The Sierpinsky Gasket is a fractal object built by effacing one triangle in the middle of each existing triangle.

a) Find the fractal dimension $D_f$ of the Sierpinsky Gasket.

b) We built an Ising model without magnetic field on a lattice formed on the Sierpinsky Gasket: there is one spin on each corner which interacts only with nearest neighbors.

Renormalise the system by summing over $s_2$, $s_4$, $s_6$, as in figure 1. Pose $\beta J = K$ and $\exp(Ks_1s_2) = \cosh(K)(1+s_1s_2 \tanh(K))$.

![Figure 1](image)

c) Write in an implicit form (i.e $h_i(v', C') = f_i(v, C)$ with $i = 1, 2$) the two equations of renormalisation for the coupling constants $v = \tanh(K)$ and the multiplicative $C$.

d) Show that $v = 0$ and $v = 1$ are two fixed points of the transformation.

e) If $v \neq 1$ we can calculate the ratio between the two equations obtained at point b) which makes the constant $C$ to disappear. We obtain an equation $h(v') = f(v)$. Assuming that $h(v')$ is invertible over $[0, 1]$ (you can see this with Mathematica or analyzing the function...) show that $v = 0$ is a stable point, that is $| (h^{-1}(f(v = 0)))' | < 1$.

f) By numerically solving the equation we can show that a third fixed point exists, $v^*$ with $0 < v^* < 1$ in such a way that $h(v^*) = f(v^*)$, but $h'(v^*) \neq f'(v^*)$. Say why this implies the existence of a phase transition in $v^*$. 