In[169]:=
```
MatrixForm[U = 1 / 2 {{1, 1, 1, 1}, {1, I, -1, -I}, {1, -1, 1, -1}, {1, -I, -1, I}}]
```

Out[169]//MatrixForm=

$$\begin{pmatrix} \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{i}{2} & -\frac{1}{2} & -\frac{i}{2} \\ \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} & -\frac{1}{2} \\ \frac{1}{2} & -\frac{i}{2} & -\frac{1}{2} & \frac{i}{2} \end{pmatrix}$$

In[124]:=
```
f[x_, y_] := Conjugate[x] / Sqrt[Abs[x]^2 + Abs[y]^2]
g[x_, y_] := Conjugate[y] / Sqrt[Abs[x]^2 + Abs[y]^2]
```

In[126]:=
```
a = f[1 / 2, 1 / 2]
```

Out[126]=
$$\frac{1}{\sqrt{2}}$$

In[127]:=
```
U1 = {{a, a, 0, 0}, {a, -a, 0, 0}, {0, 0, 1, 0}, {0, 0, 0, 1}}
```

Out[127]=
$$\left\{\left\{\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0, 0\right\}, \left\{\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}, 0, 0\right\}, \{0, 0, 1, 0\}, \{0, 0, 0, 1\}\right\}$$

In[128]:=
```
MatrixForm[U1p = U1.U]
```

Out[128]//MatrixForm=

$$\begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{\frac{1}{2}+\frac{i}{2}}{\sqrt{2}} & 0 & \frac{\frac{1}{2}-\frac{i}{2}}{\sqrt{2}} \\ 0 & \frac{\frac{1}{2}-\frac{i}{2}}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{\frac{1}{2}+\frac{i}{2}}{\sqrt{2}} \\ \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} & -\frac{1}{2} \\ \frac{1}{2} & -\frac{i}{2} & -\frac{1}{2} & \frac{i}{2} \end{pmatrix}$$

In[129]:=
```
a = f[1 / Sqrt[2], 1 / 2]
```

Out[129]=
$$\sqrt{\frac{2}{3}}$$

In[130]:=
```
b = f[1 / 2, 1 / Sqrt[2]]
```

Out[130]=
$$\frac{1}{\sqrt{3}}$$

In[131]:=
```
MatrixForm[U2 = {{a, 0, b, 0}, {0, 1, 0, 0}, {b, 0, -a, 0}, {0, 0, 0, 1}}]
```

Out[131]//MatrixForm=

$$\begin{pmatrix} \sqrt{\frac{2}{3}} & 0 & \frac{1}{\sqrt{3}} & 0 \\ 0 & 1 & 0 & 0 \\ \frac{1}{\sqrt{3}} & 0 & -\sqrt{\frac{2}{3}} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

In[132]:=
```
MatrixForm[U2p = U2.U1p]
```

Out[132]//MatrixForm=

$$\begin{pmatrix} \frac{\sqrt{3}}{2} & \frac{i}{2\sqrt{3}} & \frac{1}{2\sqrt{3}} & -\frac{i}{2\sqrt{3}} \\ 0 & \frac{\frac{1}{2}-\frac{i}{2}}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{\frac{1}{2}+\frac{i}{2}}{\sqrt{2}} \\ 0 & \frac{\frac{3}{2}+\frac{i}{2}}{\sqrt{6}} & -\frac{1}{\sqrt{6}} & \frac{\frac{3}{2}-\frac{i}{2}}{\sqrt{6}} \\ \frac{1}{2} & -\frac{i}{2} & -\frac{1}{2} & \frac{i}{2} \end{pmatrix}$$

In[133]:=
```
a = f[Sqrt[3] / 2, 1 / 2]
```

Out[133]=
$$\frac{\sqrt{3}}{2}$$

In[134]:=

In[135]:=
```
b = f[1 / 2, Sqrt[3] / 2]
```

Out[135]=
$$\frac{1}{2}$$

In[136]:=
```
MatrixForm[U3 = {{a, 0, 0, b}, {0, 1, 0, 0}, {0, 0, 1, 0}, {b, 0, 0, -a}}]
```

Out[136]//MatrixForm=

$$\begin{pmatrix} \frac{\sqrt{3}}{2} & 0 & 0 & \frac{1}{2} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \frac{1}{2} & 0 & 0 & -\frac{\sqrt{3}}{2} \end{pmatrix}$$

In[137]:=

In[138]:=
```
MatrixForm[U3p = U3.U2p]
```

Out[138]//MatrixForm=

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{\frac{1}{2}-\frac{i}{2}}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{\frac{1}{2}+\frac{i}{2}}{\sqrt{2}} \\ 0 & \frac{\frac{3}{2}+\frac{i}{2}}{\sqrt{6}} & -\frac{1}{\sqrt{6}} & \frac{\frac{3}{2}-\frac{i}{2}}{\sqrt{6}} \\ 0 & \frac{i}{4\sqrt{3}}+\frac{i\sqrt{3}}{4} & \frac{1}{4\sqrt{3}}+\frac{\sqrt{3}}{4} & -\frac{i}{4\sqrt{3}}-\frac{i\sqrt{3}}{4} \end{pmatrix}$$

In[139]:=
```
a = f[(1 - I) / (2 Sqrt[2]), (3 + I) / (2 Sqrt[6])]
```

Out[139]=
$$\left(\frac{1}{4}+\frac{i}{4}\right)\sqrt{3}$$

In[140]:=
```
b = g[(1 - I) / (2 Sqrt[2]), (3 + I) / (2 Sqrt[6])]
```

Out[140]=
$$\frac{3}{4}-\frac{i}{4}$$

In[141]:=
```
MatrixForm[Simplify[
  U4 = {{1, 0, 0, 0}, {0, a, b, 0}, {0, Conjugate[b], -Conjugate[a], 0}, {0, 0, 0, 1}}]]
```

Out[141]//MatrixForm=

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \left(\frac{1}{4}+\frac{i}{4}\right)\sqrt{3} & \frac{3}{4}-\frac{i}{4} & 0 \\ 0 & \frac{3}{4}+\frac{i}{4} & \left(-\frac{1}{4}+\frac{i}{4}\right)\sqrt{3} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

In[142]:=
```
MatrixForm[Simplify[U4p = U4.U3p]]
```

Out[142]//MatrixForm=

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \sqrt{\frac{2}{3}} & \frac{i}{\sqrt{6}} & \frac{1}{\sqrt{6}} \\ 0 & 0 & \frac{1}{\sqrt{2}} & \frac{i}{\sqrt{2}} \\ 0 & \frac{i}{\sqrt{3}} & \frac{1}{\sqrt{3}} & -\frac{i}{\sqrt{3}} \end{pmatrix}$$

In[143]:=
```
a = Simplify[f[Sqrt[3/2]/4 + 5/(4 Sqrt[6]), i/(4 Sqrt[3]) + i Sqrt[3]/4]]
```

Out[143]=
$$\sqrt{\frac{2}{3}}$$

In[144]:=
```
b = Simplify[g[Sqrt[3/2]/4 + 5/(4 Sqrt[6]), i/(4 Sqrt[3]) + i Sqrt[3]/4]]
```

Out[144]=
$$-\frac{i}{\sqrt{3}}$$

In[146]:=
```
MatrixForm[
 U5 = {{1, 0, 0, 0}, {0, a, 0, b}, {0, 0, 1, 0}, {0, Conjugate[b], 0, -Conjugate[a]}}]
```

Out[146]//MatrixForm=

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \sqrt{\frac{2}{3}} & 0 & -\frac{i}{\sqrt{3}} \\ 0 & 0 & 1 & 0 \\ 0 & \frac{i}{\sqrt{3}} & 0 & -\sqrt{\frac{2}{3}} \end{pmatrix}$$

In[147]:=
```
MatrixForm[Simplify[U5p = U5.U4p]]
```

Out[147]//MatrixForm=

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \frac{1}{\sqrt{2}} & \frac{i}{\sqrt{2}} \\ 0 & 0 & -\frac{1}{\sqrt{2}} & \frac{i}{\sqrt{2}} \end{pmatrix}$$

In[148]:=
```
a = f[1/Sqrt[2], -1/Sqrt[2]]
```

Out[148]=
$$\frac{1}{\sqrt{2}}$$

In[149]:=
```
b = g[1/Sqrt[2], -1/Sqrt[2]]
```

Out[149]=
$$-\frac{1}{\sqrt{2}}$$

In[151]:=
```
MatrixForm[U6 = {{1, 0, 0, 0}, {0, 1, 0, 0}, {0, 0, a, b}, {0, 0, b, -a}}]
```

Out[151]//MatrixForm=

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ 0 & 0 & -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix}$$

In[153]:=
```
MatrixForm[Simplify[U6p = U6.U5p]]
```

Out[153]//MatrixForm=

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -i \end{pmatrix}$$

In[154]:=
```
MatrixForm[U7 = {{1, 0, 0, 0}, {0, 1, 0, 0}, {0, 0, 1, 0}, {0, 0, 0, I}}]
```

Out[154]//MatrixForm=

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & i \end{pmatrix}$$

In[157]:=
```
MatrixForm[Simplify[U7p = U7.U6p]]
```

Out[157]//MatrixForm=

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

In[160]:=
```
MatrixForm[U1^†]
```

Out[160]//MatrixForm=

$$\begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 & 0 \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

In[161]:=
```
MatrixForm[U2^†]
```

Out[161]//MatrixForm=

$$\begin{pmatrix} \sqrt{\frac{2}{3}} & 0 & \frac{1}{\sqrt{3}} & 0 \\ 0 & 1 & 0 & 0 \\ \frac{1}{\sqrt{3}} & 0 & -\sqrt{\frac{2}{3}} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

In[164]:=
```
MatrixForm[U3^†]
```

Out[164]//MatrixForm=

$$\begin{pmatrix} \frac{\sqrt{3}}{2} & 0 & 0 & \frac{1}{2} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \frac{1}{2} & 0 & 0 & -\frac{\sqrt{3}}{2} \end{pmatrix}$$

In[165]:= **MatrixForm[U4†]**

Out[165]//MatrixForm=

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \left(\frac{1}{4} - \frac{i}{4}\right)\sqrt{3} & \frac{3}{4} - \frac{i}{4} & 0 \\ 0 & \frac{3}{4} + \frac{i}{4} & \left(-\frac{1}{4} - \frac{i}{4}\right)\sqrt{3} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

In[166]:= **MatrixForm[U5†]**

Out[166]//MatrixForm=

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \sqrt{\frac{2}{3}} & 0 & -\frac{i}{\sqrt{3}} \\ 0 & 0 & 1 & 0 \\ 0 & \frac{i}{\sqrt{3}} & 0 & -\sqrt{\frac{2}{3}} \end{pmatrix}$$

In[167]:= **MatrixForm[U6†]**

Out[167]//MatrixForm=

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ 0 & 0 & -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix}$$

In[168]:= **MatrixForm[U7†]**

Out[168]//MatrixForm=

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -i \end{pmatrix}$$

In[170]:= **MatrixForm[Simplify[U - U1†.U2†.U3†.U4†.U5†.U6†.U7†]]**

Out[170]//MatrixForm=

$$\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$