Advanced Quantum Field Theory

Exercise 5

Consider a real massive scalar field with quartic interaction:

$$\mathcal{L} = \frac{1}{2}\partial_\mu \phi \partial^\mu \phi - \frac{1}{2} m^2 \phi^2 - \frac{\lambda}{4!}\phi^4 \tag{1}$$

Add to the Lagrangian the 1 loop counter-terms:

$$\Delta\mathcal{L} = \frac{1}{2}\delta_z \partial_\mu \phi \partial^\mu \phi - \frac{1}{2}\delta_m \phi^2 - \frac{\delta_\lambda}{4!} \tag{2}$$

Compute the 1PI four point function at 2 loops. Schematically it will have the following form:

$$\text{2 loop :} \quad \frac{-i\lambda^3}{(4\pi)^2}\left(\frac{1}{\epsilon}a_2 + \frac{1}{\epsilon^2}b_2 + c_2 \log\frac{s}{\mu^2} + d_2\left(\log\frac{s}{\mu^2}\right)^2\right) + (s \to t) + (s \to u) \tag{3}$$

where $s = (p_1 + p_2)^2$ is the total energy. Recall the result at 1loop:

$$\text{1 loop :} \quad \frac{-i\lambda^2}{(4\pi)^2}\left(\frac{1}{\epsilon}b_1 + d_1 \log\frac{s}{\mu^2}\right) + (s \to t) + (s \to u) \tag{4}$$

Show and justify the following statements:

- The coefficient b_2 is local in the external momenta (depend only polynomially on s), once the contribution of the counterterms to the 2 loop 1PI is taken into account. Why is this important?
- $b_2 = (b_1)^2$
- $d_2 = (d_1)^2$