Quantum field theory
Exercises 6.
2005-12-05

• **Exercise 6.1.**

Derive the equations of motion for the complex scalar field with the folowing action

$$S = \int d^4x \left(\partial^\mu \phi \partial_\mu \phi^* - \frac{\lambda}{2}(\phi\phi^* - v^2)^2 \right) .$$

• **Exercise 6.2.**

Check explicitly in the model of one scalar field with potential $V(\phi)$

$$S = \int d^4x \left(\frac{1}{2}\partial^\mu \phi \partial_\mu \phi - V(\phi) \right)$$

that the energy

$$E = \int d^3x \left(\frac{1}{2}\dot{\phi}^2 + \frac{1}{2}\partial_i\phi\partial_i\phi + V(\phi) \right)$$

is conserved ($dE/dt = 0$) on the equations of motion.

• **Exercise 6.3.**

Find the dimensions of scalar, spinor, and gauge fields in space-time of dimension $d = 4$. A useful starting point is the fact that dimension of action S is the dimension of the Plank's constant, i.e. it is dimensionless in $\hbar = 1$ units. Repeat the anaysis in arbitrary number of dimensions d, using action $S = \int d^dx \mathscr{L}$ with usual expressions for Lagrangian $\mathscr{L}$.