Quantum Field Theory

Set 1

Exercise 1: System of natural units

Using the *system of natural units* ($\hbar = c = \mu_0 = 4\pi\epsilon_0 = 1$), express the dimensionality of the following quantities in powers of electronVolts (eV):

- Mass (M)
- Length (L)
- Time (T)
- Velocity (v)
- Force (F)
- Electric charge (Q)
- Electric Field (E)
- Magnetic Field (B)

Exercise 2: System of natural units

Work out the exact relations between two different systems of units: cgs vs. system of natural units:

- 1 cm $\to$ # $(\text{GeV})^a$
- 1 sec $\to$ # $(\text{GeV})^b$
- 1 statCoulomb $\to$ # $(\text{GeV})^c$

(Hint: recall the useful relations $\hbar = \frac{h}{2\pi} \simeq 1.05 \times 10^{-34}$ J$\cdot$s $= 1.05 \times 10^{-27}$ erg$\cdot$s $= 0.66 \times 10^{-15}$ eV$\cdot$s, $c \simeq 3 \times 10^{10}$ cm/s, $\hbar\, c \simeq 197$ MeV $\cdot$ fm $= 1.97 \cdot 10^{-5}$ eV $\cdot$ cm. Also recall that 1 statCoulomb is the unit of charge in cgs, also called esu, such that $1\, statC \,=\, 1\,\sqrt{erg \cdot cm}$.)

Exercise 3: Kinematics of two body decay

Consider a particle of mass M and energy $\tilde{E}$ in the laboratory frame moving along the $\hat{z}$ direction. Compute its velocity $\vec{v} \equiv \vec{p}/\tilde{E}$. What is the energy of the particle in the center of mass frame? The particle decays into two new particles of mass m_1 and m_2. Compute the energies and the momenta of the two particles in the center of mass frame.
Call $\tilde{\theta}$ the angle, measured in the laboratory frame, between the $\hat{z}$ direction and the momentum of one of the particles produced in the decay. Compute the energies of the particles in the laboratory frame as a function of $\tilde{\theta}$.
Suppose $m_1 = m_2$: compute the maximum and minimum energies the particles can have in the laboratory frame.
Suppose now $m_1 = m_2 = 0$: compute the minimum angle between the momenta of the particles in the laboratory frame.

Exercise 4: Kinematics of $2 \to 2$ scattering

Consider a particle with mass m_a and energy E_a that strikes a particle at rest with mass m_b and produces two particles with mass m_c and m_d. Show that, as in the previous exercise, energies and momenta of the final particles in the center of mass frame are completely fixed once masses and total energy are known. Compute them.
Suppose that the LHC, instead of being a collider with two proton beams circulating in opposite directions with an energy of 7 TeV each, were a collider with fixed target and an incoming beam with an energy E_a. What should E_a be in order to have the same energy in the center of mass frame?
Assuming that all the masses are known, show that the number of independent degrees of freedom needed to describe the process of scattering is 2. What is a convenient choice for these parameters?

Exercise 5

The Large Hadron Collider (LHC) started its operations on 10th September 2008, completing the first entire revolution of the 27 km long ring at 10.28 a.m. The proton beam coming from the SPS has been injected inside the LHC with an energy of 450 GeV per proton. Compute the velocity of the beam in units of c.
At full performance, the nominal luminosity of the machine will be such that collisions occur every 25 ns. In order to achieve this result the LHC ring will be filled with two beams traveling in opposite directions containing approximately 2800 bunches of 10^{11} protons each. These will travel at a velocity of $0.999999991 \cdot c$. Compute the energy of a single proton in GeV and the total energy of the beam in Joules. Show that this energy corresponds to the kinetic energy of a running TGV (400 tons) and compute its velocity in km/h. Finally calculate the total current circulating in the LHC ring in Amperes.

Exercise 6

Consider the process $\gamma + e^- \to \gamma + e^- + e^+ + e^-$. In the frame where the electron is at rest (laboratory frame) compute the minimum photon energy E_γ necessary for this process to happen.

Exercise 7

Estimate the numerical value of the following quantities:

- Fine structure constant α
- Bohr radius r_B
- Rydberg constant R
- Compton length λ_e
- Electron classical radius r_e

(Hints: express everything in terms of known quantities such as $e = 1.602 \cdot 10^{-19}$ C, $m_e = 0.5$ MeV$/c^2$, $1\,\mathrm{eV} = 1.602 \cdot 10^{-19}$ J, $1\,\mathrm{C} = 2.998 \cdot 10^9$ esu(statC))

Additional exercises

No correction will be provided for these exercises.

Decays

1. Consider a particle of mass M and energy E in the laboratory frame which moves along the $\hat{z}$ direction. The particle decays into two new particles of mass m_1 and m_2. Find the relation between the energy of the particles produced and the angle of emission measured with respect to the $\hat{z}$ direction in the center of mass frame.

2. Consider a particle of unknown mass which moves along the $\hat{z}$ direction and decays into two bodies. We can measure the masses m_1 and m_2, energies E_1 and E_2 in the laboratory frame and the angle θ_1 that the first particle form with the $\hat{z}$ direction. Find the mass M of the initial particle.

3. Consider a particle of mass M at rest which decays into three particles $m_1 = m_2 = m_3$. What is the maximum energy that each particle can have? What kinematic configuration does it correspond to?

Scatterings

1. Consider a light wave (a photon, $m_1 = 0$) with energy $E_1 = h\nu_1$ striking an electron with mass $m_2 = m_e$ at rest. Suppose the final products of the scattering are still a photon and an electron (Compton Scattering). Find the relation between the energy/frequency of the outcoming photon and the angle of emission with respect to the $\hat{z}$ direction.

2. Suppose that two particles of equal mass $m_a = m_b = m$ annihilate and produce two photons $m_c = m_d = 0$. In the laboratory frame one of the incoming particles has energy E_a while the other is at rest. Assuming that one of the photons is emitted at 90 degrees with respect to the direction of motion of the particle a, compute the energy and the direction of the other photon (in the laboratory frame).

3. Consider the process $a + b \to a + b + X$ where $m_a = m_b = m$ and $m_X = M$. In the rest frame where the particle b is at rest compute the minimum energy E_a of particle a necessary for the process to happen.