Quantum field theory
Exercises 10.
2006-03-27

• **Exercise 10.1.**
Show that under charge conjugation $\bar{\Psi}\gamma^{\mu}\Psi$ changes sign (Ψ is the fermionic field operator).

• **Exercise 10.2.**

1. Consider a massive vector field in $n+1$ dimensions with the action

$$S = \int d^{n+1}x \left(-\frac{1}{4} F_{\mu\nu}F^{\mu\nu} + \frac{m^2}{2} A_{\mu}A^{\mu} \right) , \qquad (\mu, \nu = 0, \ldots, n) .$$

Find the equations of motion and give the general solution for $n = 1, 2, 3$. How many degrees of freedom has this model?

2. Construct the polarization fourvectors $\varepsilon^l_{\mu}(k)$, $l = 1, 2, 3$ for the massive vector field in 3+1 dimensions

$$A_{\mu}(x) = \int \frac{d^3\mathbf{k}}{(2\pi)^3 2k_0} \sum_{l=1}^{3} \varepsilon^l_{\mu}(k) \left[a_{\mathbf{k},l} e^{ikx} + a^{\dagger}_{\mathbf{k},l} e^{-ikx} \right]$$

so that

$$k^{\mu}\varepsilon^l_{\mu}(k) = 0 ,$$

and they satisfy the orthonormality relation

$$\varepsilon^l_{\mu}(k\varepsilon)^{n\mu}(k) = -\delta^{ln}$$

and completeness relation

$$-\sum_{l=1}^{3} \varepsilon^l_{\mu}(k)\varepsilon^l_{\nu}(k) = g_{\mu\nu} - \frac{k_{\mu}k_{\nu}}{m^2} .$$

• **Exercise 10.3.**
Check the following relations

1. $[AB, C] = [A, C]B + A[B, C]$ (for some operators A, B, C)
2. $[a^{\dagger}a, a] = -a$
3. $[a^{\dagger}a, a^{\dagger}] = a^{\dagger}$

• **Exercise 10.4.**
One defines a function of an operator using the Taylor series expansion:

$$f(\hat{a}) = f(0) + f'(0)\hat{a} + \frac{1}{2} f''(0)\hat{a}^2 + \ldots .$$

Find the following commutators:

1. $[\hat{a}^{\dagger}, f(\hat{a})]$
2. $[\hat{a}, f(\hat{a}^{\dagger})]$