Natural and un-natural mass hierarchies
Particle mass versus size

- Classical computation

\[ \Delta m \sim \frac{e^2}{4\pi} \frac{1}{R} \sim \frac{e^2}{4\pi} \Lambda \]

- Quantum result

\[
\begin{align*}
\text{scalar} & \quad m^2 = m_0^2 + \frac{3e^2}{16\pi^2} \Lambda^2 + O(e^4) \\
\text{fermion} & \quad m = m_0 \left( 1 + \frac{3e^2}{8\pi^2} \ln \frac{\Lambda}{m_0} + O(e^4) \right)
\end{align*}
\]

For a fermion only a mild logarithmic divergence remains !!

concrete example:
\[
\frac{e^2}{16\pi^2} \ln \frac{M_{\text{Planck}}}{m_{\text{electron}}} \sim 0.37 = O(1)
\]

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Cancellation is due to virtual positron contribution to mass

\[ \Delta m_e = + \frac{e^2}{4\pi} \Lambda - \frac{e^2}{4\pi} \Lambda = 0 \]

This result is more directly understood in terms of symmetries
Naturally small masses ↔ Symmetry

1) Fermion:  $$\mathcal{L}_{\text{electron}} = i \bar{e}_L \gamma^\mu D_\mu e_L + i \bar{e}_R \gamma^\mu D_\mu e_R + m_e \bar{e}_L e_R + m_e \bar{e}_R e_L$$

Limit:  $$m_e = 0$$ respects chiral symmetry

$$\lim_{m_e \to 0} \mathcal{L}_{\text{electron}}$$

$$e_L \to e_L$$
$$e_R \to e^{i\theta} e_R$$

$$\delta m_e \propto \frac{\alpha}{4\pi} m_e$$
$$\propto \frac{\alpha}{4\pi} \Lambda$$

2) Vector gauge symmetry:  $$A_\mu \to A_\mu + \partial_\mu \alpha$$

Mass term:  $$m_\gamma^2 A_\mu A^\mu$$ not invariant

$$m_\gamma = 0$$

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3) interacting massless scalar

\[ \mathcal{L}_\varphi = \partial_\mu \varphi \partial^\mu \varphi + \lambda \varphi^4 \]

\[ \int d^4 x \, \mathcal{L}_\varphi \] is classically invariant under dilatations \( \varphi(x) \rightarrow k \varphi(kx) \)

however the very existence of any UV scale explicitly breaks dilatations

\[ \delta m^2_\varphi \sim \frac{\lambda}{16\pi^2} \Lambda^2 \]
3a) Nambu-Goldstone boson \( \varphi \rightarrow \varphi + c \)

\[
\mathcal{L} = \mathcal{L}(\partial \varphi) = (\partial_\mu \varphi)^2 + \frac{1}{\Lambda^4} (\partial_\mu \varphi)^2 (\partial_\nu \varphi)^2 + \ldots
\]

\( E \ll \Lambda \) the scalar becomes a free particle

The Higgs looks only mildly like a NG boson!

\[
\mathcal{L}_{\text{top}} = \lambda_t \bar{Q}_L H t_R + h.c.
\]

\[
\delta m_H^2 \sim \frac{3 \lambda_t^2}{8\pi^2} \Lambda^2
\]

OK as long as \( \Lambda \lesssim 1 \text{ TeV} \)

still interesting to build models at weak scale (see Pomarol)
No *ordinary* symmetry can protect the mass of an interacting scalar particle
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...we must speculate
Try to make Higgs scalar naturally light by following positron example: add new particles

Ex: top quark contribution

\[ \delta \mu^2 = \frac{-3\lambda_t^2}{8\pi^2} \Lambda^2 \]  
fermion

\[ + \frac{3\lambda_{\tilde{t}}}{8\pi^2} \Lambda^2 \]  
bose

Fermion and boson loops cancel each other for

\[ \lambda_t^2 = \lambda_{\tilde{t}} \]
needs a symmetry relating bosons to fermions

Does such a symmetry exist?
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Does such a symmetry exist?

YES!
\[ \lambda_i^2 = \lambda_i \]

needs a symmetry relating bosons to fermions

Does such a symmetry exist?

YES!

SuperSymmetry

Volkov, Akulov 1973
Wess, Zumino 1974
A more `mature’ and general viewpoint on mass hierarchies
(based on Renormalization Group)
\[ \Lambda_{UV} \sim \text{scale invariant dynamics} \]

\[ \Lambda_{IR} \sim \text{conformal invariance} \]
stability of $\Lambda_{IR} \ll \Lambda_{UV}$ characterized by dimensionality of perturbations at fixed point

\[ \Delta \mathcal{L} = \lambda \mathcal{O} \]

\[ \lambda(E) = \lambda(\Lambda_{UV}) \left( \frac{E}{\Lambda_{UV}} \right)^{d_{\mathcal{O}} - 4} \]

- $d_{\mathcal{O}} - 4 > 0$: irrelevant
- $d_{\mathcal{O}} - 4 = 0$: marginal
- $d_{\mathcal{O}} - 4 < 0$: relevant

Ex. scalar mass $\lambda(E) = \left( \frac{m}{E} \right)^2$
Natural Hierarchy
A. There exists no strongly relevant operator

most relevant \( 4 - d_O = \epsilon \ll 1 \)

\[ \lambda(E') = \lambda_0 \left( \frac{\Lambda_{UV}}{E} \right)^\epsilon \]

\[ \Lambda_{IR} \leftrightarrow \lambda(\Lambda_{IR}) \sim 1 \]

\[ \Lambda_{IR} \sim \Lambda_{UV} \lambda_0^{1/\epsilon} \]

exponential hierarchy
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\[ \Lambda_{IR} \leftrightarrow \lambda(\Lambda_{IR}) \sim 1 \rightarrow \Lambda_{IR} \sim \Lambda_{UV} \lambda_0^{1/\epsilon} \]

B. Strongly relevant operators exist, but can be controlled by a symmetry

- **quark mass in QCD** \[ d_\mathcal{O} = 3 \]
  - controlled by chiral symmetry

- **scalar masses in MSSM** \[ d_\mathcal{O} = 2 \]
  - SUSY + chiral symm
Natural Hierarchy

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Ex.

\[ \star \text{scalar masses in MSSM} \quad d_\mathcal{O} = 2 \quad \text{SUSY + chiral symm} \]

The Standard Model belongs to neither category