• Exercise 4.1.

- In the rest frame $E = m$, $p = 0$. Performing a boost with rapidity $\eta$ (along $x$ axis for example) we have $E = mc^\eta, \ p_x = m \sinh \eta$. Thus, $(E + p)/(E - p) = e^{2\eta}$.

- Performing a second boost with rapidity $\eta'$ along $x$ we get $E \rightarrow E \cosh \eta' + p \sinh \eta'$ and $p \rightarrow E \sinh \eta' + p \cosh \eta'$, so

$$e^{2\eta} \rightarrow \frac{(E \cosh \eta' + p \sinh \eta') + (E \sinh \eta' + p \cosh \eta')}{(E \cosh \eta' + p \sinh \eta') - (E \sinh \eta' + p \cosh \eta')} = e^{2\eta} \frac{E + p}{E - p} = e^{2(\eta + \eta')}.$$

• Exercise 4.2.

Let $v^0 = \xi^R \sigma^i \psi_R$ and $v^i = \sigma^i \sigma^j \psi_R$. Using the rules for spinor transformations under infinitesimal boosts and rotations

$$\psi_R \rightarrow e^{-(i\theta_i + \eta_i) \sigma^i} \psi_R = (1 + (-i\theta_i + \eta_i) \frac{\sigma^i}{2} + \ldots) \psi_R,$$

we get for $v^0$

$$v^0 \rightarrow \xi^R (1 + (i\theta_i + \eta_i) \frac{\sigma^i}{2}) (1 + (-i\theta_i + \eta_i) \frac{\sigma^i}{2}) \eta_R = \xi^R (1 + \eta_i \sigma^i + \ldots) \eta_R$$

and for $v^i$

$$v^i \rightarrow \xi^R (1 + (i\theta_j + \eta_j) \frac{\sigma^j}{2}) \sigma^i (1 + (-i\theta_k + \eta_k) \frac{\sigma^k}{2}) \eta_R = \xi^R (\sigma^i - \theta_j \epsilon^{ijk} \sigma^k + \eta_j \delta^{ji} + \ldots) \eta_R$$

This is exactly transformation properties of a (contravariant) four-vector under infinitesimal boost $\eta_i$ (boost of rapidity $|\eta|$ in the direction of $\eta_i/|\eta|$). We also checked that it has correct properties under usual rotations $\theta^i$.

Calculations for $\xi^L \bar{\sigma}^\mu \eta_L$ are completely analogous.

You can also check everything explicitly for finite boosts, using the formula

$$e^{\eta_i \sigma^i} = \cosh \eta + \frac{\eta_i}{\eta} \sigma^i \sinh \eta,$$

where $\eta = \sqrt{\eta^i \eta^i}$ is rapidity of the boost and $\eta_i/\eta$ — its direction.

• Exercise 4.3.
Transformations of the double-contravariant tensor are
\[ F^\mu \nu \rightarrow \Lambda^\mu \rho \Lambda^\nu \sigma F^{\rho \sigma} , \]
where \( \Lambda^\mu \rho = \exp\{-i/2 \omega_{\alpha \beta} (J^{\alpha \beta})^\mu \rho \} \), and
\[ (J^{\alpha \beta})^\mu \rho = i(\eta^{\alpha \mu} \delta^\beta \rho - \eta^{\beta \mu} \delta^\alpha \rho) . \]

Then, to the first order
\[ \delta F^{\mu \nu} = \omega^\mu \rho F^{p \nu} - \omega^\nu \rho F^{p \mu} . \]
Using the explicit form \( \omega_{i0} = -\eta^i \) and \( \omega_{ij} = \epsilon^{ijk} \theta^k \) one gets for \( B \) and \( E \)
\[ \delta E = -[\eta \times B] + [\theta \times E] , \]
\[ \delta B = +[\eta \times E] + [\theta \times B] . \]