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Relativity and Cosmology

CMI 100
$15^{15} - 17^{00}$ (c)
$17^{15} - 19^{00}$ (e)

Lecture # 1.

Semester I:
Special and general relativity

Semester II:

General relativity and cosmology

Why special relativity?

- Basis for particle physics, electrodynamics, accelerator physics, etc

Why general relativity?

- Basis for cosmology and astrophysics

Plan for the 1st semester

Chapter I: Special relativity

- Reminder of results from electrodynamics
  - (i) Lorentz transformations
  - (ii) 4-vectors, tensors
  - (iii) energy, momentum
- Special relativity in arbitrary coordinate systems
- Acceleration and SR

Chapter II: Formulation of GR

- Newtons gravity
- equivalence principle
- Gravitational forces
- Newton approximation
- tensorial analysis
- Curvature
- Flat versus curved space-time

Chapter $\overline{IV}$:

Tests and predictions of GR

- Schwarzchild Solution
- light bending in GR
- Perihelium of Mercury
- gravitational waves
- binary systems
- black holes

Chapter $\overline{\underline{III}}$

- Action principle for GR
- Einstein equations
- Energy-momentum tensor

Books :

- Gravitation and Cosmology

  S. Weinberg , 1972

- Landau, Lifshits, vol $\overline{II}$

  Field Theory

- B. Schutz

  A first course in general relativity

  Cambridge U press, 2007

  (first published 1985)

Assistants: PhD students at LPPC

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Part II

- Cosmological Solutions of GR
- Hubble expansion, accelerating universe
- Physics of the early universe
    - relic radiation
    - Big-bang nucleosynthesis
    - Baryogenesis
- Inflationary Universe

Note about system of units

- in General physics one normally use SI : meter, kilo, sec, Amper

our choice:

High energy physics (HEP)

forget about ε_0, μ_0, c, ets!

(1) Units of HEP

We have 4 main physical units:

- length : cm, m
- time : s
- mass : g, kg
- temperature: °K

and 4 fundamental constants of nature

- speed of light: $c = 299\,792\,458\,00$ cm/s
- Planck constant: $\hbar = 1.054 \cdot 10^{-27}$ erg$\cdot$s
- Newton constant: $G = 6.67 \cdot 10^{-8} \dfrac{cm^3}{g s^2}$
- Boltzmann constant: $k_B = 1.38 \cdot 10^{-16}$ erg/K

to simplify equations (we do not want to write $c, \hbar, G, k_B$ all the time!)

We can choose system of units in which one or many fundamental constants are equal to 1.

- The extreme choice: $c = \hbar = G = k_B = 1$
  Every quantity is dimensionless
  This system is used in quantum gravity, string theory
- Our (HEP) choice: $c = \hbar = k_B = 1$, since we will not consider gravity
  We are free to choose only one physical unit. In HEP, the unit of energy is selected:

$$1\,\text{GeV} = 10^3\,\text{MeV} = 10^9\,\text{eV}$$

1eV: energy, which electron gets after passing a difference of potentials = 1V

Conversion factors:

- length: $[\text{GeV}]^{-1} = 1.97 \cdot 10^{-14}\,\text{cm}$
- time: $[\text{GeV}]^{-1} = 6.58 \cdot 10^{-25}\,\text{s}$
- mass: $[\text{GeV}] = 1.78 \cdot 10^{-24}\,\text{g}$
- temperature: $\text{GeV} = 1.16 \cdot 10^{13}\,\text{K}$

Chapter I: Special relativity

Summary of what we have learned from ED, 6th semester of EPFL

- Look at lecture notes at the web.

– Summary from ED lectures, Newtonian dyn.

Important fact: Dynamical equations can be used to establish symmetries of space and time.

Example 1: Newtonian dynamics of particles interacting with central potential

N - particles, $\vec{x}_i$

Kinetic energy: $\sum_{i=1}^{N} \frac{m_i \dot{\vec{x}}_i^2}{2}$

Potential energy: $V = \frac{1}{2} \sum_{i \neq j} u(|\vec{x}_i - \vec{x}_j|)$

Newtons equations:

$$m_i \ddot{\vec{x}}_i = - \frac{\partial V}{\partial \vec{x}_i}$$

Equations do not change if we:

(i) $\vec{x}_i \Rightarrow \vec{x}_i + \vec{x}_0$ – translation $\Rightarrow$ space is translationary invariant

(ii) $(\vec{x}_i)^\alpha = D^\alpha_\beta (x_i)^\beta \quad DD^T = 1 \Rightarrow$

space is isotropic

(iii) $\vec{x}_i \to \vec{x}_i + \vec{v} t \Leftarrow$ Galileo transformation

$t \to t$

Physics is the same in all inertial coordinate systems, time is absolute

Example II: most important to us – electrodynamics

$$\mathrm{div}\,\vec{E} = \rho \qquad -\frac{\partial \vec{E}}{\partial t} + \mathrm{rot}\,B = \vec{j}$$
$$\mathrm{div}\,\vec{B} = 0 \qquad \frac{\partial \vec{B}}{\partial t} + \mathrm{rot}\,\vec{E} = 0$$

$\bar{E}$ – electric field, $\bar{B}$ – magnetic field,
ρ – density of electric charge,
$\vec{j}$ – density of electric current

These equations are not invariant under Galileo transformations. But invariant under new transformation, called Lorentz transformation:

$v \parallel$ axe x:

$$\begin{pmatrix} t' \\ x' \end{pmatrix} = \begin{pmatrix} \gamma & -\gamma v \\ -\gamma v & \gamma \end{pmatrix} \begin{pmatrix} t \\ x \end{pmatrix}$$

remember: $c = 1$!

$$y' = y$$
$$z' = z$$

$$\gamma = \frac{1}{\sqrt{1 - v^2}}$$

$$\begin{pmatrix} \rho' \\ j_x' \end{pmatrix} = \begin{pmatrix} \gamma & -\gamma v \\ -\gamma v & \gamma \end{pmatrix} \begin{pmatrix} \rho \\ j_x \end{pmatrix} \qquad j_y' = j_y \qquad j_z' = j_z$$

Electric and magnetic field have somewhat more complicated expressions

Conclusions: Physics is the same in all inertial systems, but the transition from one reference frame to another is given by L transformation rather than G transformation. Time is not absolute. Speed of light (radio-waves) is the same (1) in all coordinate systems.

We developed 4-dimensional formalism to deal with Lorentz transformations

Definitions:

- 4-vector (contravariant) is a set of 4 quantities which transform as

  $x^\mu \to x'^\mu = \Lambda^\mu{}_\nu x^\nu$ if we go from one coordinate system to another, $\mu = 0,1,2,3$
  0 – time

- metric tensor $\eta_{\mu\nu} = \mathrm{diag}(1,-1,-1,-1)$

- Definition of $\Lambda^\mu{}_\nu$: any matrix with property $\eta_{\mu\nu} = \Lambda^\alpha{}_\mu \Lambda^\beta{}_\nu \eta_{\alpha\beta}$

- 4-vector (covariant) can be constructed with the help of $\eta_{\mu\nu}$ as:

  $x_\mu = \eta_{\mu\nu} x^\nu$

- 2 other metric tensors: $\eta^{\mu\nu} = (1,-1,-1,-1)$

  $\delta_\mu^\nu = \eta_{\mu\alpha}\eta^{\alpha\nu} = \mathrm{diag}(1,1,1,1)$