

## Quantum field theory

### Exercises 7.

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- **Exercise 7.1.**

Derive equations of motion for the scalar field with the action

$$S = \int d^4x \left( -\frac{1}{2} \phi \partial^\mu \partial_\mu \phi - \frac{m^2}{2} \phi^2 \right)$$

- **Exercise 7.2.**

Consider the model of a massless scalar field

$$S = \int d^4x \frac{1}{2} \partial^\mu \phi \partial_\mu \phi$$

and the *dilatation* transformations

$$\begin{aligned} x^\mu &\rightarrow x'^\mu = e^\alpha x^\mu, \\ \phi(x) &\rightarrow \phi'(x') = \phi(x) e^{-d_\phi \alpha}. \end{aligned}$$

1. Show that this transformation is really a symmetry of the action for an appropriate choice of  $d_\phi$ . Find the corresponding Noether current and verify explicitly that it is conserved on the equations of motion.
2. Show that the mass term spoils the symmetry.
3. Show that the potential term of the form  $V(\phi) = \lambda \phi^4$  does not spoil the dilatation symmetry.