Baryon asymmetry of the Universe

- **Main questions:** Why do the Earth, the Solar system and our galaxy consists of of matter and not of antimatter?

- Why we do not see any traces of antimatter in the universe except of those where antiparticles are created in collisions of ordinary particles?

- This looks really strange, as the properties of matter and antimatter are very similar.

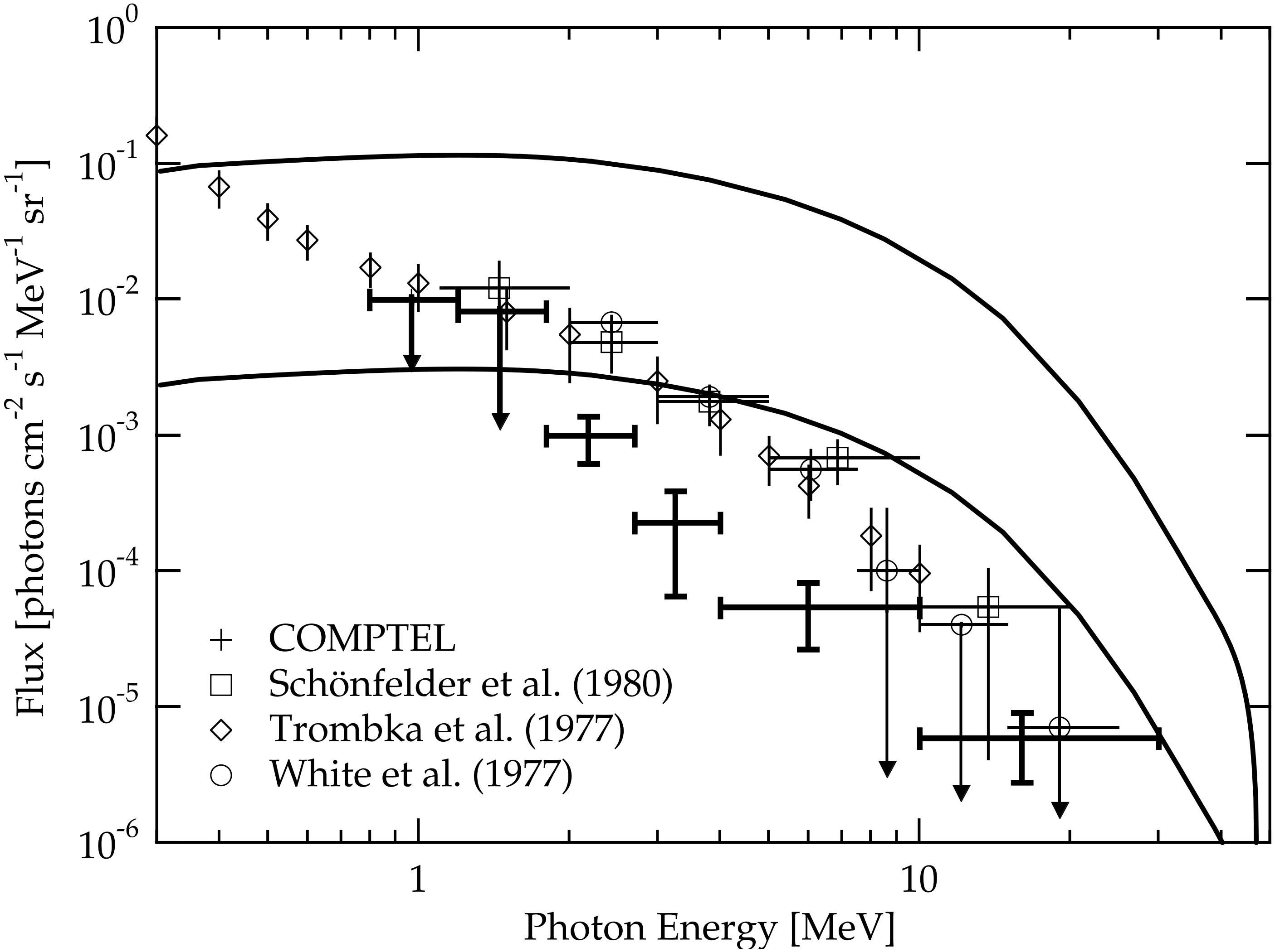
Baryon asymmetry of the Universe

There are two possibilities:

- Observed universe is asymmetric and does not contain any antimatter
- The universe consists of domains of matter and antimatter separated by voids to prevent annihilation. The size of these zones should be greater than 1000 Mpc, in order not to contradict observations of the diffuse γ spectrum.

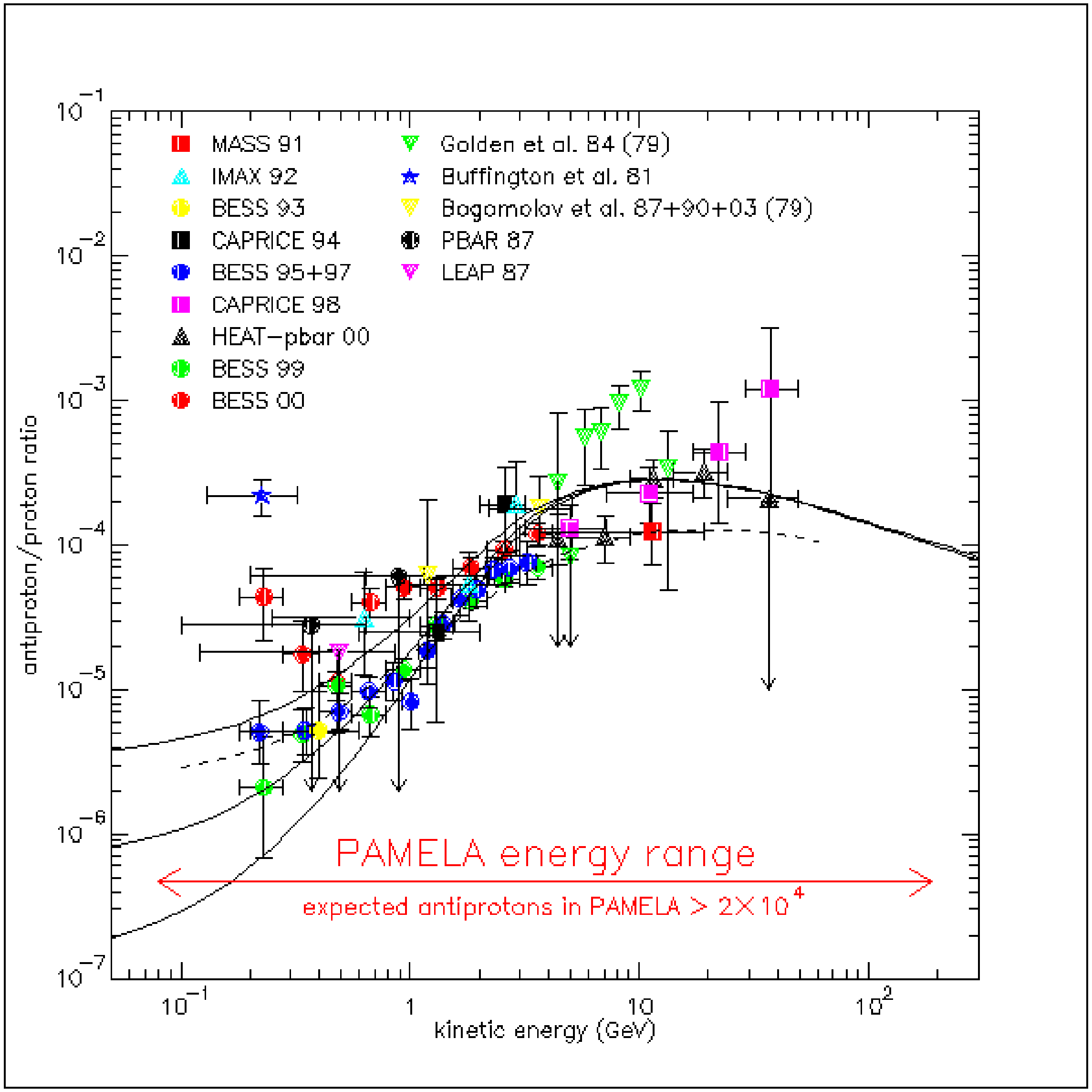
The second option, however, contradicts to the large scale isotropy of the cosmic microwave background.

Thus, we are facing the question: Why the universe is globally asymmetric?

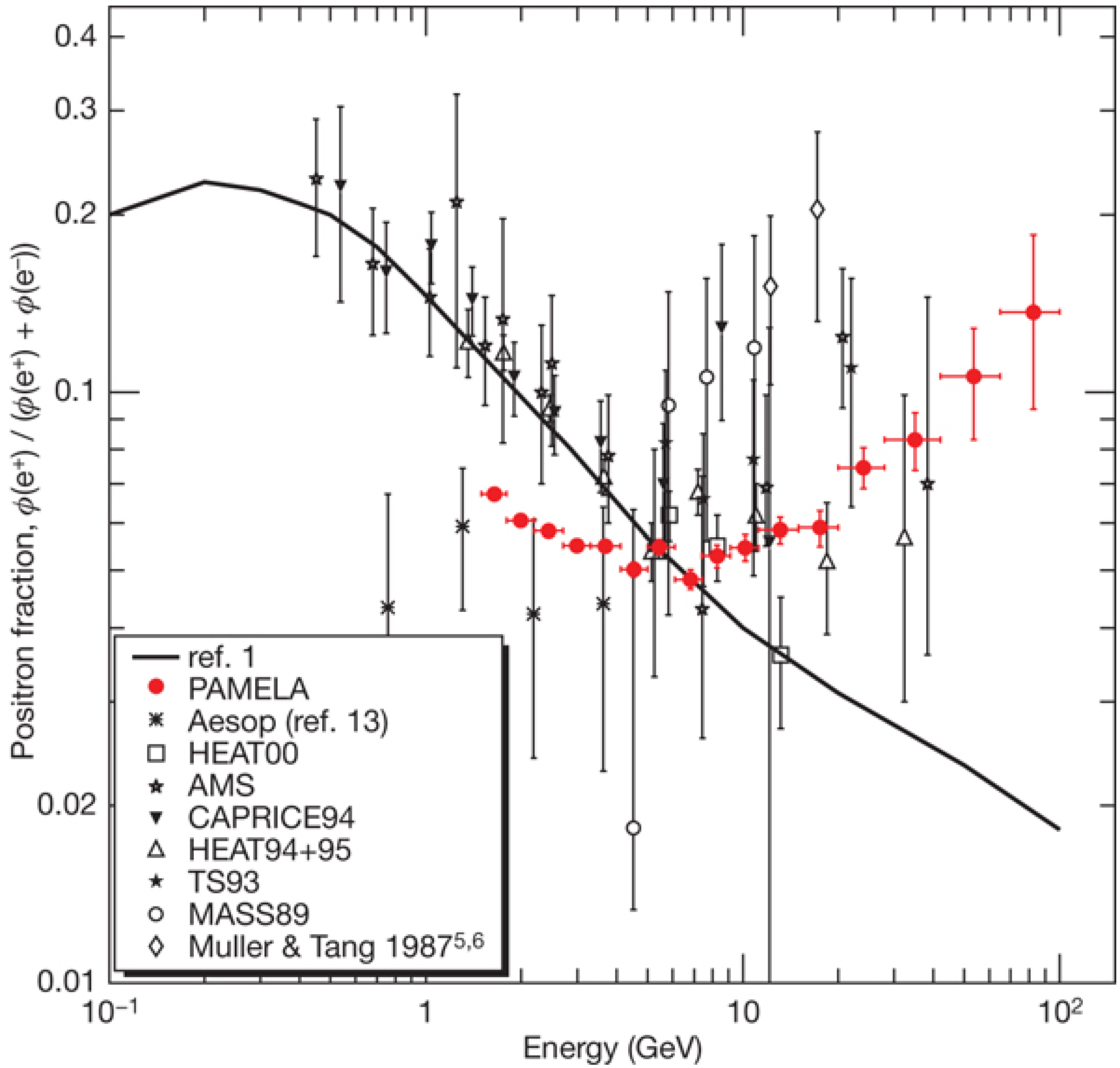


Data and expectations for the diffuse γ-ray spectrum (upper curve d = 20 Mpc, lower curve d = 1000 Mpc)

Antiprotons in the universe



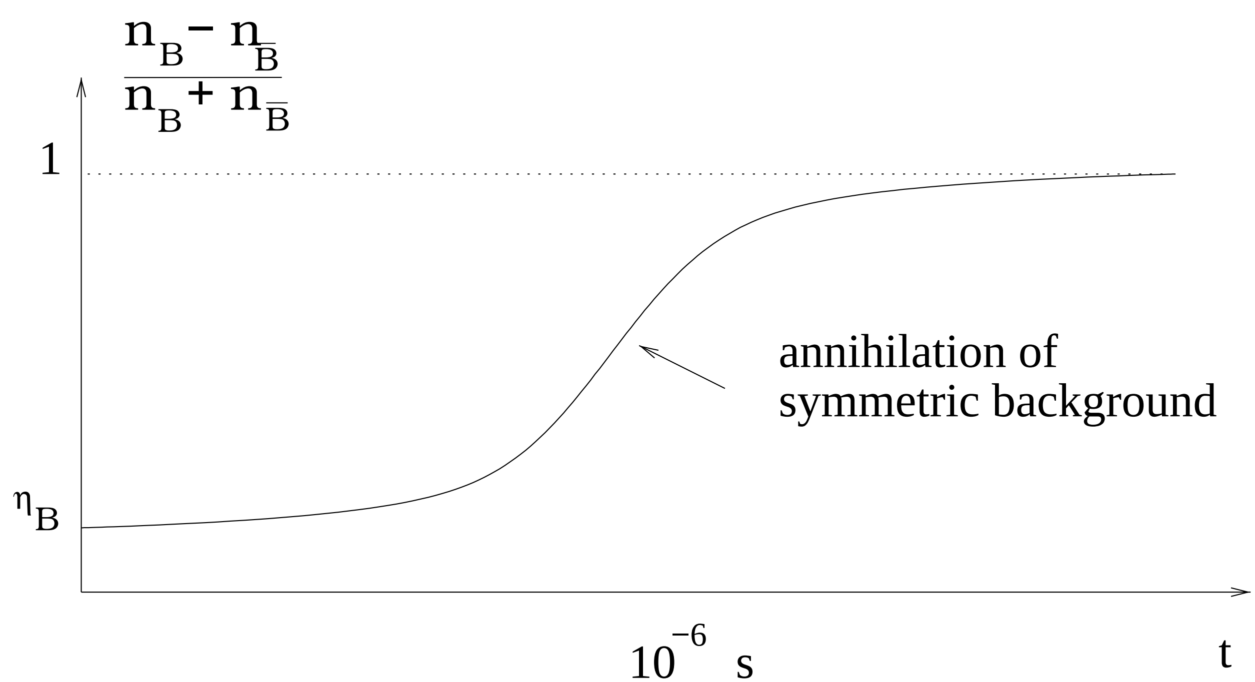
Example: antiproton-to-proton fraction in GeV: $10^{-7} - 10^{-3}$



Example: positron-to-electron fraction in GeV: $0.02 - 0.2$

If there are no anti-baryons now, what kind of asymmetry this implies in the early Universe?

- The baryon asymmetry $\boxed{\Delta_B = \frac{n_B - n_{\bar{B}}}{n_B + n_{\bar{B}}}}$ **today** is very close to 1
- How did it evolve? What was its value in the earlier times?
- Any quark (actually, any particle) is present in the plasma if $T \gtrsim m_q$, because of the annihilation processes $q+\bar{q} \leftrightarrows \gamma+\gamma$ (or $q+\bar{q} \leftrightarrows g+g$)



Thermal history of baryon asymmetry

- At early times for each 10^{10} quarks there is $10^{10} - \mathbf{1}$ antiquark.

- Symmetric quark-antiquark background annihilates into photons and neutrinos while the asymmetric part survives and gives rise to galaxies, stars, planets.

Sakharov: To generate baryon asymmetry of the Universe **3 conditions** should be satisfied

I. Baryon number should not be conserved

II. C-symmetry and CP-symmetry must be broken

III. Deviation from thermal equilibrium in the Universe expansion

Sakharov conditions-I

- **If baryon number is conserved**, then in every process
$$\psi_1 + \psi_2 + \cdots \to \chi_1 + \chi_2 + \ldots$$
left hand side and right hand side contain equal number of (baryons - anti-baryons)

- Experimentally we see that baryon charge is conserved in particle physics processes.

- As a consequence proton (the latest baryon) is stable (proton lifetime $> 6.6 \times 10^{33}$ years for decays such as $p \to \pi^0 + e^+$ or $p \to \pi^0 + \mu^+$. This bound is 5×10^{23} times longer than the age of the Universe

- The conservation of baryon number would mean that the total baryon charge of the Universe remains constant in the process of evolution.

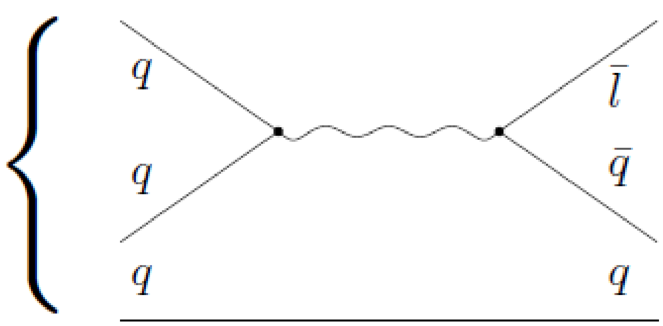
- **If** initial conditions were matter-antimater symmetric – no baryon asymmetry could have been generated

Problem. Taking the lower bound on the proton's lifetime as its actual lifetime, compute the amount of water one would need to observe 1 proton decay during 10 years.

- Postulate new interaction mediated but a massive gauge boson (X-**boson**), transforming quarks to leptons: $X \to q + \ell$ (similar to W boson in electroweak theory where $W \to e + \bar{\nu}_e$)

- As a consequence, the processes with X-boson exchange **violate** the baryon number

- For example, the protons may **decay**.



The proton decays mediated by X-boson:

$\Rightarrow \quad p \to e^+ + \pi^0$

$\Rightarrow \quad p \to \bar{\nu}_e + \pi^+$

New interactions violating baryon number

- The proton lifetime can be estimated as (similar to muon decay):

$$\boxed{\tau_p^{-1} \sim \left(\frac{\alpha_X}{M_X^2}\right)^2 m_p^5} \qquad \tau_\mu^{-1} \sim \left(\frac{\alpha_W}{M_W^2}\right)^2 m_\mu^5$$

- Existing experimental bounds on the proton lifetime: $\tau_p \gtrsim 10^{33}$ yrs gives $M_X \gtrsim 10^{16}$ GeV.

- Yukawa couplings may violate CP (**Sakharov conditions**).

- However, this mechanism requires **new physics** at $E \sim M_X \ldots$

Can we generate baryon number at **lower** energies?
YES!

Quantum anomalies

(violation of classical symmetries at quantum level)

- Massless fermions can be left and right-chiral (left and right moving):

$$(i\gamma^\mu\partial_\mu - \cancel{m})\psi = \begin{pmatrix} \cancelto{0}{-m} & i(\partial_t + \vec{\sigma}\cdot\vec{\nabla}) \\ i(\partial_t - \vec{\sigma}\cdot\vec{\nabla}) & \cancelto{0}{-m} \end{pmatrix} \begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix} = 0$$

  where $\gamma_5\psi_{R,L} = \pm\psi_{R,L}$ and $\gamma_5 = i\gamma_0\gamma_1\gamma_2\gamma_3$

- Two global symmetries: $\psi_L \to e^{i\alpha}\psi_L$ and $\psi_R \to e^{i\beta}\psi_R$

- According to the Nöther theorem, one can define two **independently** conserved charges that we call the number of left-movers $N_L = \int d^3x\, \psi_L^\dagger\psi_L$ and the number of right-movers $N_R = \int d^3x\, \psi_R^\dagger\psi_R$.

  Linear combinations of these charges are known as **fermion number** $N_L + N_R$ (current $\bar\psi\gamma^\mu\psi$) and **axial fermion number** $N_L - N_R$ (current $\boldsymbol{\bar\psi\gamma^\mu\gamma_5\psi}$). Again, both are conserved independently in the free theory

- Gauge interactions respects chirality ($D_\mu = \partial_\mu + eA_\mu$)...

$$\begin{pmatrix} 0 & i(D_t + \vec{\sigma}\cdot\vec{D}) \\ i(D_t - \vec{\sigma}\cdot\vec{D}) & 0 \end{pmatrix} \begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix} = 0$$

The symmetry $\psi \to e^{i\alpha(x)}\psi$ is *gauged*, but the Lagrangian seemingly still preserves both global symmetries: $\psi_L \to e^{i\alpha}\psi_L$ and $\psi_R \to e^{i\beta}\psi_R$

- ...but the difference of left and right-movers is **not conserved anymore**:

$$\frac{d(N_L - N_R)}{dt} = \int d^3\vec{x}\left(\partial_\mu j^5_\mu\right) = \frac{e^2}{2\pi^2}\int d^3\vec{x}\,\vec{E}\cdot\vec{B} \neq 0$$

...if one takes into account quantum corrections

- Recall: particle in the uniform magnetic field , parallel to z axis: $\vec{B} = (0, 0, B)$
- Take Dirac equation coupled to the gauge potential $\vec{A} = (0, Bx, 0)$
- Conserved quantities: energy, momenta p_y, p_z
- Take the square of the Dirac equation to get:

$$\left(-\frac{d^2}{dx^2} + (eBx - p_y)^2 - 2eBs_z\right)\phi = (E^2 - p_z^2)\phi \tag{1}$$

Spin projection $s_z = \pm\frac{1}{2}$

- The l.h.s. of (1) is just a Schödinger equation for the harmonic oscillator with the frequency $\omega = 2eB$, whose origin is shifted by $\pm eB$
- The energy levels of harmonic oscillators $\epsilon_n = \omega(n + \frac{1}{2})$, $n \geq 0$

- Therefore, the spectrum of Eq. (1) is given by

$$E_n^2 - p_z^2 = eB(2n+1) + 2s_z eB \qquad (2)$$

- Spectrum has three quantum numbers:

  ▷ $n = 0, 1, 2\dots$
  ▷ $-\infty \le p_z \le +\infty$
  ▷ $s_z = \pm\frac{1}{2}$

- Consider $n = 0$. For $s_z = -\frac{1}{2}$ the spectrum (2) becomes

$$E^2 = p_z^2 \qquad \text{massless 1-dimensional fermion} \qquad (3)$$

  for $s_z = +\frac{1}{2}$ there is **no** massless mode

- For $n > 0$ there is no cancellation between $eB(2n+1)$ and $2s_z eB$ term

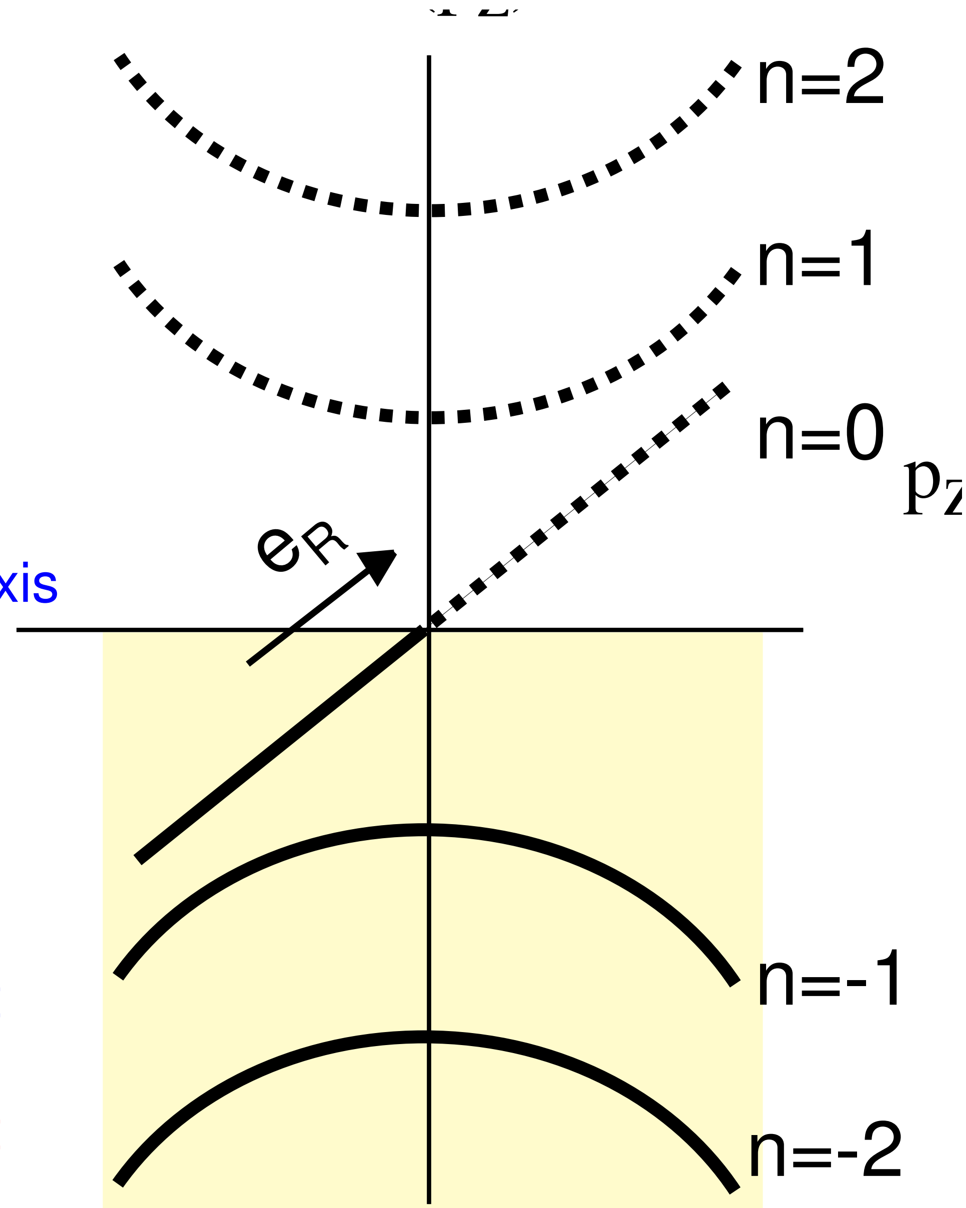
- Consider Landau levels:

$$E^2 = p_z^2 + eB(2n+1) + 2e\vec{B}\cdot\vec{s}$$

- Particles with $\vec{B}\cdot\vec{s} < 0$ have **massless** branches:

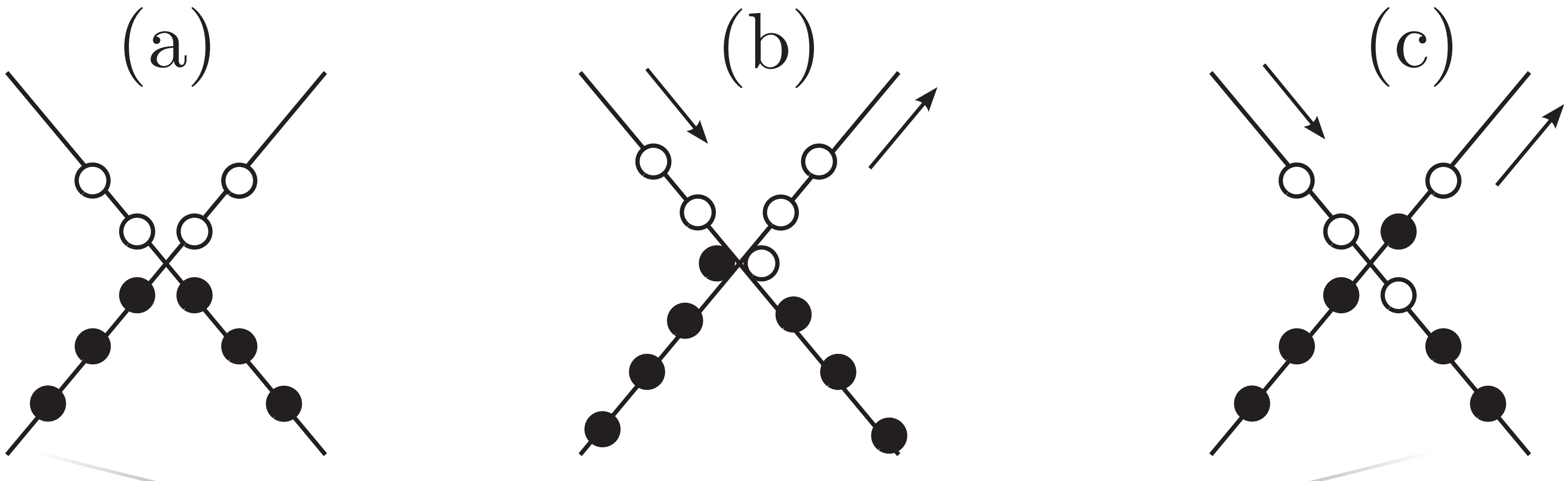
$$E = \begin{cases} -p_z & \text{move down along z-axis} \\ p_z & \text{move up along z-axis} \end{cases}$$

- Dirac vacuum $\leftrightarrow$ all states $E < 0$ are filled:
  - ▷ $E = -p_z < 0 \Rightarrow \vec{p}\cdot\vec{s} < 0$ – left particles
  - ▷ $E = p_z < 0 \Rightarrow \vec{p}\cdot\vec{s} > 0$ – right particles



- Electric field $\vec{E} = E\hat{z}$ **creates** right particle (because $p_z(t) = p_z(0) + eEt$)

- For particles of the other chirality the situation is opposite: such electric field destroys such particles (creates **hole** in the Dirac sea)

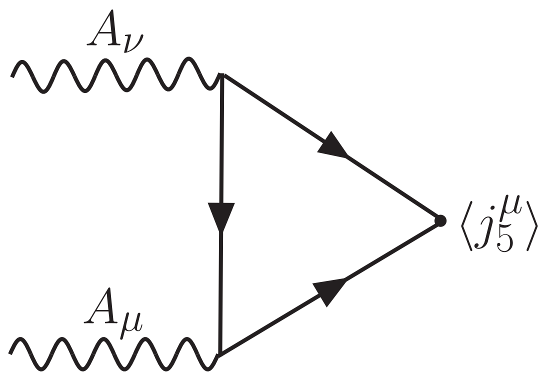


- As a result:
  - **Total number** of **particles** does not change
  - **Difference** of left minus right appears – **chiral anomaly!**

- This only happens when $\vec{E} \parallel \vec{B}$ (proportional to $\vec{E}\cdot\vec{B}$)

- One can see this for example by computing the diagram with two electromagnetic vertices and one axial current

Adler, Bell, Jackiw



- This diagram changes sign if we change all **left particles** into **right particles** (Furry theorem). Therefore it is proportional to

$$\text{triangular diagram} = \Big[e^2(-1) - e^2(+1)\Big] F(p, \epsilon_\mu, \epsilon_\nu)$$

- In general, any triangular diagram factorizes into some momentum integral, multiplied by

$$\text{Anomaly cancellation condition} = \boxed{\left(\sum_{left} e_L^2 Q_L - \sum_{right} e_R^2 Q_R\right)}$$

where e_L, e_R are **gauge charges** of left/right particles and Q_L, Q_R are charges with respect to the **global symmetry**

- **Anomaly** means that the notion of left and right chirality for massless fermions is not **gauge invariant**
- What if current in the vertex is the gauge current? Anomaly of a **gauge symmetry** renders theory inconsistent (non-unitary):

$$\partial_\nu\left(\partial_\mu F^{\mu\nu} = j^\nu\right) \Longrightarrow 0 = \partial_\nu j^\nu$$

- Gauge anomaly cancels if charges of "vector-like" (for each fermion $e_L = e_R$). For example, electrodynamic is vector like.

- Consider two Dirac fermions $b = (b_L, b_R)$ and $\ell = (\ell_L, \ell_R)$, charged with respect to a gauge group U(1) in the following way:

$$
\begin{aligned}
-Q(b_L) &= Q(\ell_L) = e \\
Q(b_R) &= Q(\ell_R) = 0
\end{aligned}
$$

- The Lagrangian has the form

$$\mathcal{L} = \bar{b}\Big(\not{\partial} - \frac{e}{2}\not{A}(1-\gamma_5)\Big)b + m_b\bar{b}b + \bar{\ell}\Big(\not{\partial} + \frac{e}{2}\not{A}(1-\gamma_5)\Big)\ell + m_\ell\bar{\ell}\ell$$

– U(1) theory with **chiral gauge group**

- This theory does not have gauge anomaly, as “anomaly cancellation condition” is satisfied:

$$Q(b_L)^3 + Q(\ell_L)^3 - Q(b_R)^3 - Q(\ell_R)^3 = 0$$

- At classical level the theory has two **global** U(1) symmetries:

$$b \to e^{i\alpha} b \quad \text{and} \quad \ell \to e^{i\beta}\ell$$

with the corresponding Nöther currents:

$$J_b^\mu = \bar{b}\gamma^\mu b \quad ; \quad J_\ell^\mu = \bar{\ell}\gamma^\mu \ell$$

you can think of corresponding conserved charges as analog of **baryon** and **lepton** number

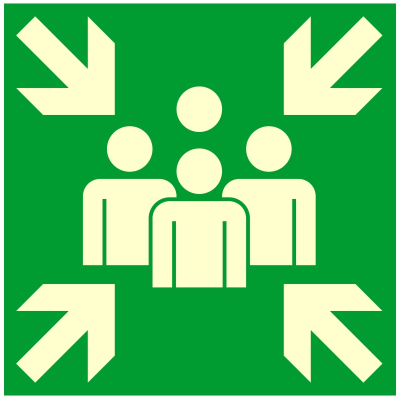
- Are these symmetries anomalous?

- Compute anomaly cancellation condition for J_b^μ:

$$\underbrace{(-e)^2(+1)}_{b_L} + \underbrace{e^2 \times 0}_{\ell_L} - \underbrace{0^2 \times (+1)}_{b_R} - \underbrace{0^2 \times 0}_{\ell_R} = -e^2 \neq 0$$

$\Rightarrow$ symmetry J_b^μ is anomalous (same is true for J_ℓ^μ)

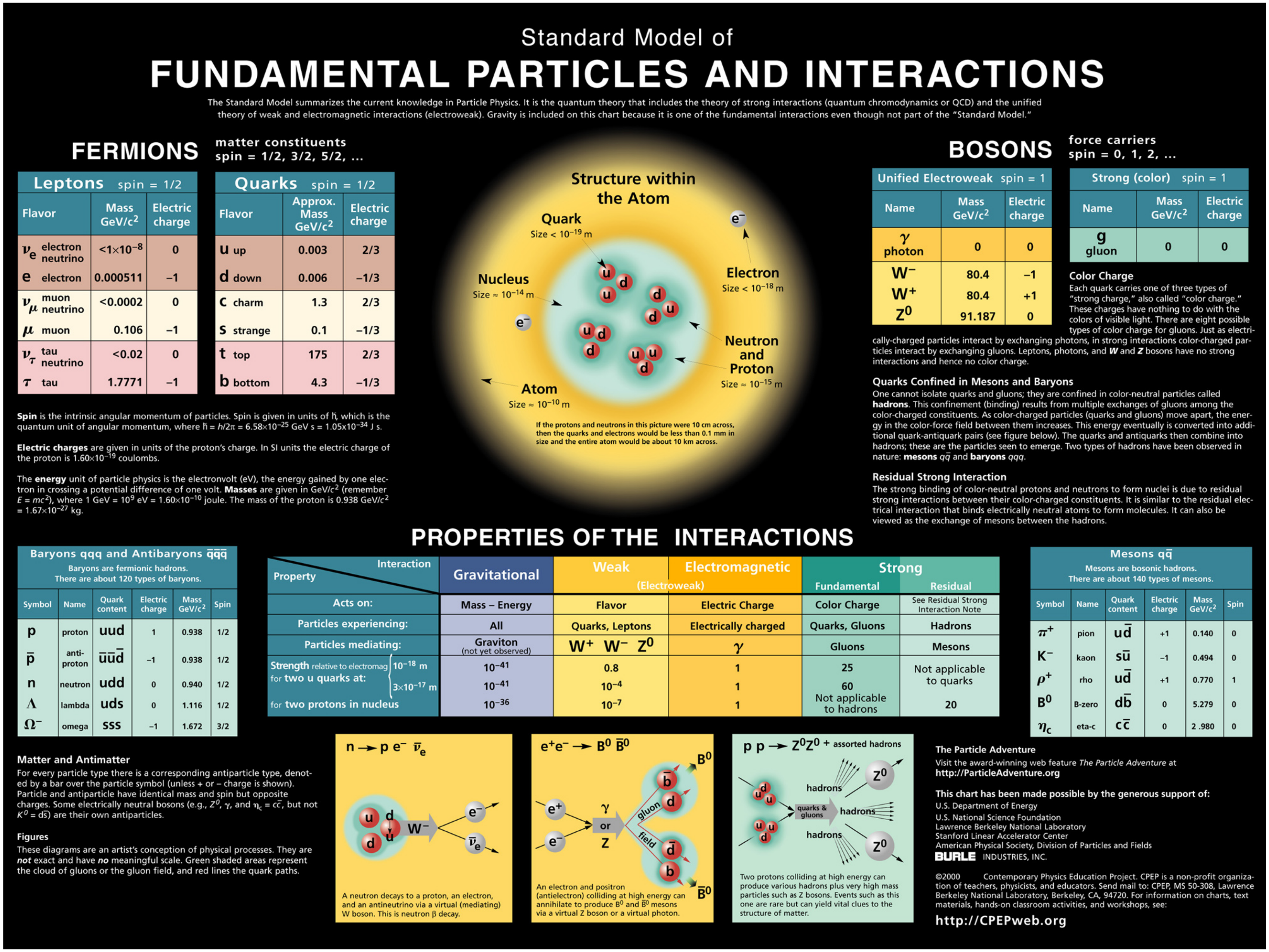
In the theory with vector-like symmetries **but** chiral gauge charges these vector-like symmetries are anomalous!
(if **both** gauge interactions **and** symmetries are vector-like $\Rightarrow$ there is no anomaly)



- Notion of chirality (left/right particles) may get incompatible with requirement of gauge invariants in **quantum** theories.

- In this case the chirality (or *the number of left minus right particles*) changes if one turns out field configurations proportional to $\vec{E}\cdot\vec{B}$.

- Anomalies in general appear when there is
  - ▷ Chiral current in the background of vector-like gauge fields
  - ▷ Vector-like current in the background of chiral gauge fields
  - ▷ Chiral current in the background of chiral gauge fields



Standard Model of

FUNDAMENTAL PARTICLES AND INTERACTIONS

The Standard Model summarizes the current knowledge in Particle Physics. It is the quantum theory that includes the theory of strong interactions (quantum chromodynamics or QCD) and the unified theory of weak and electromagnetic interactions (electroweak). Gravity is included on this chart because it is one of the fundamental interactions even though not part of the "Standard Model."

FERMIONS matter constituents spin = 1/2, 3/2, 5/2, ...

Leptons spin = 1/2

Flavor	Mass GeV/c^2	Electric charge
ν_e electron neutrino	$<1\times10^{-8}$	0
e electron	0.000511	–1
ν_μ muon neutrino	<0.0002	0
μ muon	0.106	–1
ν_τ tau neutrino	<0.02	0
τ tau	1.7771	–1

Quarks spin = 1/2

Flavor	Approx. Mass GeV/c^2	Electric charge
u up	0.003	2/3
d down	0.006	–1/3
c charm	1.3	2/3
s strange	0.1	–1/3
t top	175	2/3
b bottom	4.3	–1/3

Spin is the intrinsic angular momentum of particles. Spin is given in units of ħ, which is the quantum unit of angular momentum, where $\hbar = h/2\pi = 6.58\times10^{-25}$ GeV s = 1.05×10^{-34} J s.

Electric charges are given in units of the proton's charge. In SI units the electric charge of the proton is 1.60×10^{-19} coulombs.

The **energy** unit of particle physics is the electronvolt (eV), the energy gained by one electron in crossing a potential difference of one volt. **Masses** are given in GeV/c^2 (remember $E = mc^2$), where 1 GeV = 10^9 eV = 1.60×10^{-10} joule. The mass of the proton is 0.938 GeV/c^2 = 1.67×10^{-27} kg.

BOSONS force carriers spin = 0, 1, 2, ...

Unified Electroweak spin = 1

Name	Mass GeV/c^2	Electric charge
γ photon	0	0
W^-	80.4	–1
W^+	80.4	+1
Z^0	91.187	0

Strong (color) spin = 1

Name	Mass GeV/c^2	Electric charge
g gluon	0	0

Color Charge
Each quark carries one of three types of "strong charge," also called "color charge." These charges have nothing to do with the colors of visible light. There are eight possible types of color charge for gluons. Just as electrically-charged particles interact by exchanging photons, in strong interactions color-charged particles interact by exchanging gluons. Leptons, photons, and ***W*** and ***Z*** bosons have no strong interactions and hence no color charge.

Quarks Confined in Mesons and Baryons
One cannot isolate quarks and gluons; they are confined in color-neutral particles called **hadrons**. This confinement (binding) results from multiple exchanges of gluons among the color-charged constituents. As color-charged particles (quarks and gluons) move apart, the energy in the color-force field between them increases. This energy eventually is converted into additional quark-antiquark pairs (see figure below). The quarks and antiquarks then combine into hadrons; these are the particles seen to emerge. Two types of hadrons have been observed in nature: **mesons** $q\bar{q}$ and **baryons** qqq.

Residual Strong Interaction
The strong binding of color-neutral protons and neutrons to form nuclei is due to residual strong interactions between their color-charged constituents. It is similar to the residual electrical interaction that binds electrically neutral atoms to form molecules. It can also be viewed as the exchange of mesons between the hadrons.

PROPERTIES OF THE INTERACTIONS

Property \ Interaction	Gravitational	Weak (Electroweak)	Electromagnetic (Electroweak)	Strong: Fundamental	Strong: Residual
Acts on:	Mass – Energy	Flavor	Electric Charge	Color Charge	See Residual Strong Interaction Note
Particles experiencing:	All	Quarks, Leptons	Electrically charged	Quarks, Gluons	Hadrons
Particles mediating:	Graviton (not yet observed)	W^+ W^- Z^0	γ	Gluons	Mesons
Strength relative to electromag for two u quarks at: 10^{-18} m	10^{-41}	0.8	1	25	Not applicable to quarks
Strength relative to electromag for two u quarks at: 3×10^{-17} m	10^{-41}	10^{-4}	1	60	
for two protons in nucleus	10^{-36}	10^{-7}	1	Not applicable to hadrons	20

Baryons qqq and Antibaryons $\bar{q}\bar{q}\bar{q}$
Baryons are fermionic hadrons. There are about 120 types of baryons.

Symbol	Name	Quark content	Electric charge	Mass GeV/c^2	Spin
p	proton	uud	1	0.938	1/2
$\bar{p}$	anti-proton	$\bar{u}\bar{u}\bar{d}$	–1	0.938	1/2
n	neutron	udd	0	0.940	1/2
Λ	lambda	uds	0	1.116	1/2
Ω^-	omega	sss	–1	1.672	3/2

Mesons $q\bar{q}$
Mesons are bosonic hadrons. There are about 140 types of mesons.

Symbol	Name	Quark content	Electric charge	Mass GeV/c^2	Spin
π^+	pion	$u\bar{d}$	+1	0.140	0
K^-	kaon	$s\bar{u}$	–1	0.494	0
ρ^+	rho	$u\bar{d}$	+1	0.770	1
B^0	B-zero	$d\bar{b}$	0	5.279	0
η_c	eta-c	$c\bar{c}$	0	2 .980	0

Matter and Antimatter
For every particle type there is a corresponding antiparticle type, denoted by a bar over the particle symbol (unless + or – charge is shown). Particle and antiparticle have identical mass and spin but opposite charges. Some electrically neutral bosons (e.g., Z^0, γ, and $\eta_c = c\bar{c}$, but not $K^0 = d\bar{s}$) are their own antiparticles.

Figures
These diagrams are an artist's conception of physical processes. They are ***not*** exact and have ***no*** meaningful scale. Green shaded areas represent the cloud of gluons or the gluon field, and red lines the quark paths.

A neutron decays to a proton, an electron, and an antineutrino via a virtual (mediating) W boson. This is neutron β decay.

An electron and positron (antielectron) colliding at high energy can annihilate to produce B^0 and $\bar{B}^0$ mesons via a virtual Z boson or a virtual photon.

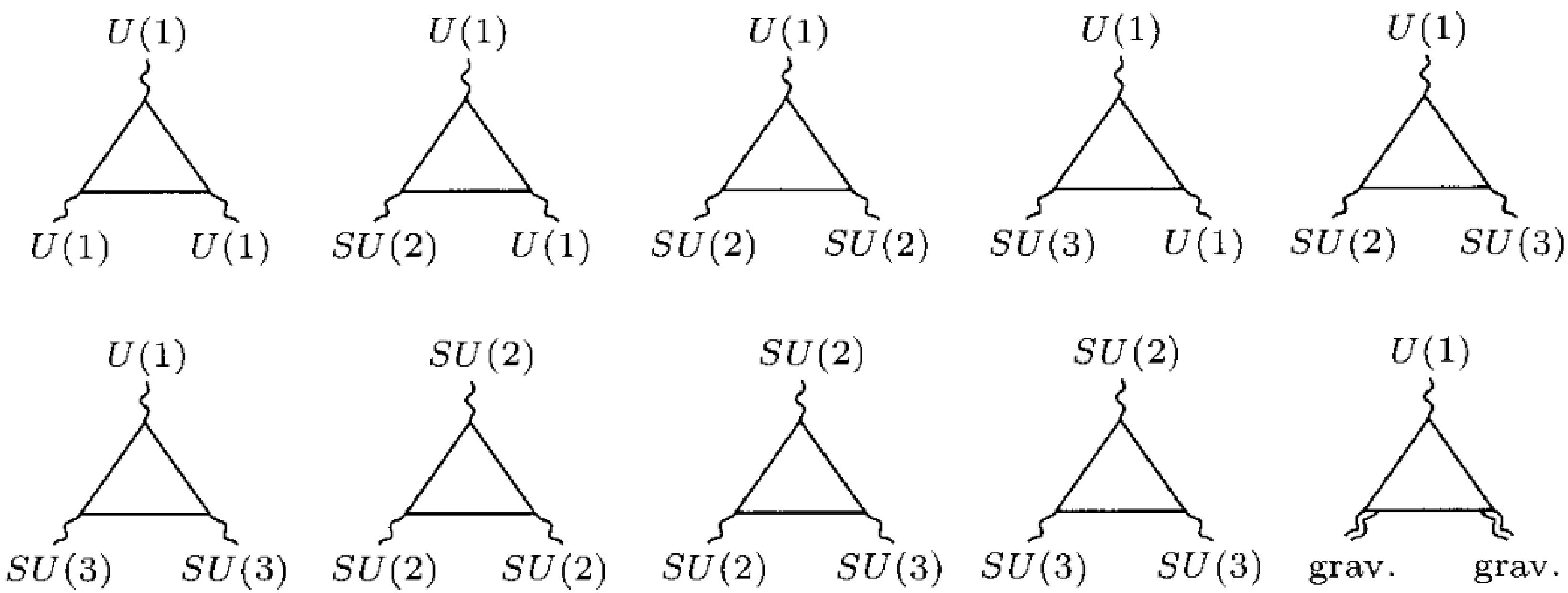
Two protons colliding at high energy can produce various hadrons plus very high mass particles such as Z bosons. Events such as this one are rare but can yield vital clues to the structure of matter.

The Particle Adventure
Visit the award-winning web feature *The Particle Adventure* at **http://ParticleAdventure.org**

This chart has been made possible by the generous support of:
U.S. Department of Energy
U.S. National Science Foundation
Lawrence Berkeley National Laboratory
Stanford Linear Accelerator Center
American Physical Society, Division of Particles and Fields
BURLE INDUSTRIES, INC.

- **BUT!** in the SM electroweak interactions are chiral. If the notion of chirality is **not-gauge invariant** – how SM can be consistent?

- There are many left and many right states in the SM. If you sum them up – they cancel all anomalies



From **Peskin & Schroeder [Sec. 20.2]**

Anomalous fermion numbers non-conservation

- Baryon number in the Standard Model:

$$J_B^\mu = \frac{1}{3}(\bar{u}\gamma^\mu u + \bar{d}\gamma^\mu d + \text{other quarks})$$

where $u = (u_L, u_R)$, etc. – vector like U(1) symmetry. Same is true for the lepton number J_L^μ

- However, **only left-chiral components** couple to the SU(2) gauge field

- SU(2) gauge fields: three gauge components A_μ^a, $a = 1, 2, 3$

- The non-conservation of baryon and lepton numbers is given by

$$\partial_\mu J_B^\mu = \partial_\mu J_L^\mu = \frac{N_f g^2}{32\pi^2}\sum_{a=1}^{3}\mathcal{F}_{\mu\nu}^a\tilde{\mathcal{F}}_a^{\mu\nu}$$

where $\mathcal{F}_{\mu\nu}$ is the SU(2) field strength:

$$\mathcal{F}^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g\epsilon^{abc} A^b_\mu A^c_\nu$$

$\tilde{\mathcal{F}}_{\mu\nu} = \frac{1}{4}\epsilon_{\mu\nu\alpha\beta}\mathcal{F}^{\alpha\beta}$ is the **dual field strength** ($\epsilon_{\mu\nu\alpha\beta}$ is the Levi-Civita tensor)

Problems: Recall that in the electrodynamics the electric and magnetic fields are defined from the field strength $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ as $E_i = F_{0i}$ and $H_i = \frac{1}{2}\epsilon_{ijk}F^{jk}$, where $i, j, k = 1, 2, 3$ are spatial indexes

- Show that the dual field strength tensor $\tilde{F}_{\mu\nu} = \frac{1}{4}\epsilon_{\mu\nu\alpha\beta}F^{\alpha\beta}$ interchanges $\vec{E} \leftrightarrow \vec{H}$, i.e. $H_i = \tilde{F}_{0i}$ and $E_i = \frac{1}{2}\epsilon_{ijk}\tilde{F}^{jk}$
- Show that $F_{\mu\nu}F^{\mu\nu} = \tilde{F}_{\mu\nu}\tilde{F}^{\mu\nu} = \vec{E}^2 - \vec{H}^2$
- Show that $F_{\mu\nu}\tilde{F}^{\mu\nu} = 4\vec{E}\cdot\vec{H}$
- Show that $F_{\mu\nu}\tilde{F}^{\mu\nu}$ is full 4-divergence, i.e. $F_{\mu\nu}\tilde{F}^{\mu\nu} = \partial_\mu K^\mu$ where K^μ is some 4-vector

- The total baryon number non-conservation is given by

$$\Delta B = \int\limits_{t_1}^{t_2} dt\, \frac{dB}{dt} = \frac{N_f g^2}{8\pi^2} \int_{t_1}^{t_2} dt \int dV \sum_{a=1}^{3} \vec{\mathcal{E}}_a \cdot \vec{\mathcal{H}}_a$$

$N_f = 3$ – number of generations of fermions	g – SU(2) coupling constant, $\frac{g^2}{4\pi} \sim \frac{1}{30}$	$\mathcal{E}_i^a \equiv \mathcal{F}_{0i}^a$; $\mathcal{H}_i^a \equiv \tilde{\mathcal{F}}_{0i}^a$

- Recall that in U(1) theory

$$\int dV\, \vec{E} \cdot \vec{B} = \frac{dN_{\text{CS}}}{dt}$$

where we defined the **Chern-Simons number**

$$N_{\text{CS}} = \frac{g^2}{96\pi^2} \int d^3x \vec{A} \cdot \vec{B}$$

a very similar formula holds for non-Abelian SU(2) fields

To generate non-zero baryon number in transitions from $t_1 \to t_2$, we need fluctuations of the SU(2) gauge that change N_{CS}:

$$B(t_2) - B(t_1) = N_f \Big[N_{\text{CS}}(t_2) - N_{\text{CS}}(t_1) \Big]$$

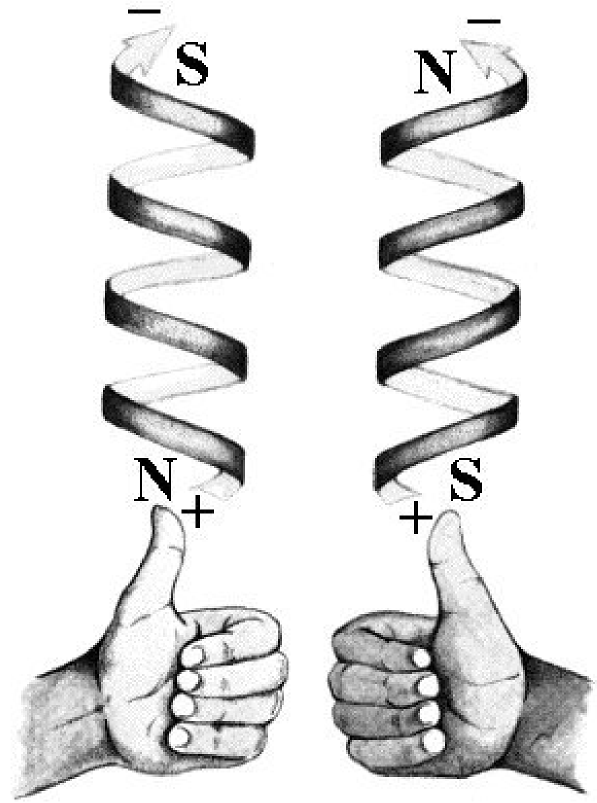
Immediate questions:

- If baryon number is not conserved in the Standard Model – why proton is stable?
- How possible are the configurations with non-zero $\vec{\mathcal{E}}_a \cdot \vec{\mathcal{H}}_a$?? How often do they occur at $T = 0$ and at high temperatures?
- $\Delta B \propto g^2 \int \vec{\mathcal{E}} \cdot \vec{\mathcal{H}} \Longrightarrow \vec{\mathcal{E}} \cdot \vec{\mathcal{H}} \propto \frac{1}{g^2}$ $\Rightarrow$ energy density of the SU(2) gauge field is $\propto \vec{\mathcal{E}}^2 + \vec{\mathcal{H}}^2 \propto \frac{1}{g^2}$. What do they mean?

Chern-Simons number

- Example: configuration with $\vec{A}(\vec{x}) = A_0\Big(\sin(kz), \cos(kz), 0\Big)$ has the magnetic field $\vec{B} = \mathrm{curl}\,\vec{A} = \vec{B}(\vec{x}) = k\vec{A}(\vec{x})$
- Magnetic energy density

$$\rho_B = \lim_{V\to\infty} \frac{1}{V}\int dV\,\frac{\vec{B}^2}{2} = \frac{1}{2}k^2 A_0^2$$



- Its Chern-Simons number density:

$$n_{\text{CS}} = \lim_{V\to\infty} \frac{1}{V}\int dV\,\vec{A}\cdot\vec{B} = kA_0^2$$

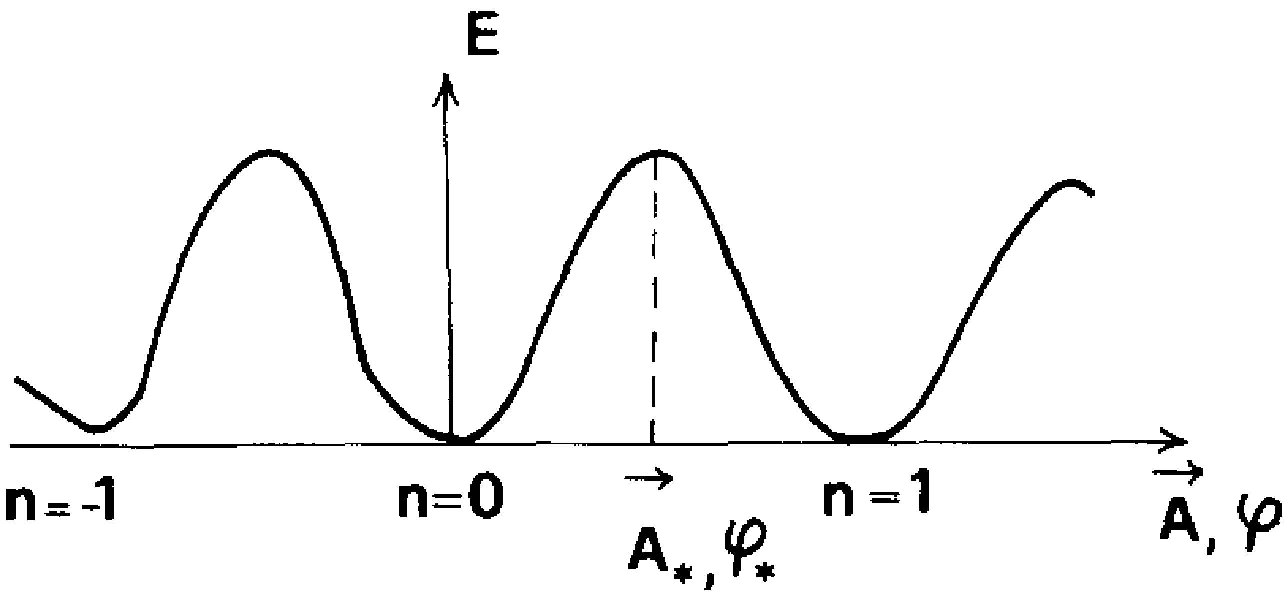
- $N_{\text{CS}} \neq 0$ means that the field is “helical”

- Notice that we can send $k \to 0$ and $A_0 \to \infty$ so that

$$n_{\text{CS}} = \text{const} \quad \text{but} \quad \rho_B \to 0$$

⇒ Configurations with $n_{\text{CS}} \neq 0$ have same energy as vacuum ($\vec{B} = 0$). Arbitrary configuration of gauge fields has higher energy

- The same is true for the SU(2) + Higgs field system:



- The height of this barrier is

$$E_{\text{barrier}} \approx \frac{2M_W}{\alpha_W} \sim 10\,\text{TeV}$$

Transitions with $\Delta N_{\text{cs}} \neq 0$

- At zero energy to change N_{CS} one needs to tunnel through the barrier. The probability is given by

$$P_{\text{tunnel}} \sim e^{-\frac{4\pi}{\alpha_W}} \sim 10^{-160}$$

– therefore proton is stable

- At finite temperatures the rate of transition becomes unsuppressed:

$$\Gamma_{\text{sph}}(T) = \begin{cases} (\alpha_W T)^4 \alpha_W \log(1/\alpha_W), & T \gtrsim E_{\text{barrier}} \\ (\alpha_W T)^4 \left(\frac{E_{\text{barrier}}}{T}\right)^7 \exp\left(-\frac{E_{\text{barrier}}}{T}\right), & T \lesssim E_{\text{barrier}} \\ \exp\left(-\frac{4\pi}{\alpha_W}\right), & T = 0 \end{cases}$$

- Each fluctuation of SU(2) field with $\Delta N_{\text{CS}} = 1$ creates 9 quarks $+$ 3 leptons violating both baryon and lepton number but leaving $B-L$ conserved

Sakharov condition-II

- C- and CP-symmetries change **all** charges including baryon number:
  $C\,|p\rangle = |\bar{p}\rangle$, $C\,|n\rangle = |\bar{n}\rangle$, $C\,|e^-\rangle = |e^+\rangle$, etc.

- If these symmetries were conserved in the early Universe this would mean that for any process, changing baryon number, there is another process, restoring baryon number. Namely, if

$$X_1 + X_2 + \dots \to Y_1 + Y_2 + \dots$$

change baryon number by $+1$, then there is a process:

$$\bar{X}_1 + \bar{X}_2 + \dots \to \bar{Y}_1 + \bar{Y}_2 + \dots$$

in which baryon number changes by -1 and **their probabilities are the same**.

- **Time reversal** T : $P_{\alpha\to\beta} \xrightarrow{T} P_{\beta\to\alpha}$

- **CP** : $|\nu(\vec{p})\rangle \xrightarrow{CP} |\bar{\nu}(-\vec{p})\rangle$

$$P_{\alpha\to\beta} \xrightarrow{CP} P_{\bar{\alpha}\to\bar{\beta}}$$

CP was believed to be the exact symmetry of nature after parity violations were discovered

- However: CP-violation in kaon decays (**1964** Cronin, Fitch,...) In a small fraction of cases ($\sim 10^{-3}$), long-lived K_L (a mixture of K^0 and $\bar{K}^0$ decays into pair of two pions, what is forbidden by CP-conservation.

- **If** CP were exact symmetry, an equal number of K^0 and $\bar{K}^0$ would produce an equal number of electrons and positrons in the reaction

$$K^0 \to \pi^- e^+ \nu_e, \quad \bar{K}^0 \to \pi^+ e^- \bar{\nu}_e,$$

- However, the number of positrons is somewhat larger ($\sim 10^{-3}$) than the number of electrons.

- **CPT**: $P_{\alpha\to\beta} \xrightarrow{CPT} P_{\bar{\beta}\to\bar{\alpha}}$

  CPT theorem: particles and antiparticles have the same mass, the same lifetime, but *all* their charges (electric, baryonic, leptonic, etc) are opposite.

- All known processes conserve CPT

- **Parity transformation** is a discrete space-time symmetry, such that all spatial coordinates flip $\vec{x} \to -\vec{x}$ and time does not change $t \to t$

- **Show** that:

P-even	P-odd
time	position $\vec{x}$
angular momentum	momentum $\vec{p}$
mass density	Force
electric charge	electric current
magnetic field	electric field

- For fermions parity is related to the notion of **chirality**

- Dirac equation without mass can be split into two non-interacting parts

$$(i\gamma^\mu\partial_\mu - \cancel{m})\psi = \begin{pmatrix} \cancelto{0}{-m} & i(\partial_t + \vec{\sigma}\cdot\vec{\nabla}) \\ i(\partial_t - \vec{\sigma}\cdot\vec{\nabla}) & \cancelto{0}{-m} \end{pmatrix} \begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix} = 0$$

Parity transformation

- **Show** that if particle moves only in one direction $\boldsymbol{p} = (p_x, 0, 0)$, then these two components $\psi_{L,R}$ are **left-moving** and **right-moving** along x-direction.

- One can define the $\gamma_5 = i\gamma_0\gamma_1\gamma_2\gamma_3$. In the above basis (Peskin & Schroeder conventions)':

$$\gamma_5 = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$$

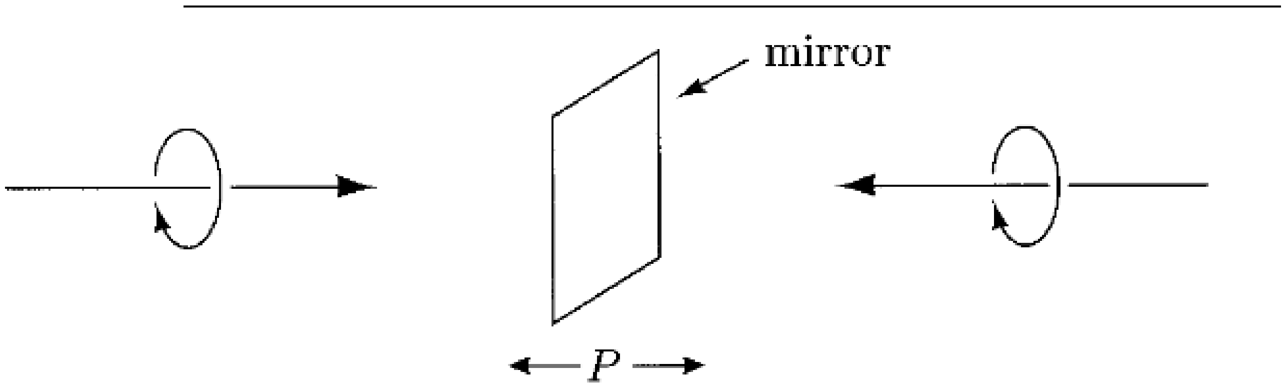
  and

$$\gamma_5 \psi_{R,L} = \pm \psi_{R,L}$$

- **Show** that for massless fermions one can define a conserved quantity **helicity**: projection of spin onto momentum:

$$h \equiv \frac{(\boldsymbol{p} \cdot \boldsymbol{s})}{|\boldsymbol{p}|}$$

  Show that left/right chiral particles have definite helicity ± 1.



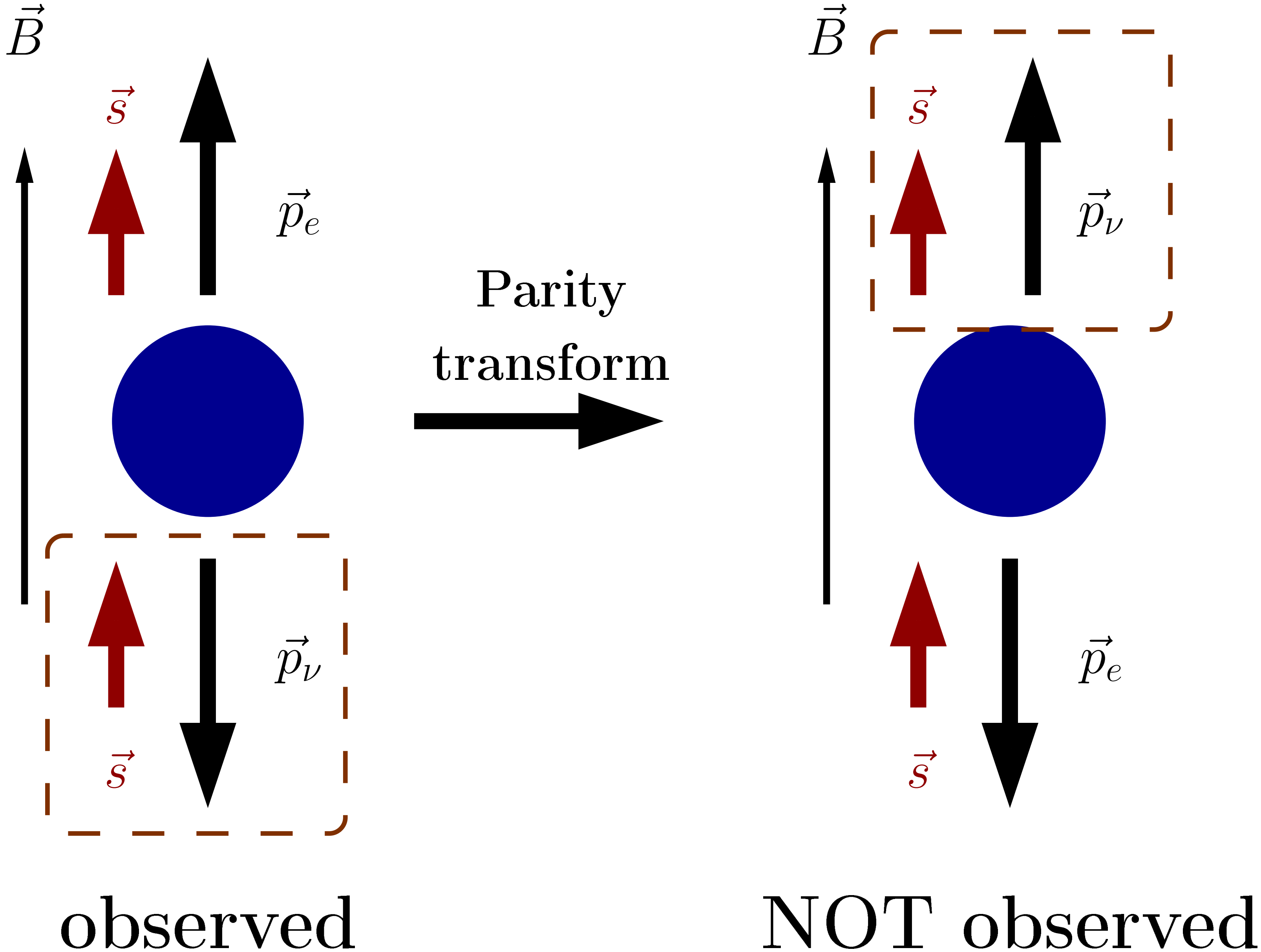
- For fermions: Left-Handed $\rightleftarrows$ Right-Handed. $|\psi_L(\vec{p})\rangle \xrightarrow{P} |\psi_R(-\vec{p})\rangle$.

- The neutrino being massless particle would have two states:
  - Left neutrino: spin **anti-parallel** to the momentum $\boldsymbol{p}$
  - Right neutrino: spin **parallel** to the momentum $\boldsymbol{p}$

- This has been tested in the experiment by Wu et al. in 1957

Parity violation in weak interactions

- Put nuclei into the strong magnetic field to align their spins
- Cool the system down (to reduce fluctuations, flipping spin of the Cobalt nucleus)
- The transition from ^{60}Co to ^{60}Ni has momentum difference $\Delta J = 1$ (spins of nuclei were known)
- Spins of electron and neutrino are parallel to each other
- Electron and neutrino fly in the opposite directions
- **Parity** flips momentum but does not flip angular momentum/spin/magnetic field


$\vec{B}$
$\vec{s}$
$\vec{p}_e$
$\vec{p}_\nu$
$\vec{s}$
Parity
transform
$\vec{B}$
$\vec{s}$
$\vec{p}_\nu$
$\vec{p}_e$
$\vec{s}$
observed
NOT observed

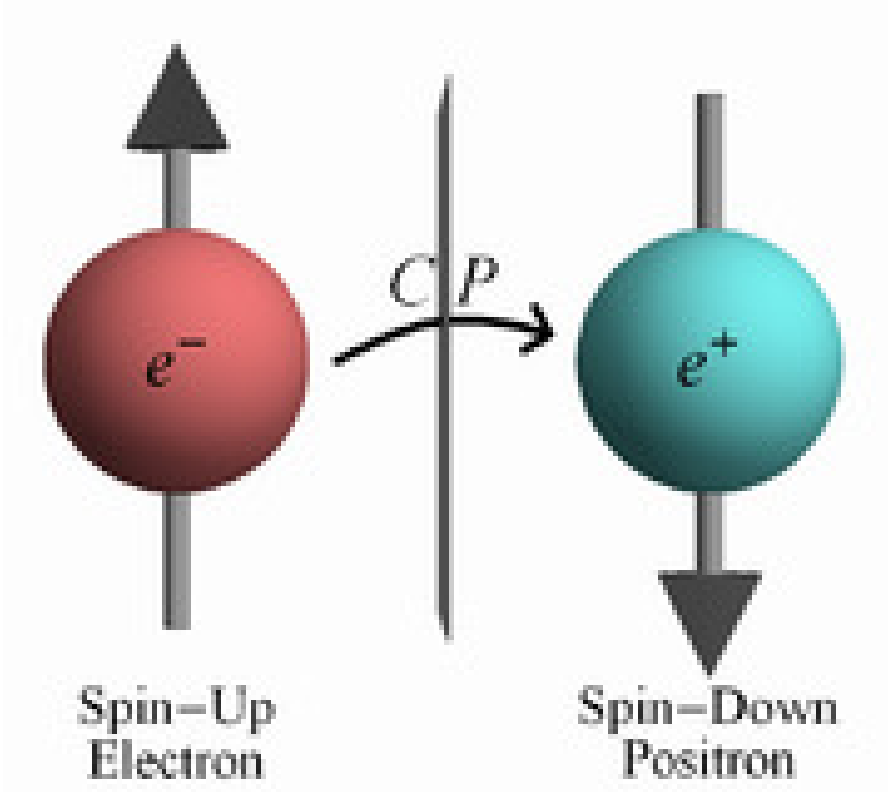
- This result means that neutrino always has spin anti-parallel to its momentum (**left-chiral particle**)

- Parity exchanges left and right chiralities. As neutrino is always left-polarized this means that

Particle looked in the mirror and did not see itself ???

- How can this be?

- Another symmetry, **charge conjugation** comes to rescue. Charge conjugation exchanges particle and anti-particle. Combine it with parity (**CP-symmetry**):



  - P: $|\nu_L\rangle \to |\nu_R\rangle$ – **impossible**
  - CP: $|\nu_L\rangle \to |\bar{\nu}_R\rangle$ – **possible**. Anti-neutrino exists and is always right-polarized
  - Life turned out to be more complicated. CP-symmetry is also broken

CP non-conservation in the SM

- All CP-non-conservation effects in the SM are in the quark sector

- These are complex phases of Cabbibo-Kobayashi-Maskawa mass matrix

- In analogy with neutrino mass matrix, one needs at least 3 flavours to have a possibility for the presence of complex phase that cannot be removed by field redefinition (CP-violation)

- There are two types of quark matrices: M_u for “up-quarks” (u,c,t) and M_d for “down-quarks” (d,s,b). Up-sector can be made diagonal, and the M_d is non-diagonal in flavour space.

- Notice, that some elements of these matrices do not play role in physical processes and can be reabsorbed in the redefinition of fields

- The lowest order in mass CP-non-invariant expression that is invariant under all possible quark fields redefinitions is given by

$$J_{\rm CP} = \mathrm{Im}\,\mathrm{Tr}\Big(M_u^4 M_d^4 M_u^2 M_d^2\Big) \propto m_t^4 m_b^4 m_c^2 m_s^2 \sin\delta_{CP} \sim 10^4\ \text{GeV}^{12}$$

  where $\delta_{\rm CP}$ is the CP-violating phase that can be measured from kaon decays

- Notice that $J_{\rm CP}/T_{\text{sph}}^{12} \sim 10^{-20}$, where T_{sph} is a temperature of sphaleron freeze-out – a number much smaller than the baryon asymmetry (that we expect to be of the order 10^{-10})

Deviation from thermal equilibrium

- In thermal equilibrium any quantity is defined in a unique way as a function of **temperature** and possibly a number of some **conserved charges** Q or corresponding **chemical potentials**

- To any equilibrium process (changing C, CP, B, or any other quantity) there is a reverse process, changing any charge in the opposite direction. As a result for example, the total baryon charge $\langle B\rangle$ will approach its equilibrium value $B_{\rm eq}(T,Q,\dots)$ – a unique function of T and values of other conserved charges.

- Notice, that if a model does not have any conserved charges $Q \neq 0$, than in equilibrium baryon number (also, lepton number or any other quantum number **will be equal to zero**:

  Density matrix $\hat\varrho = e^{-\hat H/T}$ is CPT invariant (CPT theorem), while any charge Q changes sign under CPT. Therefore **in equilibrium**

$$\langle Q\rangle = \mathrm{Tr}\Big(\hat Q\hat\varrho\Big) = \mathrm{Tr}\Big(\hat Q^{(\rm CPT)}\hat\varrho\Big) = \mathrm{Tr}\Big(-\hat Q\hat\varrho\Big) \Longrightarrow \langle Q\rangle = 0$$

Problems about thermal equilibrium

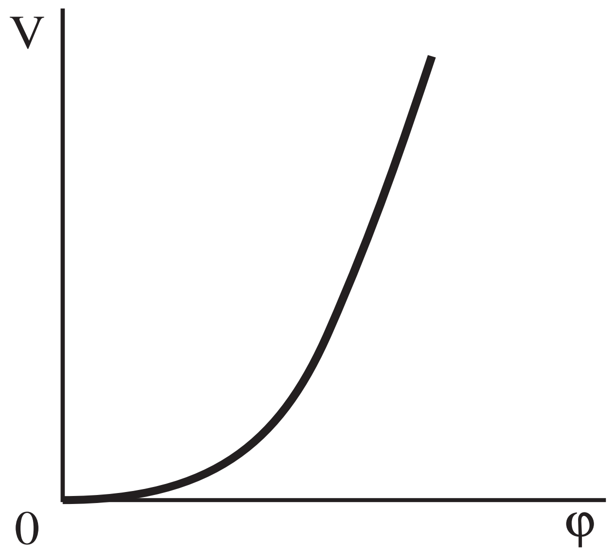
1. Estimate within the Fermi theory at what temperatures weak interactions enter thermal equilibrium? At what temperatures they go out of thermal equilibrium again?
2. At high temperatures ($T \gg 100$ GeV) one can speak about unbroken $SU(2) \times U(1)$ electroweak symmetry. The interaction is characterized by "weak oupling constant" $\alpha_W = \frac{g_W^2}{4\pi} \approx \frac{1}{30}$. Estimate, at what temperatures typical electroweak reactions are in thermal equilibrium? (Hint: use the analogy with the electromagnetic interactions)
3. Below temperatures $T \ll 100$ GeV electroweak symmetry is broken and one can speak about electromagnetic interactions. Estimate at what temperatures electromagnetic processes enter thermal equilibrium. At what temperatures they go out of thermal equilibrium again?
4. At temperatures $T \ll 100$ GeV interactions of sterile neutrino with other leptons can be described by the analog of Fermi theory (with the "sterile Fermi constant" $G'_F = \theta * G_F$, where $\theta \sim 10^{-5}$). At what temperatures such sterile neutrinos enter thermal equilibrium and go out of thermal equilibrium.

Thermal equilibrium in the SM

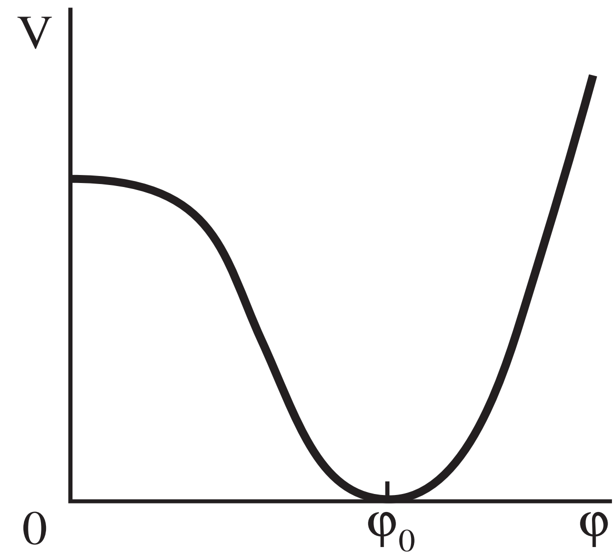
- We saw that all the SM processes containing quarks (particle, carrying baryon number) are in thermal equilibrium in the early Universe down to temperatures ~ 1 MeV when proton and neutron freeze-out. Therefore, they cannot be responsible for generation of baryon asymmetry of the Universe?

- What are physical processes can violate thermal equilibrium conditions?

Phase transitions in the early Universe



a



b

Massive scalar

$$V(\phi) = \frac{m^2}{2}\phi^2 + \frac{\lambda}{4}\phi^4$$

Spontaneous symmetry breaking

$$V_{\rm SSB}(\phi) = -\frac{m^2}{2}\phi^2 + \frac{\lambda}{4}\phi^4 + V_0 \qquad (4)$$

Minimum at $\boxed{\phi_0^2 = \frac{m^2}{\lambda}}$

1. Expand the potential (4) around the true minimum ϕ_0 and find the mass of the particle. Notice that it is not equal to m^2!

2. Generalize the potential (4) for the case of U(1) charged scalar field with the charge q (i.e. ϕ is the complex field and gauge transformation acts as $\phi \to e^{iq\xi}\phi$)

3. Introduce the gauge field A_μ and write the kinetic term for the charged scalar field ϕ (changing ordinary derivative ∂_μ to covariant derivative $D_\mu = \partial_\mu + qA_\mu$). Show that at the minimum of the potential $V_{\rm SBB}$ the gauge field becomes massive. Find the mass of the gauge field

4. Introduce the charged fermions, coupled to the field ϕ via **Yukawa interaction** $f\bar{\psi}\phi\psi$. Show that when ϕ is at the minimum of the potential (4) the Yukawa term becomes the Dirac mass term for the fermion.

Symmetry breaking at finite temperatures

- What happens with the system at finite temperatures (e.g. when $T^4 \sim V_0$)?

- Consider **free energy per unit volume** of the scalar field in the homogeneous and isotropic Universe:

$$\boxed{V_{\mathsf{EFF}}(\phi, T) \equiv \text{Free energy}/\text{Volume}}$$

- At low temperatures $V_{\mathsf{EFF}}(\phi, T) \approx V_{\mathrm{SBB}}(\phi)$

- The minimum of effective potential $\langle\phi\rangle_T$ is determined from the usual condition of minimum $\frac{\partial V_{\mathsf{EFF}}(\phi,T)}{\partial\phi} = 0$ while $\frac{\partial^2 V_{\mathsf{EFF}}(\phi,T)}{\partial\phi^2} > 0$

- What is the form of $V_{\text{EFF}}(\phi, T)$? Consider the situation when the temperature is high $T \gg \langle\phi\rangle_T$. Qualitatively in this situation all the particles have masses $m(\phi) \propto \langle\phi\rangle_T$ and $m(\phi) \ll T$. In this limit one would expect that the change in free energy density (is given by)

$$V_{\text{EFF}}(\phi, T) \approx V_{\text{SBB}}(\phi) + T^4 \sum_{\text{all massive particles}} c_i \frac{m_i^2(\phi)}{T^2}$$

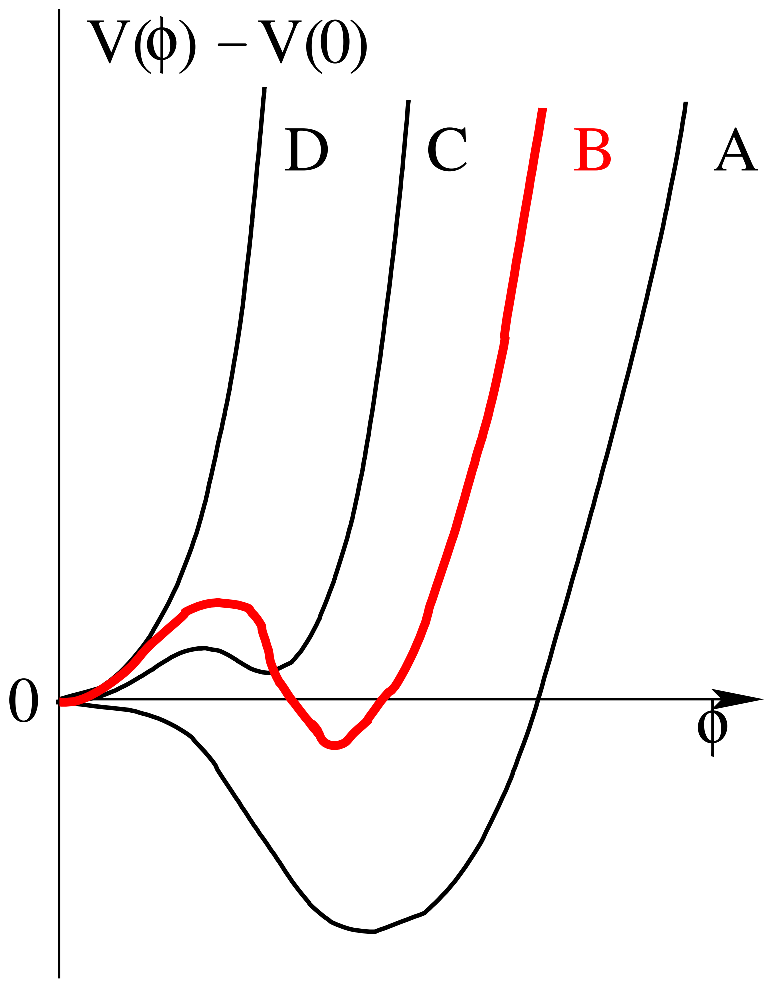
- Then we have

$$V_{\text{EFF}}(\phi, T) \approx (-\lambda v^2 + \alpha T^2)\phi^2 + \lambda\phi^4$$

where

$$\alpha = \frac{1}{12v^2}(6M_W^2 + 3M_Z^2 + 6m_t^2)$$ – heaviest particles to which Higgs couples

- If $T > \sqrt{\frac{\lambda}{\alpha}} v$ then $\phi = 0$ is the minimum of the system



Proper treatment of quantum corrections gives

$$V_{\text{EFF}} = \frac{\alpha}{24}(T^2 - T_{c2}^2)\phi^2 - \gamma T\phi^3 + \lambda\phi^4$$

D: single minimum at $\phi = 0$ **symmetric phase**

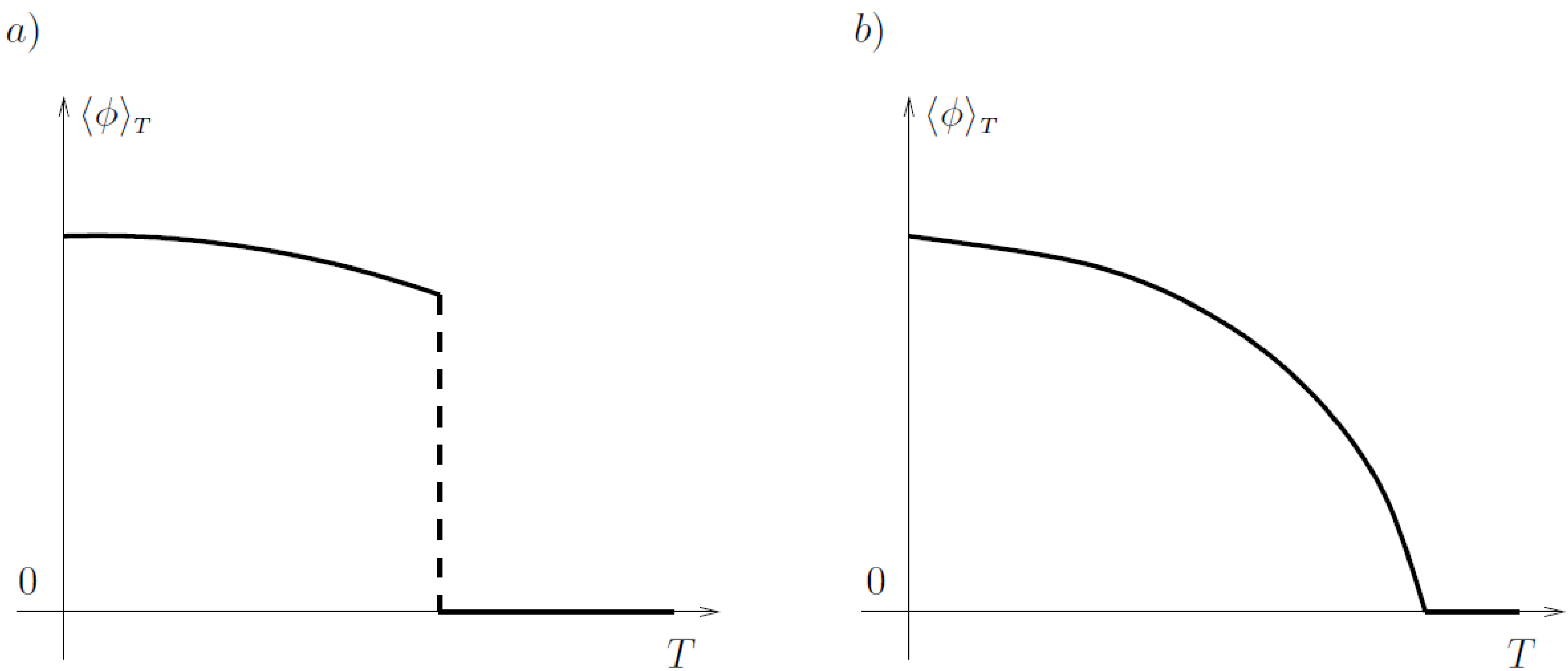
C: second (local) minimum appears at $\phi \neq 0$. The distance between two minima always finite!

B: Minimum at $\phi \neq 0$ is a true minimum separate from the metastable vacuum at $\phi = 0$ by potential barrier

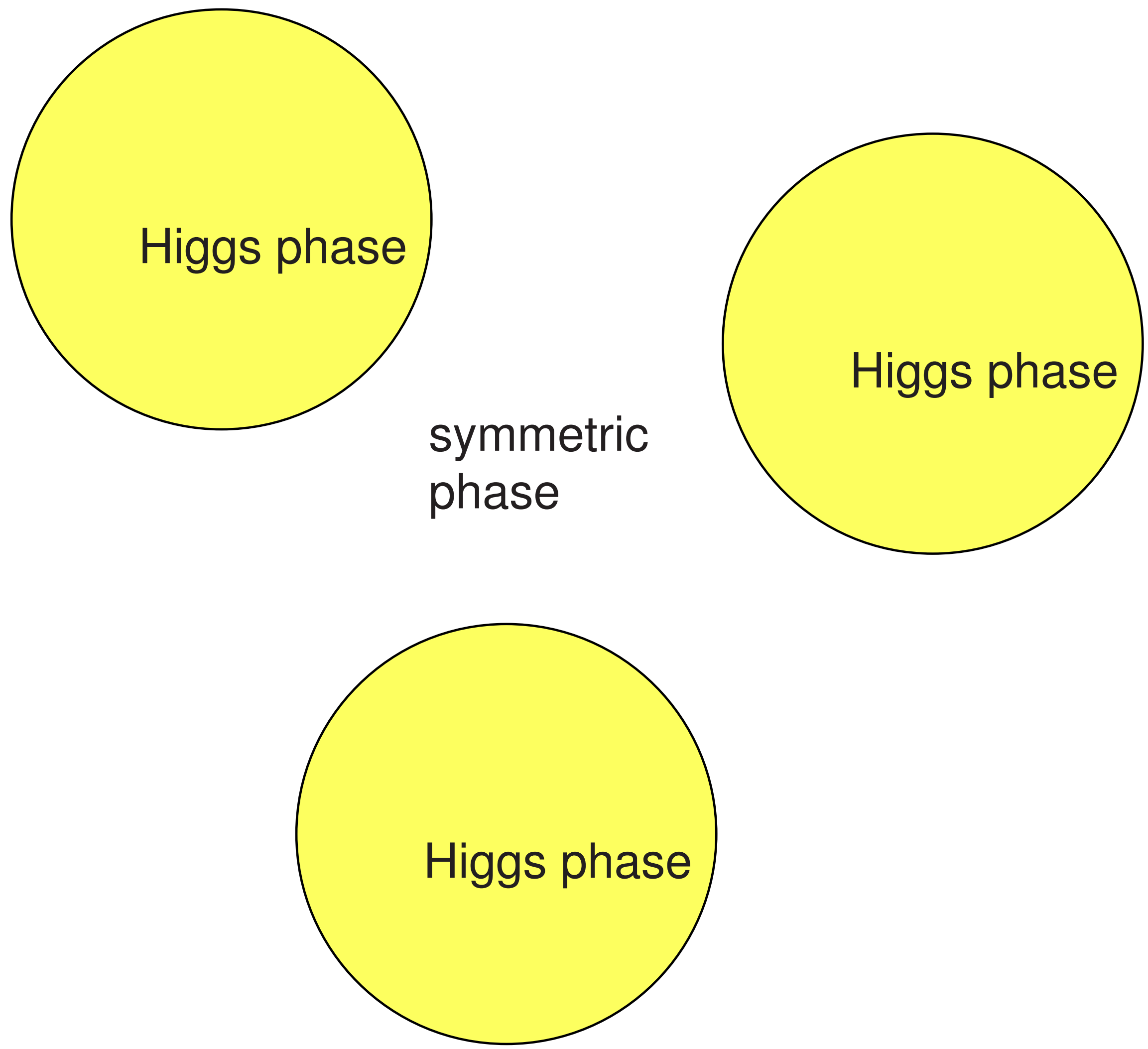
A: $T = 0$, $\phi_{min} = \phi_0$

Two main types of phase transitions:

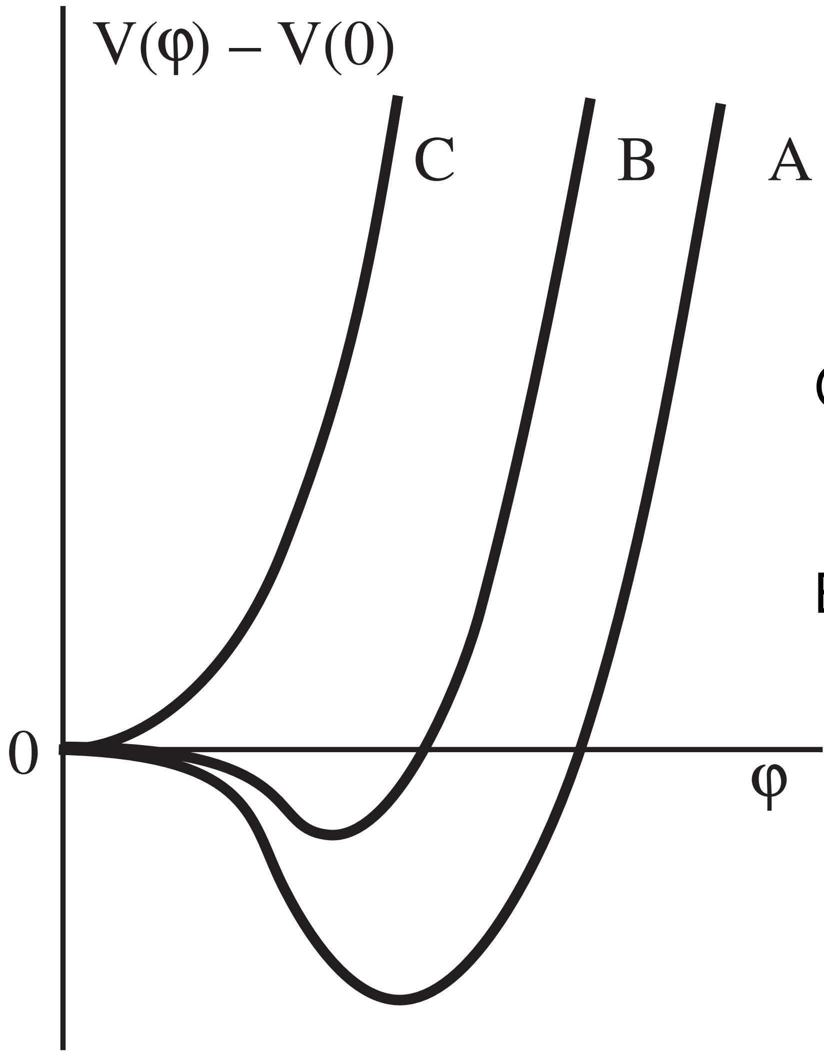
- **I order** – Discontinuity of $\frac{\partial F}{\partial T} \propto \langle\phi\rangle_T$ (**left**).
- **II order** – No discontinuity of $\langle\phi\rangle_T$ (**right**).



First-order phase transition



2nd order phase transition



$$V_{\text{EFF}} = -\frac{m^2}{2}\phi^2 + \frac{\lambda}{4}\phi^4 + \frac{\lambda T^2}{2}\phi^2$$

$$T_c = \frac{m}{\sqrt{\lambda}}$$

C: $T > T_c$ single minimum at $\phi = 0$
symmetric phase

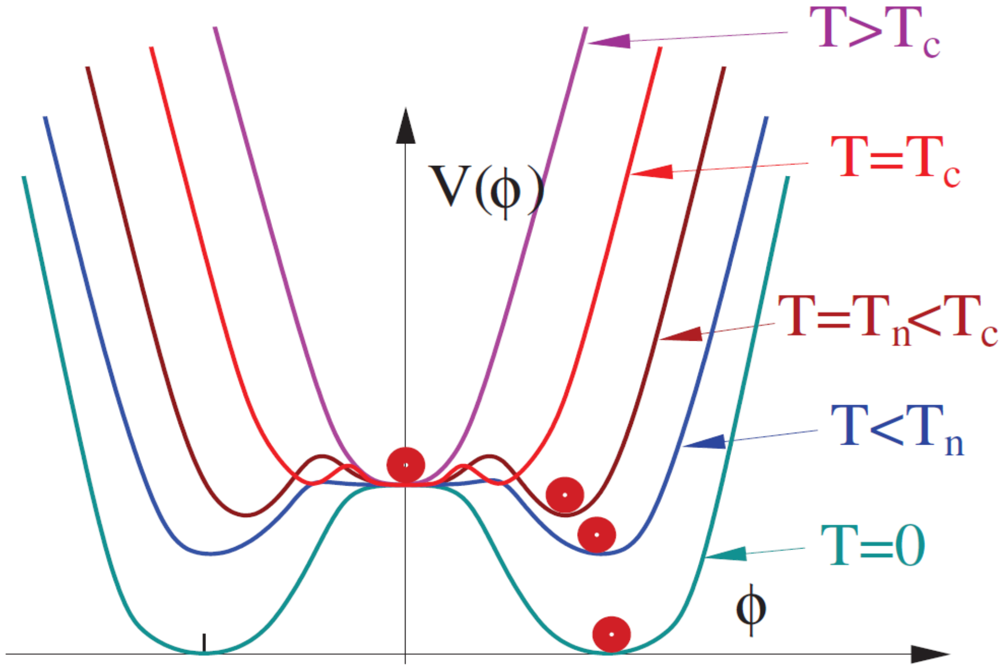
B: At $T \leq T_c$ a minimum at $\phi \neq 0$ appears, $\phi = 0$ becomes maximum. $\langle\phi\rangle_T = \sqrt{\frac{T_c^2 - T^2}{\lambda}}$. Notice that $\langle\phi\rangle_T \propto \sqrt{T_c - T}$ – starts from 0 at $T = T_c$ (unlike the 1st order P.T.)

A: $T = 0$, $\phi_{min} = \phi_0$

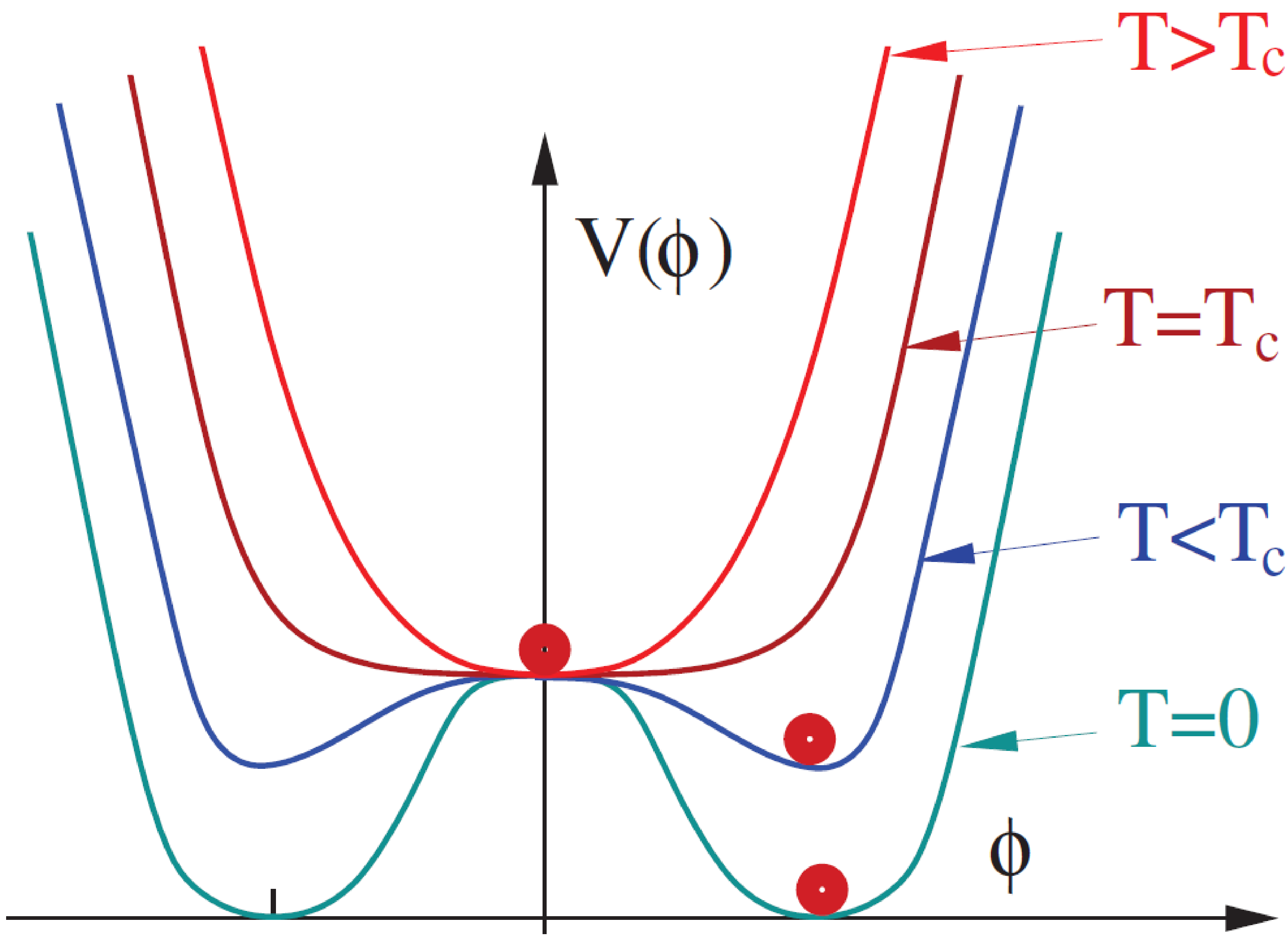
In the presence of the temperature, the potential for the field ϕ can change:

$$V_{\mathsf{EFF}}(\phi, T) = \lambda \left[\frac{\phi^4}{4} + \frac{\phi^2}{2}\left(\frac{T^2}{4} - v^2\right)\right]$$

$T_c = 2v$

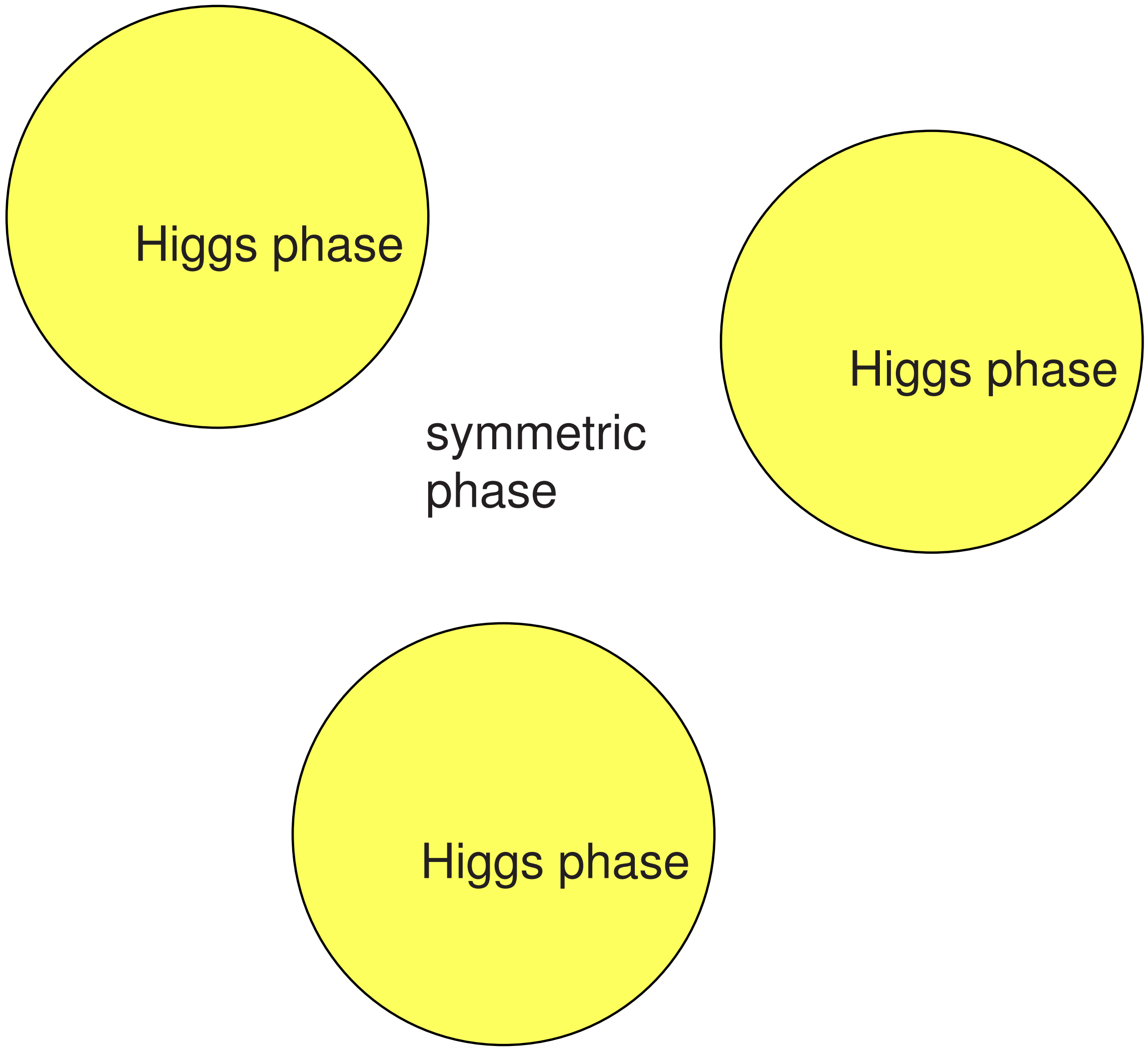


From http://www.phys.uu.nl/~prokopec



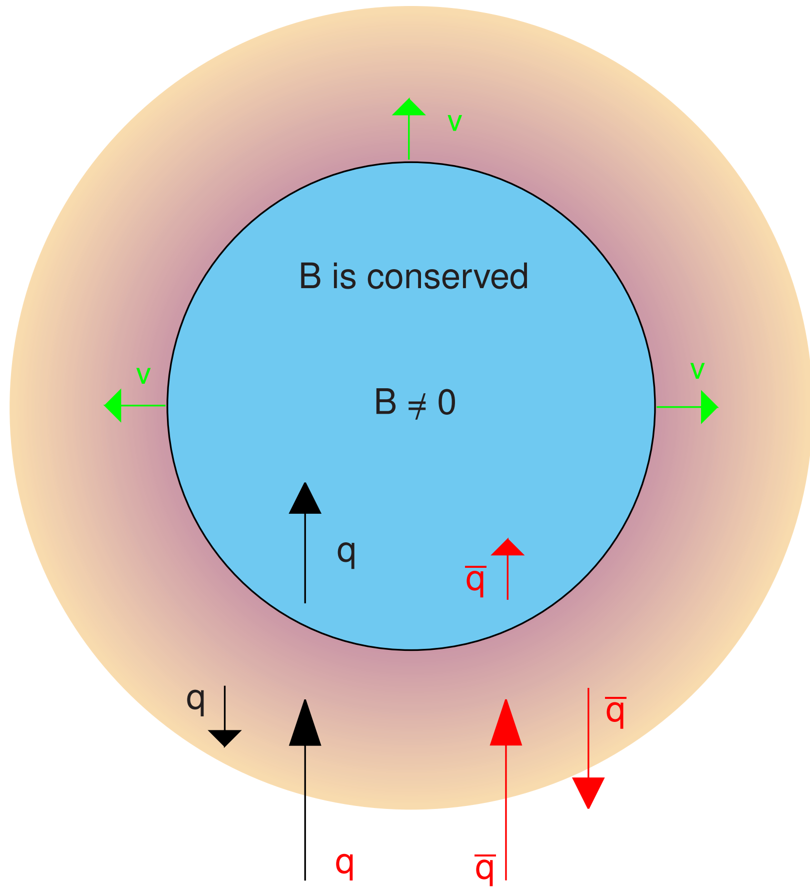
Back to Sakharov

First-order phase transition



B is not conserved

B = 0



- In the SM all the conditions seems to be satisfied:
  - CP is violated
  - Baryon number may not be non-conserved: it can be created from lepton number by non-perturbative processes active at high temperature
  - There may be phase transitions (EW, QCD).

- However, experimental bounds on the SM parameters show that this does not happen!

- Sphaleron processes violate $B+L$ but do not affect $B-L$ charge.
- If at $T > T_{\text{sph}}$ you generate non-zero **lepton number** via some process (so that $B-L$ becomes $\neq 0$) ...
- ...then sphalerons will "**transform L into B**" (so that, for example, in the SM plasma one gets $B = \frac{28}{79}(B-L)$ and $L = -\frac{51}{79}(B-L)$)

Khlebnikov & Shaposhnikov, "*The Statistical Theory of Anomalous Fermion Number Nonconservation*" Nucl. Phys. B308 (1988) 885-912

This class of scenarios is called LEPTOGENESIS

Sterile neutrinos and leptogenesis

There exist three classes of leptogenesis scenario related to sterile neutrinos:

Thermal leptogenesis: **Fukugita & Yanagida'86**

Works for $M_N \sim 10^{12}$ GeV

Resonant leptogenesis: **Pilaftsis, Underwood'04–'05**

Works for $M_{N_1} \approx M_{N_2} \sim M_W$ and $|M_{N_1} - M_{N_2}| \ll M_{N_1,N_2}$

Leptogenesis via oscillations: **Akhmedov, Smirnov & Rubakov'98**

Asaka &Shaposhnikov'05

Works for $M_{N_1} \approx M_{N_2} \lesssim M_W$ and $|M_{N_1} - M_{N_2}| \ll M_{N_1,N_2}$

- "Sufficiently heavy" sterile neutrinos can decay into left leptons + Higgs

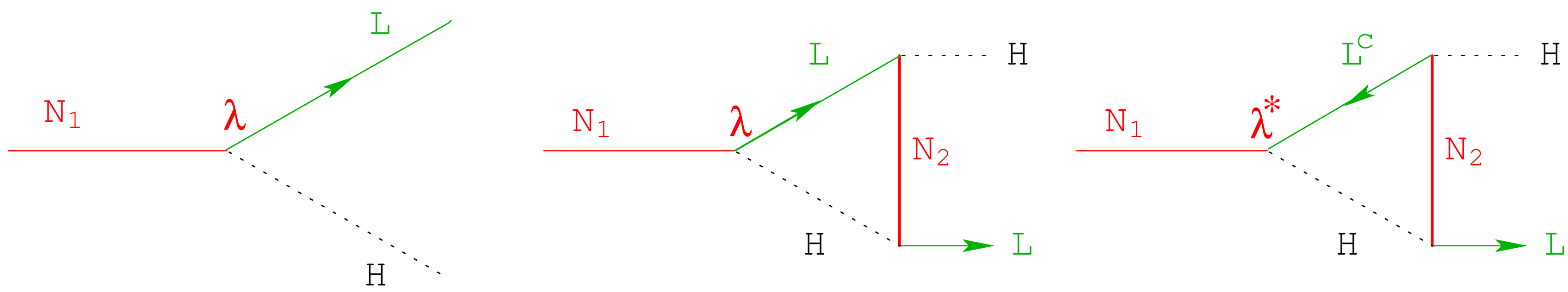


Fig. from **Strumia & Vissani**

- Due to their Majorana mass these decays break **lepton number** ($N \to L + H$ and $N \to \bar{L} + \bar{H}$)

- Decay rate $\Gamma_{tot} \propto |F|^2 M_N$

- Tree level decay of $N_1 \to L + H$ (the first graph):

$$\Gamma = \frac{|F|^2 M_1}{8\pi}$$

– complex phase does not contributes, so not satisfied **2nd Sakharov condition**

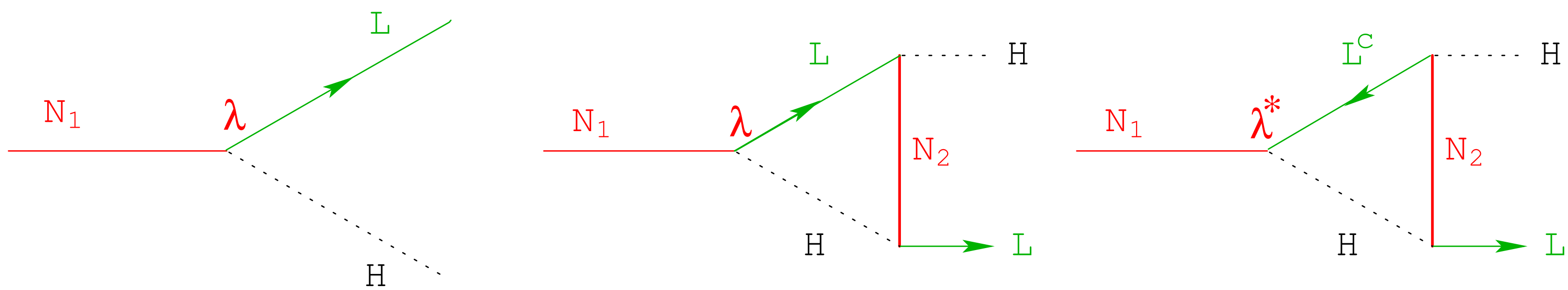
- Need to take into account loop effects (graphs 2 and 3)

- The resulting $\eta_B \propto 10^{-3}|F|^2 \Rightarrow |F|^2 \sim 10^{-7} \Rightarrow \boxed{M_N \sim 10^{12}\text{ GeV}}$

- Such sterile neutrinos would add corrections to the Higgs mass of the order of $|F|^2 M_N^2 \sim 10^{14}$ GeV$^2 \gg M^2_{\text{Higgs}}$ — **gauge hierarchy problem!**

- Can lighter sterile neutrinos provide leptogenesis?

- **Yes!** but still $\eta_B \propto |F|^2$ and one needs to compensate smaller Yukawas (the smaller is the mass, the smaller are the Yukawa couplings)



- If masses of two sterile neutrinos **approximately equal**, then in the last diagram the production of lepton number is enhanced by $\dfrac{M_1\Gamma_{tot}}{(M_1 - M_2)^2 + \Gamma_{tot}^2}$

- Leptogenesis possible for $M_N \sim M_W$!

Leptogenesis via oscillations

- As M_N decreases, the Majorana nature of particles plays lesser role. Can one get a leptogenesis for $M_N \ll T_{\text{sph}}$

Akhmedov, Smirnov, Rubakov'98

Asaka, Shaposhnikov '05

- Recall, that sphalerons (SU(2) gauge configurations) convert **lepton number stored in left lepton doublets** into the baryon number.

- Can it be that **total lepton number** $= 0$ but is distributed between sectors:

$$\text{Lepton \# of left } \nu = -\text{Lepton \# of } N_I \neq 0$$

  sterile neutrinos are Majorana particles, so for them role of lepton number is played by helicity

- Need at least **two sterile neutrinos** with $M_{N_1} \approx M_{N_2}$

Leptogenesis via oscillations

Akhmedov, Smirnov, Rubakov'98

1) Need to choose **at least two** sterile neutrinos that **do not thermalize** until T_{sph}

- At $T > m_t$ thermalization goes via Higgs exchange $N + t \leftrightarrow \nu + t$ or $H \leftrightarrow N + \nu, \ldots$
- $\Gamma_{therm} \sim \frac{9|F|^2 f_t^2}{64\pi^3}$ compares to $H(T)$ at $T_{eq} \sim 5M_N$.
- Therefore, if $M_N < M_W$ – particles not thermalized until sphalerons freeze-out

2) Sterile neutrinos are produced (e.g. via $H \to N + \nu$)

no lepton number is any sectors so far!

3) Sterile neutrinos **oscillate into each other** in the CP-violating way

recall, that we can have CP-phases in the Yukawa matrix of sterile neutrinos!

4) This generate some effective “lepton number” in sterile (and, therefore, in active) sectors

Leptogenesis via oscillations

- The frequency oscillation between two neutrinos with masses $M_1 \approx M_2$ is given by

$$\omega_{\text{osc}} \sim \frac{M_1^2 - M_2^2}{E_N} \approx \frac{M_N \Delta M(T)}{T}$$

- If $\omega_{\text{osc}} \gg H(T_{\text{sph}})$ – many oscillations had occurred by the time of sphaleron freeze-out and lepton number is “washed out”

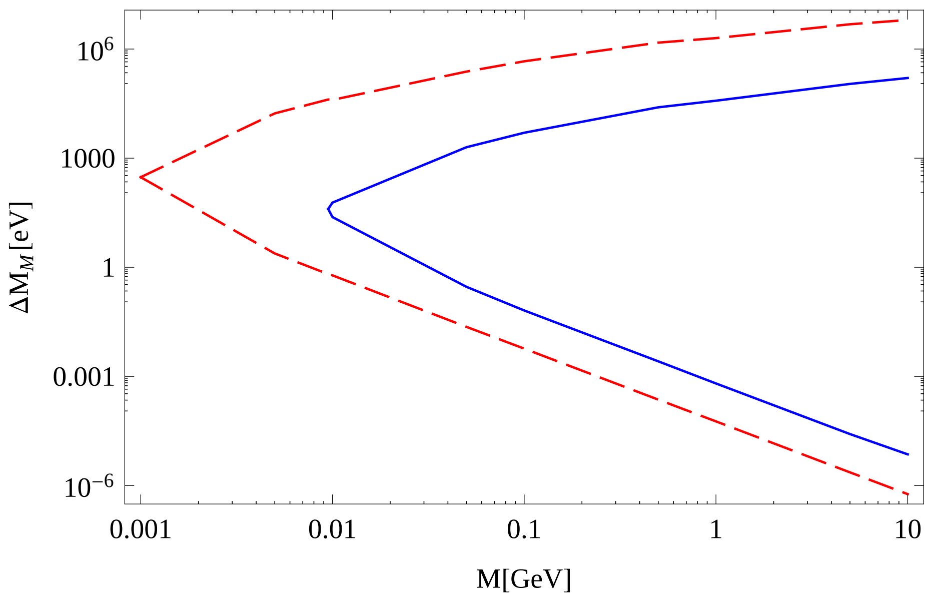
- If $\omega_{\text{osc}} \ll H(T_{\text{sph}})$ – essentially no oscillations had occurred and lepton number in the sterile sector did not have time to develop

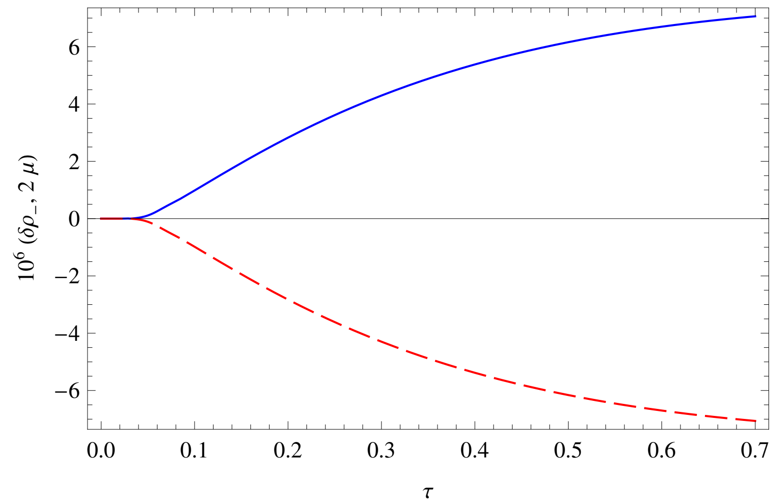
- Optimal condition:

$$\omega_{\text{osc}} \sim \frac{T_{\text{sph}}^2}{M_*} \Longrightarrow \big(M_* M_N \Delta M(T)\big)^{1/3} \sim T_{\text{sph}}$$

Leptogenesis via oscillations

Canetti & Shaposhnikov [1006.0133]





- Mechanism works down to $M_N \sim 1$ MeV
- Roughly $\Delta M(T_{\text{sph}}) \sim m_{\text{atm}} \left(\frac{1\ \text{GeV}}{M_N}\right)$
- This leptogenesis do no have to stop at $T \sim T_{\text{sph}}$. Lepton asymmetry continues to be generated **below sphaleron freeze-out**

Baryon asymmetry in the Universe

- Sterile neutrinos with the masses from 1 MeV to 10^{12} GeV can be responsible for generation of baryon asymmetry of the Universe through leptogenesis

- Heavy particles ($M_N \sim 10^{12}$ GeV) would lead to the gauge hierarchy problem

- Almost degenerate particles with the masses from $M_N \sim M_W$ can produce baryon asymmetry through either **resonant leptogenesis** (Majorana nature of particles plays crucial role) or via **coherent CP-violating oscillations** – total lepton number is non-zero but the active neutrinos acquire an effective lepton number

- The latter mechanism allows for lepton asymmetry to be generated below the sphaleron freeze-out temperature. Therefore, it is possible that $\eta_L \gg \eta_B$ in such models