Quantum Field Theory

Set 24

Exercise 1: Decay of a massive scalar particle

Consider the Lagrangian

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}m^2\phi^2 + \frac{1}{2}\partial_\mu\Phi\partial^\mu\Phi - \frac{1}{2}M^2\Phi^2 - \frac{\lambda}{2}\phi^2\Phi,$$

with $M > 2m$ and λ a real, positive parameter. Using the Lagrangian above, compute the lifetime τ of the particle of mass M.
To perform the computation, use the relation between $\mathcal{M}_{\Phi\to 2\phi}$, entering the definition of the decay width, and the S-matrix element, and also the explicit expression of the latter in terms of the matrix element of the interaction Hamiltonian between initial and final states. In order to evaluate this, expand the scalar fields in terms of creation and annihilation operators.

Exercise 2: Microcausality

Consider a Klein-Gordon real scalar field ϕ.

- By using the decomposition in of the field ϕ in terms of ladder operators, express the commutator $[\phi(x), \phi(y)]$ in terms of the function $D(x-y)$:

$$D(x) \equiv \int d\Omega_p e^{-ipx}\,, \quad (p_0 \equiv \Omega_p) \tag{1}$$

- Check the Lorentz transformation properties of $D(x)$
- Use Lorentz invariance to show that $[\phi(x), \phi(y)] = 0$ when $x-y$ is space-like.

  This implies immediately that also $[\phi(x), \pi(y)] = 0$ for $x-y$ space-like.

Exercise 3: Scattering in $\lambda\phi^4$

Consider the Lagrangian

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{\lambda}{4!}\phi^4.$$

Using the above Lagrangian and Wick's theorem, compute the matrix element $\mathcal{M}_{2\phi\to 2\phi}$ and the total cross section for the scattering process $\phi\phi \to \phi\phi$.