Consider the scalar QED with quartic self-interaction:

\[ \mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + D_\mu \phi \bar{D}^\mu \phi - \frac{\lambda}{4!} (|\phi|^2)^2 \]  

(1)

where the covariant derivative is defined as \( D_\mu \phi = \partial_\mu \phi + ie A_\mu \phi \). The Feynman rules are:

\[ \begin{align*}
\gamma \mu & = -ie(p + p')_\mu \\
\lambda & = -i\lambda \\
\mathcal{G} & = 2ie\eta^{\mu\nu}
\end{align*} \]

Notice the use of the Landau gauge for the photon propagator, since it enforces cancellations that simplify the calculations.

Compute the beta functions for both the gauge and the quartic coupling, \( \beta_e, \beta_\lambda \) and the anomalous dimension of the scalar field \( \gamma_\phi \).