Series 10

We apply a renormalization group decimation technique to the system of $2N$ spins on two lines with one interaction costant (K_1) between spins in the same line and another (K_2) between the spins of the two lines as in Fig. 1.

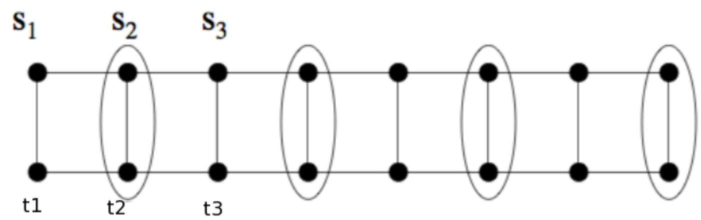

Figure 1: Ising model on two coupled lines.

Consider the following Hamiltonian:

$$-\beta H = \sum_{i=1}^{N} [K_1(s_i s_{i+1} + t_i t_{i+1}) + K_2(s_i t_i + s_{i+1} t_{i+1}) + 2C]$$

The decimation procedure consists into integrate one spin over two, for example the spins with *even* index i. The renormalized Hamlitonian $-\beta H'$ is defined from:

$$e^{-\beta H'(\{s_i,t_i\},\text{odd i})} = \sum_{\{s_i,t_i\}\text{even i}} e^{-\beta H}$$

1. Show that
$$-\beta H' = \sum_{\text{odd i}} [K_2(s_i t_i + s_{i+2} t_{i+2}) + 4C + \widetilde{H}_i]$$
where
$$\widetilde{H}_i = \log[e^{2K_2} 2\cosh(K_1(s_i + t_i + s_{i+2} + t_{i+2})) + e^{-2K_2} 2\cosh(K_1(s_i - t_i + s_{i+2} - t_{i+2}))]$$

2. the term $\widetilde{H}_i$ is a function of only $(s_i + s_{i+2})$ and $(t_i + t_{i+2})$; moreover it is invariant the transformation $(s,t) \to (-s,-t)$ as well as the transformation $(s,t) \to (t,s)$. We want to rewrite $\widetilde{H}_i$ in a more convenient form; given those symmetry considerations we can guess a form like:
$$\widetilde{H}_i = D + B(s_i + s_{i+2})(t_i + t_{i+2}) + A(s_i + s_{i+2})^2 + A(t_i + t_{i+2})^2 + E(s_i + s_{i+2})^2(t_i + t_{i+2})^2 \quad (1)$$
Show that the four constants D, B, E, A are enough for taking into account all the 16 cases for $\{s_i, s_{i+2}, t_i, t_{i+2}\}$.

The renormalized Hamiltonian has more coupling constants: in particular, devoloping the equation (1) you see that the *diagonal interactions* $(s_i t_{i+2} + s_{i+2} t_i)$ and the *square interactions* $(s_i t_{i+2} s_{i+2} t_i)$ appeared. This is a typical feature of the renormalization procedures, named *proliferation of couplings*.

Verify that the introduced new coupling constants are enough for defining a complete set of renormalization equations without further proliferation

3. starting from the Hamiltonian

$$-\beta H = \sum_{i=1}^{N} [K_1(s_i s_{i+1} + t_i t_{i+1}) + K_2(s_i t_i + s_{i+1} t_{i+1}) + K_3(s_i t_{i+1} + s_{i+1} t_i) + K_4(s_i t_{i+1} s_{i+1} t_i) + 2C]$$

   repeat the decimation procedure;

4. find the equation of renormalization;

5. verify that the infinite temperature $K_1 = K_2 = K_3 = K_4 = 0$ is a fixed point of the system.