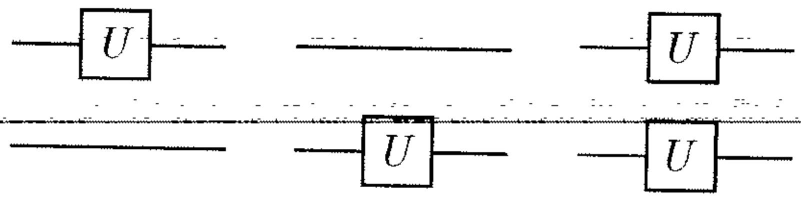
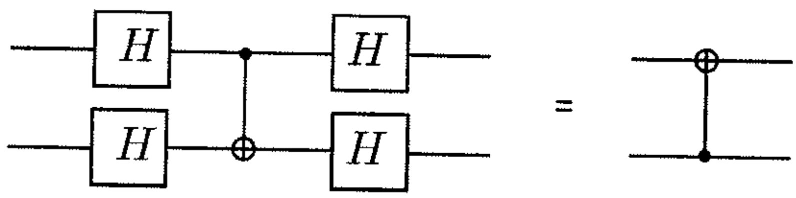
Traitement quantique de l'information I (automne 2009)
Série 15: Circuits quantiques

Exercice 1 Ecrire les matrices des opérateurs unitaires correspondants aux trois circuits dans la figure. Ici U est un opérateur unitaire arbitraire agissant sur un qu-bit. Utiliser la base computationnelle $\{|00\rangle, |01\rangle, |10\rangle, |11\rangle\}$.

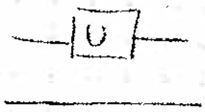


Exercice 2 Démontrer l'égalité entre les deux circuits quantiques illustrés dans la figure. Ici H est la porte logique de Hadamard.

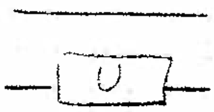


Série 15

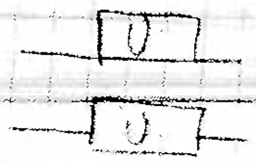
1.



$$= U \otimes I = \begin{pmatrix} U_{11} I & U_{12} I \\ U_{21} I & U_{22} I \end{pmatrix}$$

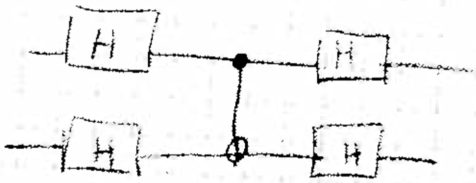


$$= I \otimes U = \begin{pmatrix} U & 0 \\ 0 & U \end{pmatrix}$$

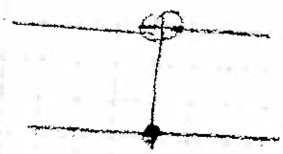


$$= U \otimes U = \begin{pmatrix} U_{11} U & U_{12} U \\ U_{21} U & U_{22} U \end{pmatrix}$$

2.



$\overset{?}{=}$



$$H^{(1)} H^{(2)} \; C \; H^{(1)} H^{(2)}$$

$$\underbrace{\frac{1}{2} \begin{pmatrix} H & +H \\ H & -H \end{pmatrix} \begin{pmatrix} I & 0 \\ 0 & X \end{pmatrix}} \begin{pmatrix} H & H \\ H & -H \end{pmatrix}$$

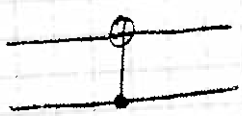
$$\frac{1}{2} \begin{pmatrix} H & HX \\ H & -HX \end{pmatrix} \begin{pmatrix} H & H \\ H & -H \end{pmatrix} = \frac{1}{2} \begin{pmatrix} H^2 + HXH & H^2 - HXH \\ H^2 - HXH & H^2 + HXH \end{pmatrix}$$

$$H^2 = \frac{1}{2}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = I$$

$$HXH = \frac{1}{2}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

$$= \frac{1}{2}\begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = Z$$

$$\text{Gate} = \begin{pmatrix} \frac{1}{2}(I+Z) & \frac{1}{2}(I-Z) \\ \frac{1}{2}(I-Z) & \frac{1}{2}(I+Z) \end{pmatrix} = \begin{array}{c} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} \\ \begin{array}{cccc} e_1 & e_4 & e_3 & e_2 \end{array} \end{array}$$



$|00\rangle \to |0\ 0\rangle = e_1$

$|01\rangle \to |1\ 1\rangle = e_4$ OK.

$|10\rangle \to |1\ 0\rangle = e_3$

$|11\rangle \to |0\ 1\rangle = e_2$