Consider the dimensionally regulated action

\[ S[\phi_0, \{\lambda_0/\Lambda^{d_I}\}] \]  

where the bare couplings are

\[ \lambda_{0I} \equiv \lambda_{0I}(\mu, \{\lambda(\mu)\}) = \mu^{-\kappa_I \epsilon} \left( \lambda_I(\mu) + \frac{1}{\epsilon} P_I^{(1)}(\{\lambda(\mu)\}) + \ldots \right) \]

with \( P_I^{(i)}(\lambda) \) polynomial series in the couplings. Here \( d_I \) and \( \kappa_I \) numerical coefficients that depend on the structure of the operator. For instance for an interaction of the type

\[ \lambda_{N,M} \Lambda^N - M^{-4} \partial^M \phi^N \]

we have \( d_{N,M} = N + M - 4 \) and \( \kappa_{N,M} = 1 + N/2 \). The independence of the bare couplings under RG transformations amounts to

\[ \lambda_{0I}(\mu, \{\lambda(\mu)\}) = \lambda_{0I}(\eta \mu, \{\lambda(\eta \mu)\}) \]

or equivalently

\[ \lambda_{0I}(\mu/\eta, \{\lambda(\mu)\}) = \lambda_{0I}(\mu, \{\lambda(\eta \mu)\}) \]

The renormalized and bare fields are related in minimal subtraction by

\[ \phi(x, \{\lambda(\mu)\}) = Z(\{\lambda(\mu)\}) \phi_0 \quad Z(\{\lambda(\mu)\}) = 1 + \frac{1}{\epsilon} Z^{(1)}(\{\lambda(\mu)\}) + \ldots \]

Since \( Z \) is dimensionless and since finite powers of \( \mu \) are not generated by loops, no dependence on \( \Lambda \) can arise in \( Z \) (this means that \( Z \) is only affected by dimension 4 interactions and by some products of relevant and irrelevant couplings, for instance \( m^2 \lambda_6/\Lambda^2 \equiv \lambda_2 \lambda_6 \)). We want to consider the scaling of correlators

\[ G(\{x\}, \mu, \{\lambda_I(\mu)/\Lambda^{d_I}\}) = \int D\phi \phi(x_1, \{\lambda(\mu)\}) \ldots \phi(x_n, \{\lambda(\mu)\}) e^{iS_0[\phi_0, \{\lambda_{0I}/\Lambda^{d_I}\}]} \]

Defining the scale tranformed field by

\[ \phi_0(x) = \eta^\Delta \bar{\phi}_0(\eta x) \]

with \( \Delta = (d - 2)/2 = 1 - \epsilon/2 \) we have, by inspection,

\[ S[\phi_0, \mu, \{\lambda_I(\mu)/\Lambda^{d_I}\}] = S[\bar{\phi}_0, \mu/\eta, \{\lambda_I(\mu)/(\Lambda/\eta)^{d_I}\}] \]
where we made explicit the dependence on \( \mu \) and on the renormalized couplings. Making the substitution 0.8 in 0.7, using the invariance of the action 0.9 and of the integration measure \( D\phi = D\tilde{\phi} \) we find

\[
G(\{x\}, \mu, \{\lambda_I(\mu)/\Lambda^{d_I}\}) = \eta^{\Delta n} G(\{\eta x\}, \mu/\eta, \{\lambda_I(\mu)/(\Lambda/\eta)^{d_I}\}) 
\tag{0.10}
\]

Now, defining

\[
\frac{\phi(x, \{\lambda(\mu)\})}{\phi(x, \{\lambda(\eta \mu)\})} = \frac{Z(\{\lambda(\mu)\})}{Z(\{\lambda(\eta \mu)\})} \equiv R(\mu, \eta \mu) 
\tag{0.11}
\]

and using the RG invariance of the bare couplings we can also write

\[
G(\{\eta x\}, \mu/\eta, \{\lambda_I(\mu)/(\Lambda/\eta)^{d_I}\}) = R^n G(\{\eta x\}, \mu, \{\lambda_I(\eta \mu)/(\Lambda/\eta)^{d_I}\}) 
\tag{0.12}
\]

which, together with 0.10 implies (as we are dealing with finite quantities \( \Delta = 1 \) can be taken)

\[
G(\{x/\eta\}, \mu, \{\lambda_I(\mu)\lambda^{d_I}\}) = [\eta R(\mu, \eta \mu)]^n G(\{x\}, \mu, \{\lambda_I(\eta \mu)/(\Lambda/\eta)^{d_I}\}) 
\tag{0.13}
\]

which implies that, apart from the field dimension factor \( (\eta R)^n \), the scaling with \( \eta \) is controlled by the running of the dimensionless couplings \( \bar{\lambda}_I \)

\[
\{\lambda_I(\eta \mu)/(\Lambda/\eta)^{d_I}\} = \bar{\lambda}_I(\mu \eta)/\mu^{d_I} 
\tag{0.14}
\]

Notice that

\[
\ln R = \int_\mu^{\eta \mu} \gamma(\{\lambda(\mu')\}) d\ln \mu' = \int_{\lambda_I(\mu)}^{\lambda_I(\eta \mu)} \frac{\gamma(\{\lambda\})}{\beta_4(\{\lambda\})} d\lambda_4. 
\tag{0.15}
\]

Thus if a fixed point is reached \( \gamma \equiv \gamma_* = \text{const} \) we have

\[
(\eta R)^n = \eta^{(1 + \gamma_*)n} 
\tag{0.16}
\]