Advanced Quantum Field Theory

Exercise 10

Consider the following Lagrangian

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + i\bar{\psi}\,\not{D}\psi - (H\bar{\psi}_L\psi_R + h.c.) + \partial_\mu H^\dagger\partial^\mu H - V(|H|) \tag{1}$$

where as usual $D_\mu\psi = \partial_\mu\psi - ieA_\mu\psi$ and $\psi_{L(R)} = \frac{1\mp\gamma_5}{2}\psi$. The theory has a $U(1)$ gauge symmetry. Show that the theory is invariant under an additional global chiral symmetry under which both ψ and H transform (recall that under a chiral symmetry the left and right components of ψ transform with opposite charge; in other words, a Dirac spinor transforms with $e^{i\gamma^5\alpha}$).

Compute the Noether current J^5_μ associated to the above symmetry.

Consider the theory in the unbroken phase, $< H >= 0$, and show that this symmetry is anomalous computing the correlator $< J^5_\mu A_\nu A_\rho >$ (you can use the standard formula for the anomalies).

Consider now the theory in the broken phase: $< H >= v/\sqrt{2}$ and $H = \frac{v+\rho(x)}{\sqrt{2}}e^{i\pi(x)/v}$. In this phase the chiral symmetry is spontaneously broken, the fermion ψ and the longitudinal component of H, ρ, acquire a mass of order v. The only massless degrees of freedom are the Goldstone boson π associated to the broken symmetry and the photon. Consider the effective Lagrangian describing these fields at energies $E \ll v$. Notice that, being π a Goldstone boson, we expect the Lagrangian to be invariant under the shift symmetry $\pi \to \pi + c$. Hence the most general renormalizable Lagrangian compatible with this symmetry is

$$\mathcal{L}_{eff} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}\partial_\mu\pi\partial^\mu\pi \tag{2}$$

Compute the Noether current associated to the shift symmetry.

The chiral symmetry of the original Lagrangian is however anomalous: this anomaly must be traced in the low energy description of the theory and must translate in an anomaly for the shift symmetry. In order to show that this is the case compute the one loop correlation function $< \pi A_\mu A_\nu >$ in the full theory. Add to the effective Lagrangian an operator (it has dimension 5) that matches the result.

Perform the variation of the effective Lagrangian under the shift symmetry and show that the Noether current has an anomaly equal to the one computed in the full theory.