Quantum field theory
Exercises 1.
2005-10-31

- **Exercise 1.1.**

Express the following in natural units (GeV to some power, $\hbar = c = \varepsilon_0 = \mu_0 = 1$)

1. 1 cm
2. 1 s
3. 1 g
4. 1 K°
5. 1.60×10^{-19} C (electron charge)
6. 1 Gauss
7. Gravitational constant $G_N = 6.67 \times 10^{-11}$ m^{3}kg^{-1}s^{-2}
8. Hubble constant $H \simeq 100$ km s^{-1}Mpc^{-1} (1 pc = 3.26 light years)
9. Critical density of the Universe $\rho_{\text{crit}} = 3H^2/(8\pi G_N)$

- **Exercise 1.2.**

The Universe is permeated by a thermal background of electromagnetic radiation at a temperature $T = 2.725(1)$ K (the cosmic microwave background radiation, CMB). Estimate with dimensional arguments the energy density of this gas of photons and compare it with the critical density for closing the Universe, ρ_{crit}.

- **Exercise 1.3.**

Model the Sun as an ionized plasma of electrons and protons, with an average temperature $T \simeq 4.5 \times 10^6$ K and an average mass density $\rho \simeq 1.4$ g/cm^3. Estimate te mean free path of photons in the Sun's interior, and compare the contribution to the mean free path coming from the scattering on electrons with that from the scattering on protons. Knowing that the radius of the Sun is $R_\odot \simeq 6.96 \times 10^{10}$ cm, estimate the total time that a photon takes to escape from the Sun.

- **Exercise 1.4.**

Suppose first that a beam of particles with mass m_1, speed v_1, number density (number of particles in unit volume) n_1^0 is colliding with a target consisting of particles with mass m_2 and number density n_2^0. Then the number of scattering events in period of time dt and in volume dV is

$$dN = \sigma v_1 n_1^0 n_2^0 dV dt \ ,$$

where σ is *cross-section* (this is in fact its definition).

Find the expression for dN in a frame of reference where the "target" is not at rest, i.e. where there are two colliding beams with velocities $\mathbf{v}_1$ and $\mathbf{v}_2$ and number densities n_1 and n_2.