

Classification of Friedmann Models

RELATIVITY & COSMOLOGY II

The initial state of the Universe remained a problem

- If Universe is filled with cosmological constant its energy density does not change
- If Universe is filled with anything with non-negative pressure: the density decreases as the Universe expands

In the past the Universe was becoming denser and denser \implies ultradense cold state of the initial Universe

High density baryonic matter — a Universe-size neutron star? Neutrons cannot decay anymore $(n \rightarrow p + e + \bar{\nu}_e)$ as there are no available Fermi levels for fermions. The state is stable and remains such until cosmological singularity ($\rho \propto 1/t^n$)

The Universe in the past

The origin of elements (Hydrogen, Helium, metals) remained a challenging problem 1920s-1930s

See e.g. Ultradense ($\rho_n \sim 1 \text{g/cm}^3$) neutron star would mean that no hydrogeneview by is left (as soon as density has dropped to allow neutron decay $n \rightarrow p + e + \bar{\nu}_e$, Zel'dovich, Section 13 each proton is bombarded by many neutrons so that $p + n \rightarrow d + \gamma$, $d + n \rightarrow d$ $t + \gamma$) Zel'dovich in



Wikipedia or

here

Binding energy



Gamow proposes a very modern idea:



See Peebles's account in [1310.2146]

Nucleosynthesis

■ If (as people thought) the density of plasma, needed for nuclear reactions to take place was $\rho \sim 10^7 \, {\rm g/cm^3} \Rightarrow$ very rapid expansion of the Universe (age $\sim 10^{-2} \, {\rm sec}$). Not enough to establish thermal equilibrium?

Paper by Gamow (1946

Paper by Gamow (1948

- Gamow: we need temperature $T_d \sim 10^9$ K (~ 100 keV) to produce deuterium, but the density need not be high. This is a non-equilibrium process!
- At such temperature the density of radiation $(\sigma_{SB}T_d^4)^1$ is much larger than the density of matter that Gamow had estimated as

$$\underbrace{(v \, n \, \sigma_{p+n \to d+\gamma})^{-1}}_{\text{(1)}} \sim t_{\text{d}}$$

time between p-n collisions

 $^{1}\sigma_{SB}$ is the StefanBoltzmann constant

Here the lifetime of the Universe has the is given by Gamow to be:

$$\rho_{\rm rad} = \sigma_{\rm SB} T^4 = \frac{3}{32\pi G_N} \frac{1}{t_d^2} = 8.4 \,\mathrm{g/cm^3} \left(\frac{T}{10^9 \,\mathrm{K}}\right)^4$$

- Using $\sigma \sim 10^{-29} \,\mathrm{cm}^2$ one gets $n_b^{(i)} \sim 10^{18} \,\mathrm{cm}^{-3}$ and therefore $\rho_b^{(i)} \sim 10^{-6} \,\mathrm{cm}^3 \Rightarrow$ the Universe was radiation dominated!
- If so, what is the temperature of radiation bath today? For Gamow (also Alpher, Herman a series of papers, see detailed account in Peebles) the density today was $n_b^{(0)} \sim 10^{-5} \,\mathrm{cm}^{-3}$

• Therefore
$$T^{(0)} \sim T_i \left(\frac{n_b^{(0)}}{n_b^{(i)}}\right)^{1/3} \sim 20 \,\mathrm{K}$$

Relic radiation?!

There should be relic radiation, filling the Universe with $\lambda \sim \frac{2.9 \text{ mm} \cdot \text{K}}{T}$



The ideas of Gamow meant

- The Universe was hot (radiation-dominated at some epoch)
- The density of radiation dropped faster than the density of matter ⇒ matter-radiation equality (at $T \sim 10^3$ K!)
- Recombination of protons and electrons
- The growth of Jeans instabilities did not start until that matterdominated epoch (see below)
- Gamow estimates the size of the instability as

Paper by Gamow (1948

$$k_B T_{\rm rec} \sim \frac{G_N \rho_{\rm matter} R^3}{R}$$

 $\Rightarrow R \sim 1$ kpc similar to a typical galaxy size(!)

In **1950s** this was not so obvious!

- There was no relict radiation from recombination
- You should get about 30% of Helium (which was considered to be wrong, as its abundance was measured $\sim 10\%$)
- In low density hot matter you cannot produce heavy nuclei (A = 5 and A = 8) in this way. With Hubble constant at that time $H_0 \sim 500 \text{km/sec/Mpc}$ the age of the Universe \approx the age of the Earth \Rightarrow heavy elements could not be produced in stars, should be in the Universe "from the very beginning".

It was concluded by many that "Hot Big Bang" is ruled out see e.g. Zel'dovich UFN 1963 Accidentally discovered by Arno Penzias and Robert Wilson:



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1965



Data from **COBE** (1989 – 1996) showed a perfect fit between the black body curve and that observed in the microwave background.

CMB spectrum

Cosmic microwave background radiation is almost perfect blackbody



CMB temperature T = 2.725 K

- Temperature of CMB T = 2.725 K
- CMB contribution to the total energy density of the Universe: $\Omega_{\gamma} \simeq 4.5 imes 10^{-5}$
- Spectrum peaks in the microwave range at a frequency of 160.2 GHz, corresponding to a wavelength of 1.9 mm.
- 410 photons per cubic centimeter
- Almost perfect blackbody spectrum ($\delta T/T < 10^{-4}$)
- COBE has detected anisotropies at the level $\delta T/T \sim 10^{-5}$



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CMB spectrum today





- CMB
- Baryon-to-photon ratio from BBN and CMB (independently)
- Primordial abundance of light elements. Most notably, ⁴He

■ To produce chemical elements one needs to pass through "deuterium bottleneck" $p + n \leftrightarrow D + \gamma$



- ⁴He is the second most abundant element (after hydrogen). Constitutes about 25%.
- At high temperatures chemical equilibrium between protons and neutrons is maintained by weak interactions $n + \nu \Leftrightarrow p + e^-$, $n + e^+ \Leftrightarrow p + \bar{\nu}$, $n \Leftrightarrow p + e^- + \bar{\nu}_e$
- These reactions go out of equilibrium at $T_{\nu} \approx 1 \text{ MeV}$
- The difference of concentrations of n and p at that time is

$$\frac{n_n}{n_p} = \exp\left(-\frac{m_n - m_p}{T_\nu}\right) \approx \frac{1}{6}$$

 $m_n - m_p = 1.2 \text{ MeV}$

• Recall:
$$\frac{n_n}{n_p} = \exp\left(-\frac{m_n - m_p}{T_\nu}\right) \approx \frac{1}{6}$$

Almost all neutrons will end up in ⁴He. The mass abundance of Helium is

$$Y_p \equiv \frac{4n_{He}}{n_n + n_p} = \frac{4(n_n/2)}{n_n + n_p} = \frac{2(n_n/n_p)}{1 + n_n/n_p}$$

• There is a time-delay between freeze-out of weak reaction and time of Helium formation. The unstable neutrons (lifetime $\tau_n \sim 900$ sec decay and therefore by the time of Helium formation $n_n/n_p \approx 1/7$, which gives $Y_p \approx 25\%$

Baryon-to-photon ratio



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- $p + n \leftrightarrow D + \gamma$. Deuterium binding energy $E_D = 2.2$ MeV. We see that $\rho_{rad} \gg \rho_b \Rightarrow$ many photons per each baryon
- To accumulate deuterium we need temperature of photons to drop down to $T \approx 10^9$ K so that number of photons with energy $E \gtrsim E_D$ becomes of the order of n_b :

$$n_b \sim n_\gamma (E > E_D) \propto e^{-E_D/T_\gamma}$$

- The reaction rate $\Gamma_D \approx n_b \langle \sigma v \rangle$ drops below the Hubble expansion rate H(t). The baryon density at that time is found to be $n_b \sim 10^{18} \mathrm{cm}^{-3}$
- The age of the Universe at that time was

$$\frac{1}{2H(t)} \equiv t \sim \left(\frac{3}{32\pi G_N T^4}\right)^{1/2} \Longrightarrow t[\text{sec}] \approx \frac{1}{(T[\text{MeV}])^2}$$

BBN-II

- To estimate the temperature today we need to know a redshift
- One way to know it : measure baryon density today and use the fact that number of baryons is conserved (i.e. comoving density does not change)
- Using baryon density today $n_b^0 \sim 10^{-7} \text{cm}^{-3} \implies z_{bbn} = (n_b/n_b^0)^{1/3} = 2 \times 10^8$.
- Temperature of the photons today: $T_{today} = \frac{T_{bbn}}{z_{bbn}} \approx \frac{10^9}{2 \times 10^8} = 5 \text{ K}$

BBN and particle physics

Nowadays BBN has become a tool to determine properties (bounds) on light particles/decaying particles/evolution of fundamental constants

- The Helium abundance is known with a precision of a few% (e.g. $Y_p = 0.2565 \pm 0.0010 (\text{stat.}) \pm 0.0050 (\text{syst,})$)
- Neutron lifetime provides a "cosmic chronometer", measuring the time between T_{\u03c0} (temperature of freeze-out of weak reactions) and T_d (temperature of deuteron production):

$$\left. \frac{n_n}{n_p} \right|_{T_d} = \frac{n_n}{n_p} \left|_{T_\nu} e^{-t/\tau_n} \right|_{T_\nu}$$

This time depends on the temperature T_d and number of relativistic species at that time

Total energy density at radiation dominated epoch (i.e. the Hubble expansion rate/lifetime of the Universe) depends on the effective number of relativistic degrees of freedom:

$$\frac{3}{8\pi G_N} H^2 = \rho_{\rm rad} = \frac{\pi^2}{30} g_* T^4 \quad {\rm or} \quad$$

where the number of relativistic degrees of freedom is given by

$$g_* = \sum_{\text{boson species}} g_i + \frac{7}{8} \sum_{\text{fermion species}} g_i$$

where relativistic species (having $\langle p \rangle \gtrsim m$) count



The primordial Helium abundance may change if

- There are more than 3 neutrino species (Roughly: one extra neutrino or a particle with similar energy density is allowed at about 2σ level)
- There was any other particle with the mass \ll MeV and lifetime of the order of seconds or more that was contributing to g_* at BBN epoch (1 second – lifetime of the Universe at $T \sim 1$ MeV)
- There were heavy particles with lifetime in the range 0.01 few seconds (that were decaying around BBN epoch)
- Newton's constant (entering Friedmann equation) changed between BBN epoch and later times (e.g. CMB or today)

Cosmology in a couple of words		
•	FRW metric: (Scale factor)	$ds^2 = -dt^2 + a^2(t)d\vec{x}^2$
•	Hubble equation	$\left[\frac{\dot{a}(t)}{a(t)}\right]^2 \equiv H^2(t) = \frac{8\pi}{3M_{\rm Pl}^2}\rho_{\rm tot}(t)$
•	Red-shift:	$1+z=\frac{E_{\rm then}}{E_{\rm now}}\gtrsim 1$
•	Expansion means cooling	Temperature $\sim \frac{1}{a(t)}$
•	Radiation dominated epoch: before recombination, $z \gtrsim 10^3$	$\rho_{\rm rad}(t) \sim \frac{1}{a^4}$
•	Matter dominated epoch: (up until now)	$\rho_{\rm mat}(t) \sim \frac{1}{a^3}$