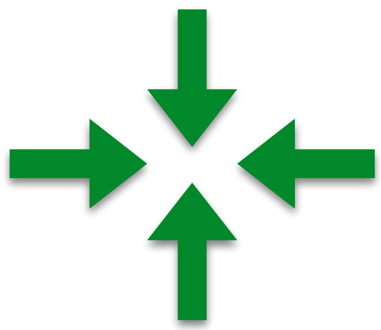
Leave the tight-binding regime to come back to a situation closer to the original quantum Hall effect. Consider a J_g=1/2 — J_e=3/2 transition (alcali-metal atom)



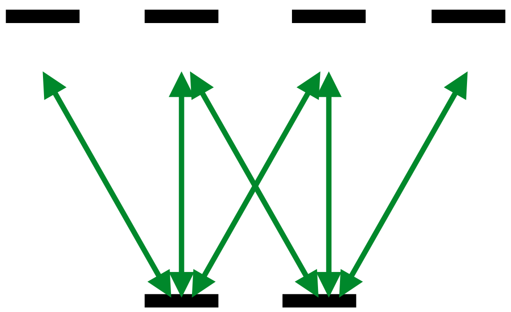


At any point $\boldsymbol{r}$, dressed states of the ground-level manifold $|\chi_{\boldsymbol{r}}^{(\pm)}\rangle$

What happens when the atom moves slowly enough to follow adiabatically one of these dressed states?



Flux lattices



The adiabatic following of a dressed state gives rise to a geometrical phase

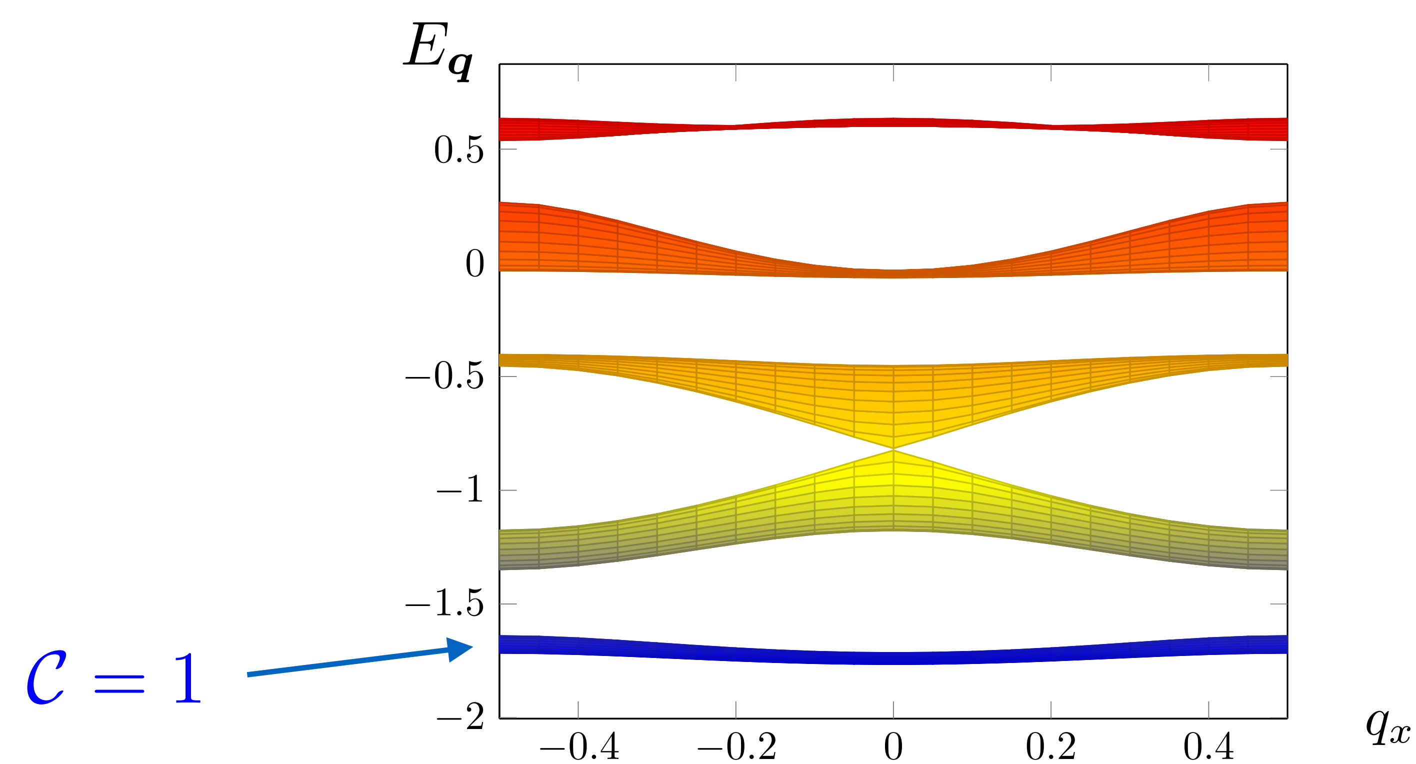
Berry connection: $\boldsymbol{\mathcal{A}}(\boldsymbol{r})$

Berry curvature: $\boldsymbol{\mathcal{B}}(\boldsymbol{r}) = \boldsymbol{\nabla} \times \boldsymbol{\mathcal{A}}(\boldsymbol{r})$

Artificial magnetic field in position space

Flux lattices: periodic lattices for which this artificial field leads to topological bands

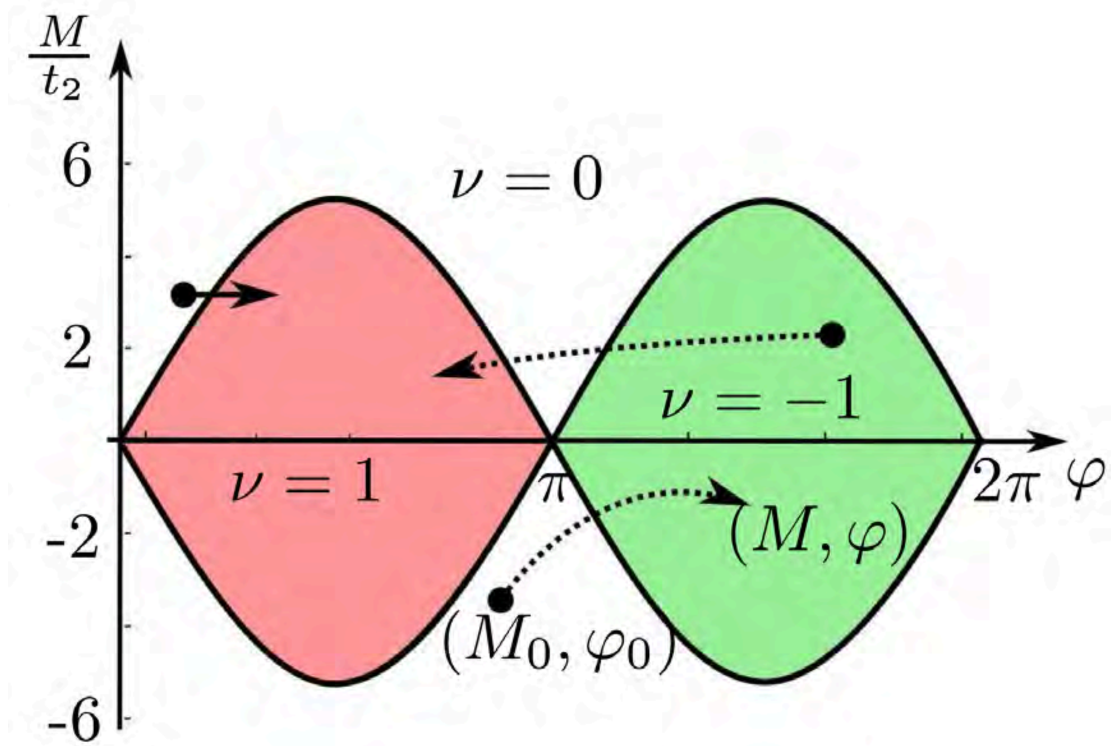
Cooper 2011,
Cooper & Dalibard 2011, 2013



Evolution of topological features in a quench

How do the topological characteristics influence the post-quench dynamics?

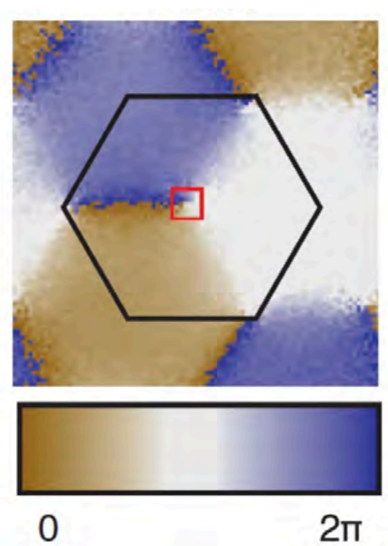
What happens to the edge excitations?



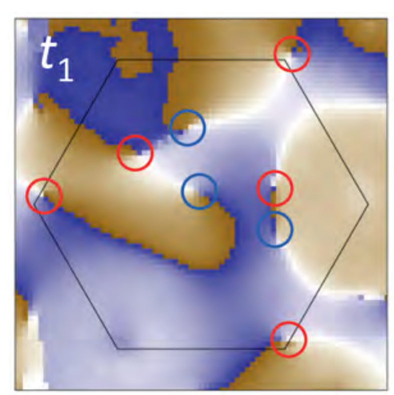
Caio, Cooper & Bhaseen, Quantum Quenches in Chern Insulators, PRL 115, 236403 (2015)

Recent experimental investigation by the Hamburg group

Measurement of the azimuthal angle ϕ_q of the vector $\boldsymbol{h}(\boldsymbol{q})$ on the Bloch sphere



Static

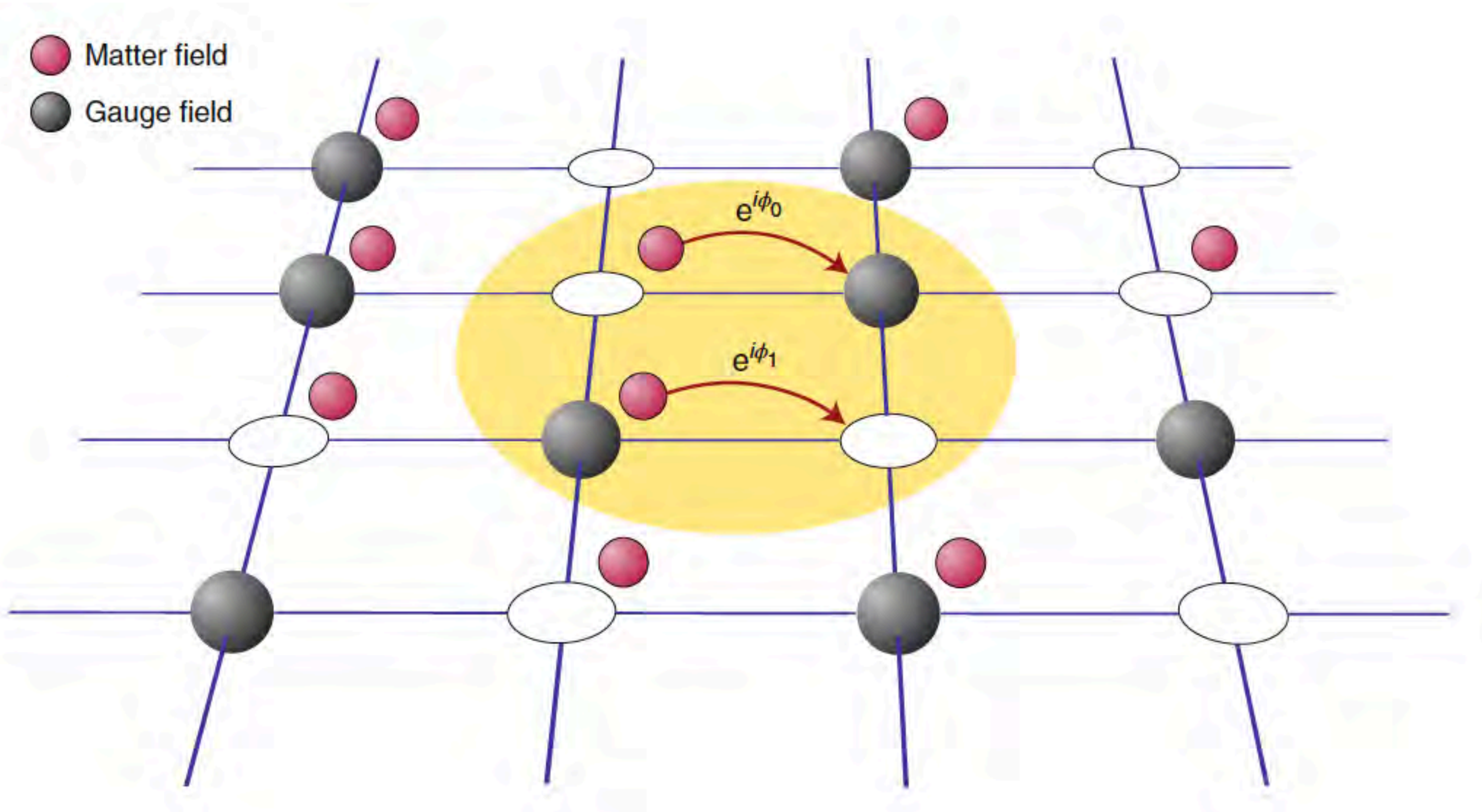


Post-quench: dynamical vorices

Fläschner et al, Nature Physics 14, 265 (2018)

Tarnowski et al, Nature Communications 10, 1728 (2019)

A new development: Dynamical gauge fields



C. Chin, Nat. Phys. **15** 1106, Nov. 2019

The phase and/or the amplitude of the tunneling for site i to site j is controlled by an atomic species, described itself by quantum mechanics

Zurich group: F. Görg et al., Nat. Phys. 15, 1161 (2019).
Munich group: C. Schweizer et al. Nat. Phys. 15, 1168 (2019)

Towards the simulation of models of particle physics, such as lattice quantum chromodynamics?

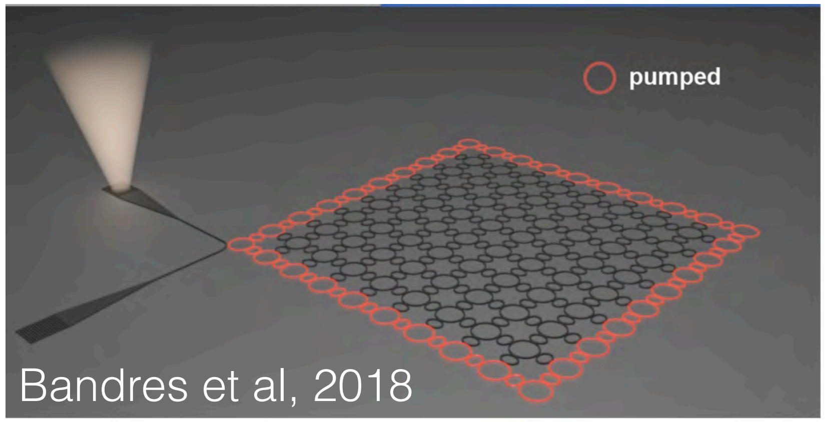
Conclusions

Objective of these first 3 weeks: Introduction to the topological classification of states of matter, very different from geometrical symmetries

- Restriction to 1D or 2D periodic systems

  Winding numbers or Chern numbers for energy bands

- Illustrations from atomic or photonic systems



What we have seen is only a small fraction of the whole zoo of possible topological insulators and superconductors

General classification of these phases in N dimensions:
the "ten-fold way" based on three (non-geometrical) symmetries

Time-reversal symmetry	Particle hole symmetry	Sub-lattice (or chiral) symmetry
T	P	S

The periodic table for topological insulators and superconductors

Adapted from Chiu et al., Rev. Mod. Phys. 88, 035005 (2016)

				Dimension of space (for a gapped bulk material)						
Class	T	C	S	1	2	3	4	5	6	7
A	0	0	0	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0
AIII	0	0	1	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$
AI	$+$	0	0	0	0	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$
BDI	$+$	$+$	1	$\mathbb{Z}$	0	0	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$
D	0	$+$	0	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$2\mathbb{Z}$	0
DIII	$-$	$+$	1	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$2\mathbb{Z}$
AII	$-$	0	0	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0
CII	$-$	$-$	1	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0
C	0	$-$	0	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0
CI	$+$	$-$	1	0	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$

SSH

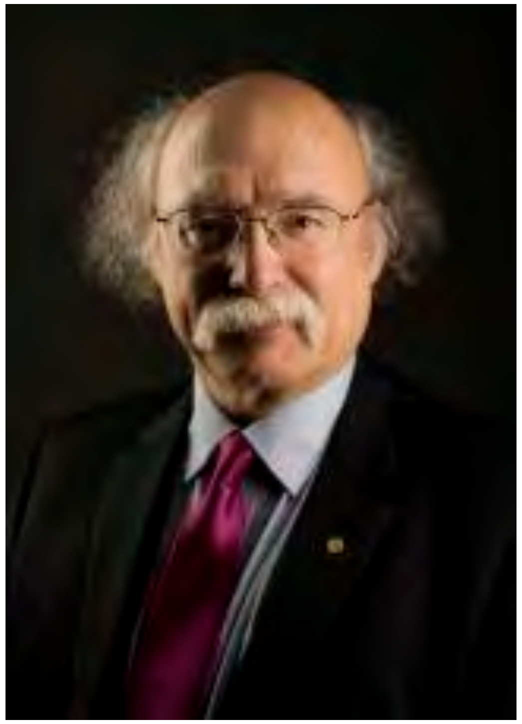
Kitaev chain

Haldane model

Physics and topology

J.M. Kosterlitz

D.J. Thouless



F.D. Haldane

Nobel prize 2016:
"for theoretical discoveries of topological phase transitions and topological phases of matter."

Next lecture

First three lectures