Quantum Field Theory

Set 11

Exercise 1: Coherent states

A state like $|\chi\rangle \equiv a^\dagger(\vec{k}_1)a^\dagger(\vec{k}_2)\dots a^\dagger(\vec{k}_n)|0>$ describes a definite number of particles with spatial momenta $\vec{k}_1, \vec{k}_2, \dots, \vec{k}_n$. Instead, a coherent state in quantum mechanics is an eigenvector of the annihilation operators, $a(\vec{q})|\psi>=\alpha(\vec{q})|\psi>$, where the eigenvalue α is a complex function of the momenta. Obviously $|\chi>$ defined above cannot be a coherent states, since it has a definite number of particles.
Using the normalization for the ladder operators $[a(\vec{q}), a^\dagger(\vec{p})] = \delta^3(\vec{k}-\vec{q})$, solve the following points:

- Find the general form of a coherent state, disregarding for the moment the normalization factor. Hint: start from the ansatz $|\psi>=\sum_{n=0}^{\infty} c_n \left(\int dk z(\vec{k})a^\dagger(\vec{k})\right)^n |0>$ with $z(\vec{k})$ a generic complex functions and the coefficients c_n to be fixed. How can the result be written in a compact way?
  Hint: Use the formula $[A, B^n] = [A, B]B^{n-1} + B[A, B]B^{n-2} + \dots + B^{n-1}[A, B]$
- Put the right normalization factor in front of $|\psi>$, so that $<\psi|\psi>=1$ (Assume that the vacuum is normalized to 1, $\langle 0|0\rangle = 1$).
  Hint: when the commutator of two operators A, B is a c-number (not an operator) then the Baker–Campbell–Hausdorff formula reduces to:

$$e^A e^B = e^{A+B} e^{\frac{1}{2}[A,B]} \tag{1}$$

- Compute the expectation value $<\psi|\phi(x)|\psi>$ of the Klein-Gordon field $\phi(x) = \int \frac{d^3k}{(2\pi)^{3/2}\sqrt{2\omega_{\vec{k}}}}\left[a(\vec{k})e^{-ikx} + a^\dagger(\vec{k})e^{ikx}\right]$. Is the result different from the case of a state with definite particle number?