Advanced Quantum Field Theory

Exercise 1

Consider a real massive scalar field with quartic interaction:

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}m^2\phi^2 - \frac{\lambda}{4!}\phi^4 \tag{1}$$

Extract the Feynman rules and write the 1PI 2 and 4 point function at 1 loop as integrals in momentum space. Perform a Wick rotation in the 0-th component of the integrated momentum

$$p^0 \longrightarrow ip^0_E\,, \qquad \vec{p} \longrightarrow \vec{p}_E \tag{2}$$

and express the integral in terms of the *euclidean* four-momuntum p^μ_E. Regularize the divergent integrals using dimensional regularization:

$$\int \frac{d^4p_E}{(2\pi)^4} \longrightarrow \int \frac{d^dp_E}{(2\pi)^d}\,, \qquad d = 4 - 2\epsilon \tag{3}$$

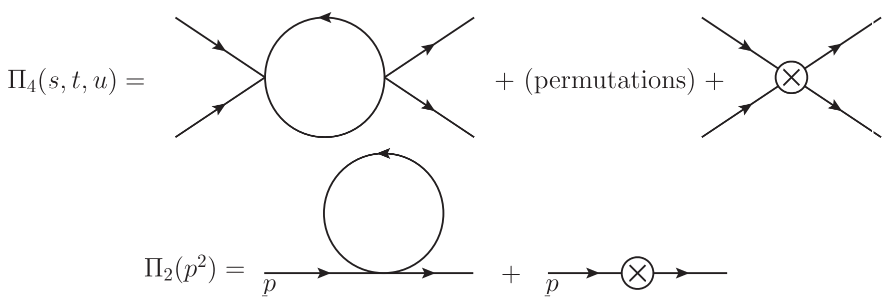
Notice that the expression for the four point function was dimensionless in 4 dimensions. Hence, introduce a scale μ (arbitrary at this stage) and multiply the integral by $\mu^{\#}$, where # is a suitable exponent that has to be chosen in order to compensate the change in dimension of the integral.

Compute the integrals and expand in powers of ϵ up to the 0-th order.

Add to the Lagrangian the following list of counter-terms:

$$\Delta\mathcal{L} = \frac{1}{2}\delta_z\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}\delta_m\phi^2 - \frac{\delta_\lambda}{4!}\phi^4 \tag{4}$$

Add to the 1PI 2 and 4 point functions computed with the original Lagrangian the tree level contributions coming from the above counter-terms:



In order to verify that all the divergences of the theory can be canceled with a suitable choice of the parameters $\delta_z\,, \delta_m\,, \delta_\lambda$ impose the *renormalization condition*:

$$\Pi_2(p^2 = m^2) = 0 \tag{5}$$
$$\Pi'_2(p^2 = m^2) = 0 \tag{6}$$
$$\Pi_4(s = 4m^2, t = u = 0) = 0 \tag{7}$$
$$\tag{8}$$

and determine $\delta_z\,, \delta_m$ and δ_λ.