Binding energy

![Graph showing the binding energy per nucleon as a function of the number of nucleons in the nucleus. The graph includes points for various isotopes such as O^{16}, C^{12}, Fe^{56}, Li^{7}, Li^{6}, H^{3}, He^{3}, H^{2}, and U^{235}, U^{238}. The x-axis represents the number of nucleons, and the y-axis represents the average binding energy per nucleon in MeV.]
To produce chemical elements one needs to pass through “deuterium bottleneck” $p + n \leftrightarrow D + \gamma$
We saw that for each baryon there were $\sim 10^{10}$ photons.

Binding energy of deuterium is $E_D = 2.2 \text{ MeV}$ (or $T_D = 2.5 \times 10^{10} \text{ K}$).

At $T = E_D$ 85% of all photons have $E > T_D \Rightarrow$ any deuterium nucleus will be quickly photo-disassociated via $D + \gamma \rightarrow p + n$.

Production of deuterium becomes efficient when temperature drops so that the number of photons with $E > E_D$ will be $\sim 10^{-10}$

\[
\frac{n_\gamma(E > E_D)}{n_\gamma_{tot}} \sim \eta_B \implies \eta_B \left( \frac{2.5T_{BBN}}{m_p} \right)^{\frac{3}{2}} e^{T_{BBN} \frac{E_D}{T_{BBN}}} \sim 1 \tag{1}
\]

$T_{BBN} \approx 70 \text{ keV}$ and $t_{BBN} = \frac{M^*_{Pl}}{2T^2_{BBN}} \approx 120 \text{ s}$
In principle, BBN is a very complicated process involving many coupled Boltzmann equations to track all the nuclear abundances. In practice, however, two simplifications will make our life a lot easier:

1. No elements heavier than helium. Essentially no elements heavier than helium are produced at appreciable levels. So the only nuclei that we need to track are hydrogen and helium, and their isotopes: deuterium, tritium, and $^3\text{He}$.

2. Only neutrons and protons above 0.1 MeV. Above $T_{\text{\Lambda}} \approx 0.1 \text{ MeV}$ only free protons and neutrons exist, while other light nuclei haven't been formed yet. Therefore, we can first solve for the neutron/proton ratio and then use this abundance as input for the synthesis of deuterium, helium, etc.

Let us demonstrate that we can indeed restrict our attention to neutrons and protons above 0.1 MeV. In order to do this, we compare the equilibrium abundances of the different nuclei:

- First, we determine the relative abundances of neutrons and protons. In the early universe, neutrons and protons are coupled by weak interactions, e.g.

$$\text{n} + \bar{\nu}_e \rightarrow \text{p} + e^- \quad \text{and} \quad \text{n} + e^+ \rightarrow \text{p} + \nu_e.$$  

We want to apply eq. (3.3.85)—nuclear statistical equilibrium—to this reaction. Assuming that the chemical potentials of electrons and neutrinos are negligibly small, we have $n_e = 64$. 

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**Figure 3.6:** Numerical results for helium production in the early universe.
BBN predictions confirmed

- Curves — theoretical predictions of Big Bang nucleosynthesis

- Horizontal stripes — values that follow from observations.

- Golden stripe — measured value of $\eta$ from CMB observations!
Dependence on number of neutrinos

<table>
<thead>
<tr>
<th>$N_\nu$</th>
<th>$g_\ast$</th>
<th>$\frac{n_n}{n_p}$</th>
<th>$Y_p$</th>
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<tr>
<td>1.</td>
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<td>6.</td>
<td>4.72529</td>
<td>0.160931</td>
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</tr>
</tbody>
</table>

Measurements of $Y_p$ have error bars $\pm 0.008$