

Quantum field theory

Exercises 4.

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- **Exercise 4.1.**

Consider a massive particle moving with velocity $v = \tanh \eta$.

- Show that, if E is the energy of the particle and p its momentum along the propagation direction, then

$$\eta = \frac{1}{2} \log \frac{E + p}{E - p}.$$

- Verify that under a boost in the direction of motion of the particle with velocity v' (and corresponding rapidity $\eta' = \operatorname{arctanh} v'$) η transforms additively

$$\eta \rightarrow \eta + \eta'.$$

- **Exercise 4.2.**

Prove that, if ψ_R and ξ_R are right-handed Weyl spinors, $\xi_R^\dagger \sigma^\mu \psi_R$ is a four-vector, and similarly for $\xi_L^\dagger \bar{\sigma}^\mu \psi_L$, where ξ_L , ψ_L are left-handed Weyl spinors.

- **Exercise 4.3.**

Find the explicit form of the variation of an antisymmetric tensor $F^{\mu\nu}$ under an infinitesimal Lorentz transformation. Writing $F^{0i} = -E^i$ and $F^{ij} = -\varepsilon^{ijk} B^k$, find the infinitesimal transformation of E^i and B^i .