Relativity & Cosmology-II

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Friedmann equations

$$H(t) \equiv \frac{\dot{a}(t)}{a(t)}$$

$$H^2(t) + \frac{\kappa}{a^2} = \frac{8\pi G}{3}\rho(t) \quad \kappa = 0, +1, -1 \qquad (1)$$

$$\frac{\partial \rho}{\partial t} + 3H(t)(\rho + p) = 0 \qquad (2)$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p) \qquad (3)$$

- **Einstein's Λ-term:** $\rho(t) = -p(t) = \mathrm{const}$
- **Matter:** $\rho(t)a^3(t) = \mathrm{const}$, $p(t) = 0$

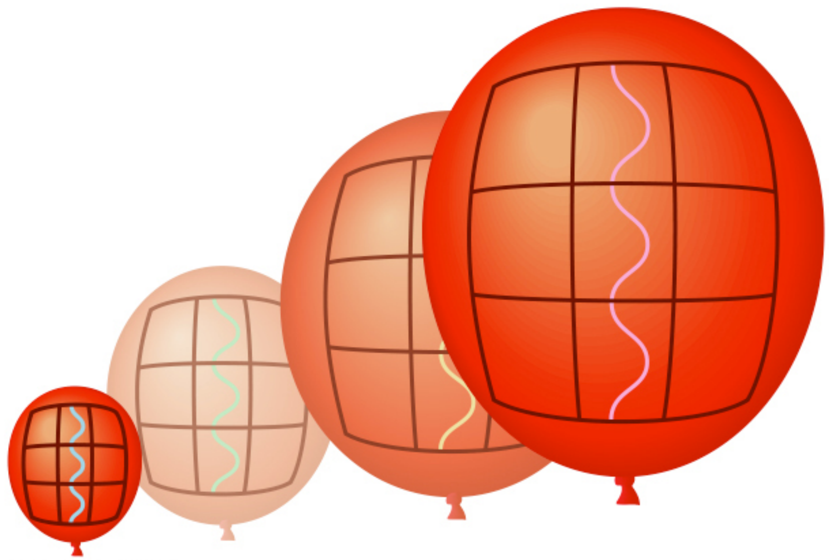
Hubble equation — interplay between **kinetic energy** $E_k = \frac{\dot{a}^2}{2}$ and **potential energy** $E_p = -\frac{GM}{a(t)}$:

$$\frac{\dot{a}^2}{2} - \frac{G\frac{4\pi}{3}\rho(t)a^3(t)}{a(t)} = -\frac{\kappa}{2}$$

Reminder: redshift

Universe stretches:

$$\color{red}{1+z} \equiv \frac{\lambda_{\text{observed}}}{\lambda_{\text{emitted}}} = \frac{a(t_{\text{obs}})}{a(t_{\text{emit}})}$$



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Doppler effect:

a galaxy is receding

$$\color{red}{1+z} \equiv \frac{\lambda_{\text{observed}}}{\lambda_{\text{emitted}}} = \sqrt{\frac{1+v/c}{1-v/c}}$$

where Hubble velocity

$v = H_0 \times$ distance

Luminosity distance

- Consider a source (star/galaxy) at redshift $z > 0$. Let it has intrinsic **luminosity** $\mathcal{L}$ (energy emitted per unit time, measured in $\mathrm{ergs/sec}$)

- In the flat non-expanding "absolute" space we would measure a **flux** (energy received per unit time per unit area, measured in $\mathrm{ergs/sec/cm^2}$):

$$\mathcal{F} = \frac{\mathcal{L}}{4\pi r^2} \qquad \text{non-expanding Universe!}$$

- What changes with expansion?

  1. Photons change their energy: a photon, emitted with energy $E_{\text{emit}} = \hbar\omega$ is detected with the energy $E_{\text{obs}} = \hbar\omega/(1+z)$
  2. Time interval changes as $\dfrac{\delta t_{\text{emit}}}{a(t_{\text{emit}})} = \dfrac{\delta t_{\text{obs}}}{a(t_{\text{obs}})}$
  3. Sphere of the area $4\pi r^2 = 4\pi r^2 a^2(t_{\text{obs}})$

Luminosity distance

- Therefore the flux received from a source at redshift z is given by

$$\mathcal{F} = \frac{\mathcal{L}}{4\pi r^2} \to \mathcal{F} = \frac{\mathcal{L}}{4\pi \underbrace{r^2 a^2(t_{\text{obs}})(1+z)^2}_{\equiv d_L^2}}$$

- and the **luminosity distance** is defined as

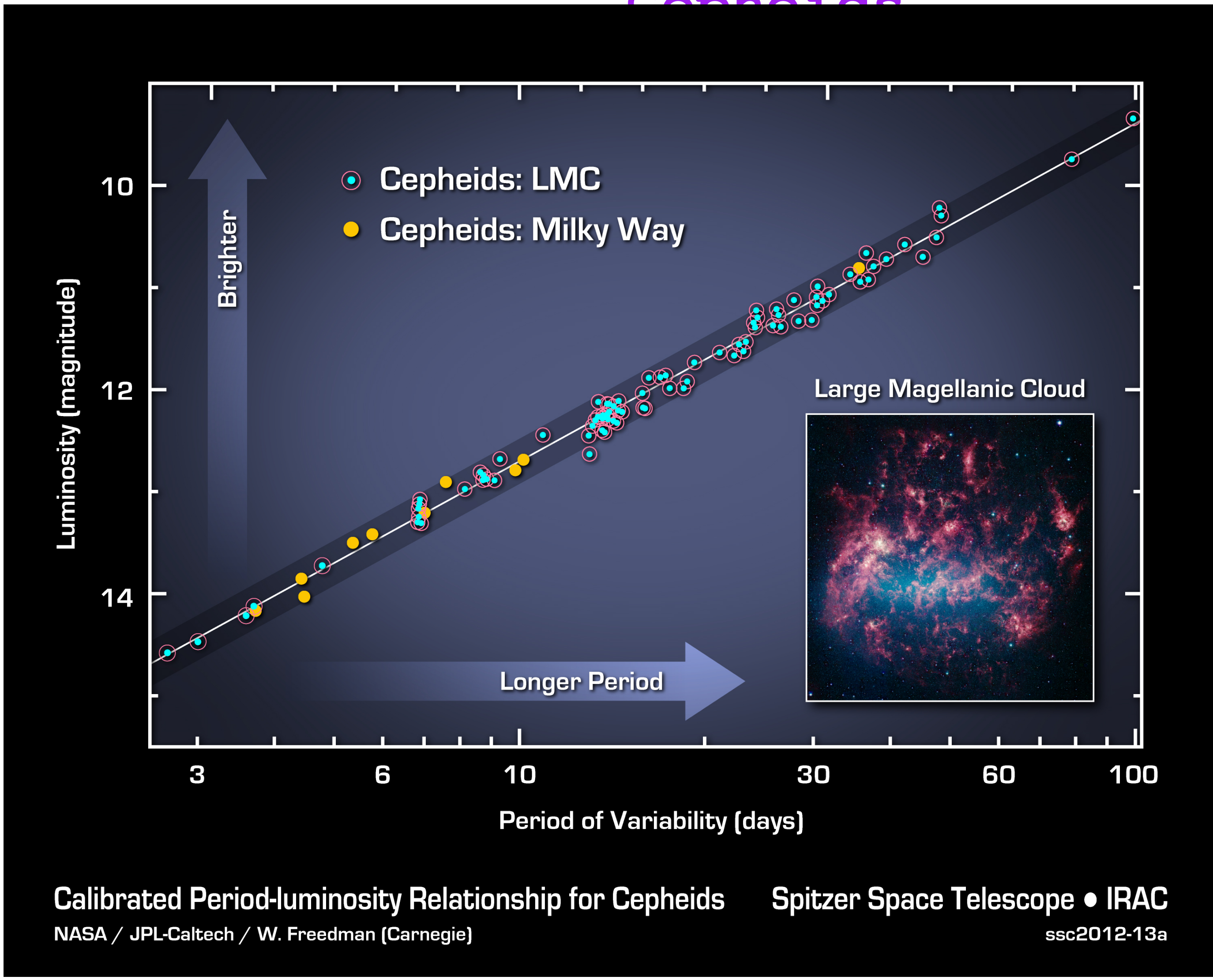
$$d_L = r a(t_{\text{obs}})(1+z) = r\frac{a^2(t_{\text{obs}})}{a(t_{\text{emit}})}$$

- when $z \ll 1$ one has

$$z = H_0 d_L$$

Check!

Cepheids


Cepheids: LMC
Cepheids: Milky Way
Brighter
Luminosity (magnitude)
10
12
14
Large Magellanic Cloud
Longer Period
3
6
10
30
60
100
Period of Variability (days)
Calibrated Period-luminosity Relationship for Cepheids
Spitzer Space Telescope • IRAC
NASA / JPL-Caltech / W. Freedman (Carnegie)
ssc2012-13a

Hubble constant measurement

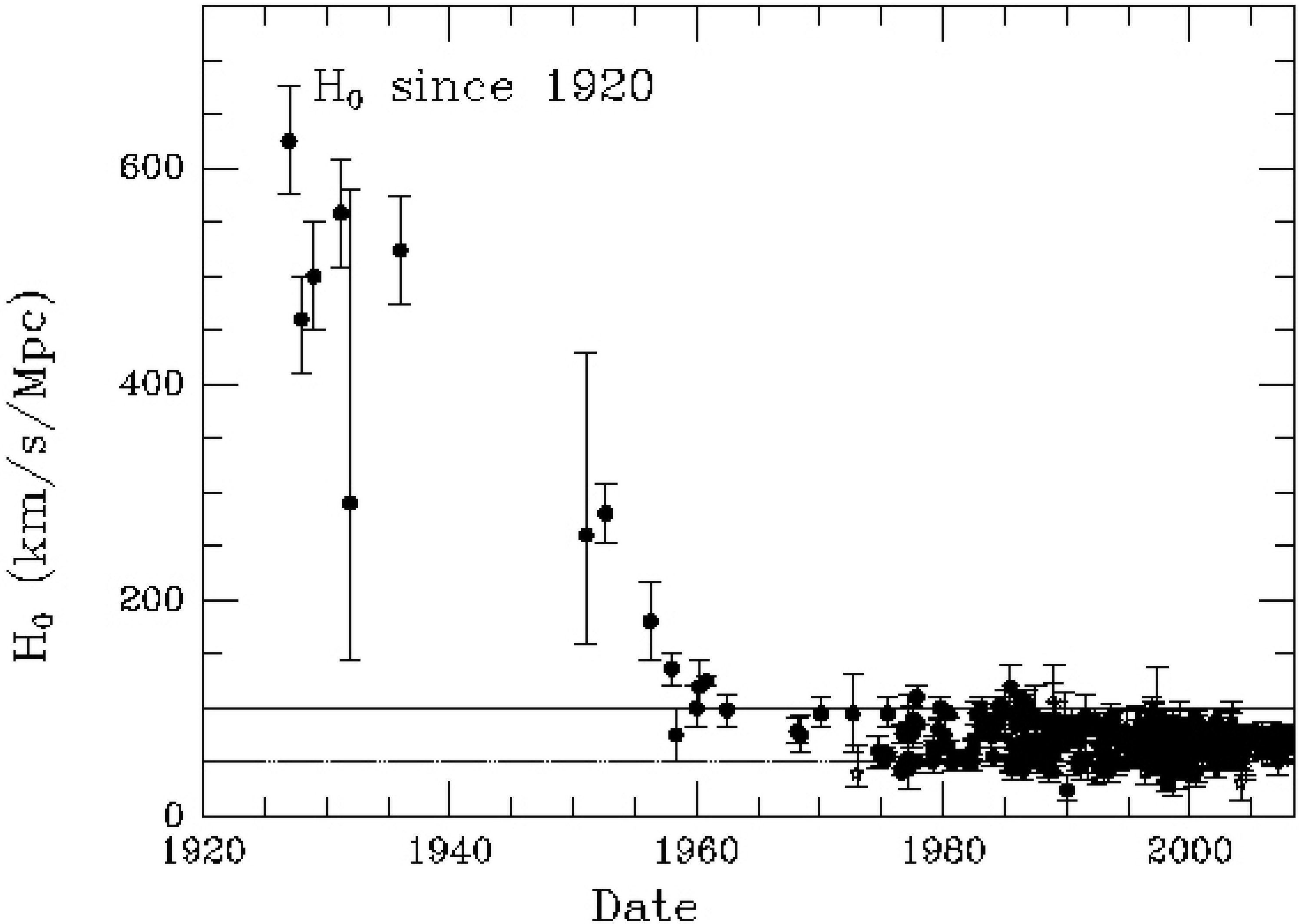
0806.3018

../images/0806_3018_fig1-eps-converted-to.pdf

μ – **distance modulus** – difference between **absolute** and **apparent** magnitude of an object

$$d = 10^{0.2\mu - 5}\,\mathrm{Mpc}$$

Hubble constant history



https://www.cfa.harvard.edu/~dfabricant/huchra/hubble

Surface of zero velocity

0811.4610

Surface of zero velocity

../images/0811_4610_fig1-eps-converted-to.pdf

Content of the Universe

- Friedmann equation (for spatially flat Universe!)

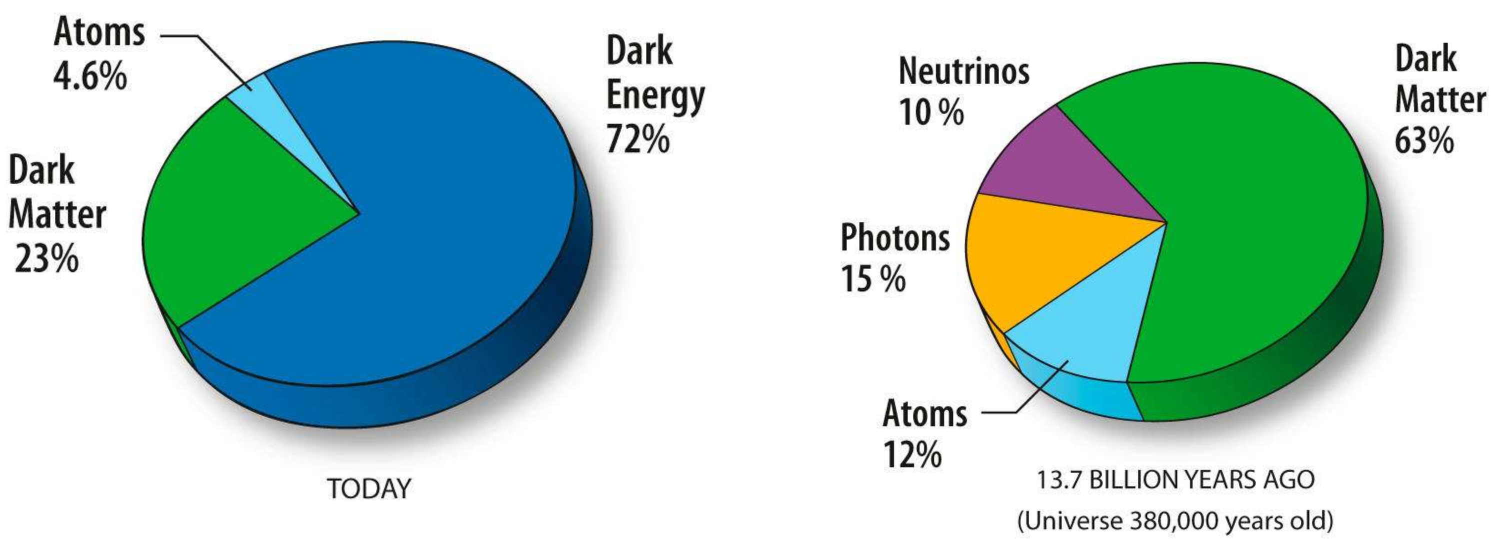
$$H^2(z) = H_0^2\Big(\Omega_\Lambda + \Omega_{\text{mat}}(1+z)^3 + \Omega_{\text{rad}}(1+z)^4\Big)$$

($z_{\text{today}} = 0$, z increases into the past)

- Ω is a fraction of a given substance in the total energy balance of the Universe **today**

- Relative importance of components changes with time (redshift)

WMAP website



Primordial plasma

$$H^2(z) = H_0^2\Big(\Omega_\Lambda + \Omega_{\mathsf{mat}}(1+z)^3 + \Omega_{\mathsf{rad}}(1+z)^4\Big)$$

Primordial plasma

../images/Lambda_matter_radiation-eps-converted-to.pdf

Primordial plasma

Λ-dominated universe and cosmology **L. M. Krauss R. J. Scherrer "The End of Cosmology?" Scientific American (2008)**

Classification of Friedmann models

../images/Solution_Friedmann_equation-eps-converted-to.pdf

Classification of Friedmann models

- Re-write Friedmann equations selecting out Λ term:

$$H^2(t) + \frac{\kappa}{a^2} = \frac{8\pi G}{3}\rho(t) + \frac{\Lambda}{3} \qquad \kappa = 0, +1, -1 \tag{4}$$

$$\frac{\partial\rho}{\partial t} + 3H(t)(\rho + p) = 0 \tag{5}$$

$$\frac{\ddot{a}}{a} = \frac{\Lambda}{3} - \frac{4\pi G}{3}(\rho + 3p) \tag{6}$$