Quantum field theory Exercises 3. 2005-11-14

• Exercise 3.1. Two body decays

Let a particle with mass M can decay into two particles with masses m_1 and m_2 .

- How many parameters describe the final state?
- Find the expression for the phase space in the rest frame of the initial particle (use the angles describing the direction of the particle m_1 as the coordinates to get the simplest possible form)
- Suppose, that the matrix element $|\mathcal{M}_{fi}|$ for this decay does not depend on direction of flight of the decay products (i.e., spherically symmetric). What is the total decay rate of the particle?

• Exercise 3.2. Three body decay

Prove, that 3-body phase space in Mandelstam variables (see ex. 2.1) is uniform

$$d\Phi^{(3)} = \operatorname{const} \cdot ds \, dt \; ,$$

where we decided to use *S* and *t* as independent variables. Find the constant in the expression. • Exercise 3.3.

Denote by $d\Phi^{(n)}(P; p_1, ..., p_n)$ the *n*-body phase space with $p_1 + \cdots + p_n = P$. Show that

$$d\Phi^{(n)}(P;p_1,\ldots,p_n) = \int_0^\infty \frac{d\mu^2}{2\pi} d\Phi^j(q;p_1,\ldots,p_j) \times d\Phi^{(n-j+1)}(P;p_{j+1},\ldots,p_n,q) ,$$

where $\mu^2 = q^2 \equiv q_0^2 - \mathbf{q}^2$. Discuss the physical meaning of this recursive representation of the phase space.

• Exercise 3.4. Do you really know Lorentz transformations? *

Find the fault in the following reasoning of two persons (say Rosenkrantz and Guildenstern) learning special theory of relativity.

R: Let us make a Gedanken experiment: Hamlet takes a 5 meter long spear and runs very fast, at speed of 0.5 (in $\hbar = c = 1$ units). Then let us put a 5 meter long house with two doors on his way, with me standing at the front door, and Guildenstern at the back door. I open the door, Hamlet runs in and I close the door. As far as he is running fast, his spear is only about 3.5 meters long due to Lorentz length reduction, and he is completely trapped in the house (and it is Guildenstern who decides whether to leave him there or let him out, ha-ha!) Oh, look, we will trap him even if his spear is 6 meters long, as far as it reduces to merely 4.2 meters!

G: Stop, Rosenkrantz! But how can that be? Let us think like Hamlet is going to think—"I am running with 5 meter sphere. A house in front of me is only 3.5 meters due to Lorentz length reduction. The spear is not going to fit there! They will not trap me, ha-ha!" See, even if he has only a 4 meter spear, we will not trap him...

So, will Hamlet's spear fit into the house, or not?