Quantum Field Theory

Set 17

Exercise 1: Multiple photons states

Consider the wave function for a state consisting of two photons:

- Show that the Bose symmetry of the photon imposes a constraint on the wave function.
- Find the condition on the wave function in order to have a physical state.
- Find the condition on the wave function in order to have a state with positive norm.

Exercise 2: Transformation properties of transverse photons

The polarization of a photon of momentum k_μ is defined by the four vector ε_μ satisfying $\varepsilon_\mu k^\mu = 0$. In the Coulomb gauge we instead use the transverse polarization $\varepsilon^\perp_\mu = (0, \vec{\varepsilon}^\perp)$. Show that the conditions $\varepsilon^\perp_0 = 0$ and $\varepsilon^\perp_i k^i = 0$ are *not* Lorentz invariant (namely, the Lorentz transform of $\varepsilon^\perp_\mu$ has in general $\varepsilon'^\perp_0 \neq 0$ and $\varepsilon'^\perp_i k'^i \neq 0$), but it is still possible to find a vector $\tilde{\varepsilon}^\perp_\mu = \varepsilon'^\perp_\mu + \alpha k'_\mu$, i.e. equal to $\varepsilon'^\perp_\mu$ up to a longitudinal component, which is satisfying the two conditions.