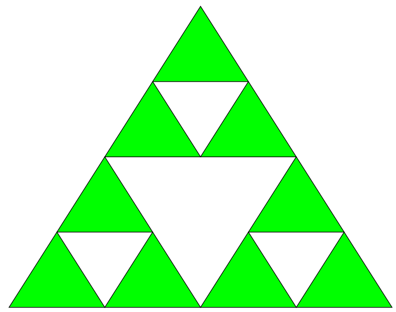
Series 11

The Sierpinsky Gasket is a fractal object built by effacing one triangle in the middle of each existing triangle.



a) Find the fractal dimension D_f of the Sierpinsky Gasket.

b) We built an Ising model without magnetic field on a lattice formed on the Sierpinsky Gasket: there is one spin on each corner which interacts only with nearest neighbors.

Renormalise the system by summing over s_2, s_4, s_6, as in figure 1. Pose $\beta J = K$ and $\exp(Ks_1s_2) = \cosh(K)(1 + s_1s_2\tanh(K))$.

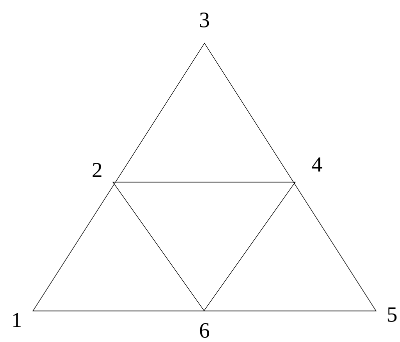


Figure 1:

c) Write in an implicit form (i.e $h_i(v', C') = f_i(v, C)$ with $i = 1, 2$) the two equations of renormalisation for the coupling constants $v = \tanh(K)$ and the multiplicative C.

d) Show that $v = 0$ and $v = 1$ are two fixed points of the transformation.

e) If $v \neq 1$ we can calculate the ratio between the two equations obtained at point b) which makes the constant C to disappear. We obtain an equation $h(v') = f(v)$. Assuming that $h(v')$ is invertible over $[0, 1)$ (you can see this with Mathematica or analyzing the function...) show that $v = 0$ is a stable point, that is $|\left(h^{-1}(f(v = 0))\right)'| < 1$.

f) By numerically solving the equation we can show that a third fixed point exists, v^* with $0 < v^* < 1$ in such a way that $h(v^*) = f(v^*)$, but $h'(v^*) \neq f'(v^*)$. Say why this implies the existence of a phase transition in v^*.