Quantum Field Theory

Set 27

Exercise 1: Parity violation in polarized Z decay

Consider the term describing the interaction between a massive neutral vector boson Z_μ and a lepton-antilepton pair:

$$\mathcal{L}_{\text{int}} = Z_\mu \bar{l}\gamma^\mu(g_V + g_A\gamma^5)l.$$

Show that if both g_V and g_A are non zero it doesn't exist a parity assignment for the field Z_μ which makes the Lagrangian invariant under parity transformations.
Consider the decay $Z \to e^+e^-$ where the initial Z is polarized in the $\hat{z}$ direction. Call θ the angle between the momentum of the electron and the $\hat{z}$ direction. Show that if the interaction is parity preserving the decay amplitude is invariant under $\theta \to \pi - \theta$.
Define the forward-backward asymmetry:

$$A = \frac{N_+ - N_-}{N_+ - N_-},$$

where N_+ (N_-) is the number of electrons emitted in the upper (lower) half space w.r.t. the $\hat{z}$ direction. This quantity must be zero in a parity invariant theory. Compute A given that:

$$A = \frac{\Gamma_{[0,\pi/2]} - \Gamma_{[\pi/2,\pi]}}{\Gamma_{[0,\pi/2]} + \Gamma_{[\pi/2,\pi]}},$$

with

$$\Gamma_{[a,b]} \equiv \int_a^b \frac{d\Gamma}{d\theta} d\theta.$$

Exercise 2: Compton Scattering in the rest frame of the initial electron

The aim of this exercise is to compute the differential cross section $\frac{d\sigma}{d\cos\theta}$ for the process $e^-\gamma \longrightarrow e^-\gamma$.
We will proceed through the following steps:

- Preliminary: deduce the famous Compton relation between the energies of the initial and final photon in the rest frame of the incoming electron.
- Draw all the Feynman diagrams contributing to the scattering.
- Using the Feynman rules for QED, write the expression for the amplitude $i\mathcal{M} \equiv i\mathcal{M}^{\mu\nu}\epsilon_\nu\epsilon^*_\mu$, where ϵ_ν and ϵ^*_μ are the polarizations of the incoming and outgoing photon.
- Verify the gauge invariance of the matrix element by checking explicitly that $k_\nu\mathcal{M}^{\mu\nu} = k'_\mu\mathcal{M}^{\mu\nu} = 0$, where k^μ (k'^μ) is the momentum of the incoming (outgoing) photon.
- Obtain the unpolarized amplitude squared by summing over all polarizations of the final particles and averaging over all polarizations of the initial particles, by making use of the formulas

  $$\sum_s u^s(p)\bar{u}^s(p) = \not{p} + m, \qquad \sum_{pol} \epsilon_\mu\epsilon^*_\rho \to -\eta_{\mu\rho},$$

  where the symbol '$\to$' means that the replacement holds up to longitudinal terms (vanishing in the amplitude by gauge invariance).

- Simplify the obtained expression using the identities:

$$\begin{aligned}
\gamma^\mu \not{a} \not{b} \not{c} \gamma_\mu &= -2 \not{c} \not{b} \not{a}, \\
\gamma^\mu \not{a} \not{b} \gamma_\mu &= 4\, a \cdot b, \\
\gamma^\mu \not{a} \gamma_\mu &= -2 \not{a}, \\
\gamma^\mu \gamma_\mu &= 4,
\end{aligned}$$

  and reduce the traces to a maximum of 4 Dirac matrices, which you can evaluate using

$$\mathrm{Tr}\left[\gamma^\mu \gamma^\nu \gamma^\rho \gamma^\sigma\right] = 4 \left(\eta^{\mu\nu}\eta^{\rho\sigma} - \eta^{\mu\rho}\eta^{\nu\sigma} + \eta^{\mu\sigma}\eta^{\nu\rho}\right).$$

- Show that the final result is

$$\frac{1}{4}\sum_{pol} |\mathcal{M}^{\mu\nu}\epsilon_\nu \epsilon^*_\mu|^2 = 2e^4 \left[\frac{p\cdot k'}{p\cdot k} + \frac{p\cdot k}{p\cdot k'} + 2m^2\left(\frac{1}{p\cdot k} - \frac{1}{p\cdot k'}\right) + m^4 \left(\frac{1}{p\cdot k} - \frac{1}{p\cdot k'}\right)^2\right],$$

  where we have denoted with p^μ (p'^μ) the 4-momentum of the incoming (outgoing) electron and with k (k') the 4-momentum of the incoming (outgoing) photon.

- Making use of the expression for the 2-body phase space compute the differential cross section $\frac{d\sigma}{d\cos\theta}$.

- Finally study the high energy ($s \gg m^2$) and low energy ($s - m^2 \ll m^2$) limits, where $s \equiv (p+k)^2$. Show that in the latter case the total cross section reduces to the Thompson cross section $\sigma_T = \frac{8\pi}{3}\frac{\alpha^2}{m^2}$ where $\alpha = e^2/(4\pi)$.