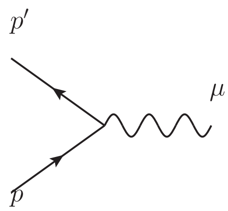
Advanced Quantum Field Theory

Exercise 6

Consider the scalar QED with quartic self-interaction:

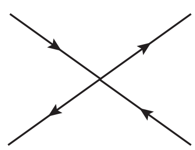
$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + D_\mu\phi^\dagger D^\mu\phi - \frac{\lambda}{4!}(|\phi|^2)^2 \tag{1}$$

where the covariant derivative is defined as $D_\mu\phi = \partial_\mu\phi + ieA_\mu\phi$. The Feynman rules are:



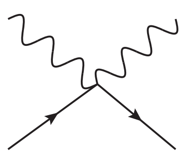
$= -ie(p+p')_\mu$

$= \frac{-i}{k^2}\left(\eta^{\mu\nu} - \frac{k^\mu k^\nu}{k^2}\right)$



$= -i\lambda$

$= \frac{i}{p^2}$



$= 2ie\eta^{\mu\nu}$

Notice the use of the Laundau gauge for the photon propagator, since it enforces cancellations that simplify the calcuations.

Compute the beta functions for both the gauge and the quartic coupling, β_e , β_λ and the anomalous dimension of the scalar field γ_ϕ.