**Quantum field theory**
**Exercises 9.**
2006-03-20

• **Exercise 9.1.**
Let $[a, a^\dagger] = 1$. Analyze the following transformation (Bogoliubov transformation)

$$b = Aa^\dagger + Ba + C\,,$$
$$b^\dagger = B^* a^\dagger + A^* a + C^*\,.$$

What conditions $A$, $B$ and $C$ must satisfy to make it possible to interpret operators $b$, $b^\dagger$ as creation–annihilation operators?

• **Exercise 9.2.**
Find the spectrum of the Hamiltonian

$$H = \omega_1 (a^\dagger)^2 + \omega_1^* (a)^2 + \omega_3 a^\dagger a$$

with $[a, a^\dagger] = 1$.