Lecture # 13

Main points of # 12

Chaotic inflation: dynamics of scalar field.

Today: gravitational instabilities.

Elements of structure formation

Jeans instabilities

Let us understand first the following simple case

- Universe is static
- filled by liquide with density ρ, pressure p and velocity v

Let us neglect all general relativity effects and consider gravitational potential φ

- Let initially $\rho_0 = \text{Const}$, $p_0 = \text{Const}$, $v = 0$, $\vec{g} = 0$

What is the behaviour of small perturbations?

Equations:

Newtonian gravity,

$$\nabla^2 \varphi = + 4\pi G \rho$$ (a'la ED equation for scalar potential)

Equation of continuity:

$$\frac{\partial \rho}{\partial t} + \vec{\nabla}\underbrace{(\rho \vec{v})}_{\text{current}} = 0$$

(analogue of electric current conservation in ED)

Also, Euler equations:

$$\frac{\partial \vec{v}}{\partial t} + (\vec{v}\cdot\vec{\nabla})\vec{v} = -\frac{1}{\rho}\vec{\nabla}p - \vec{\nabla}\varphi$$

How to derive Euler equation?

$$\underset{\text{acceleration}}{\vec{g}} = -\vec{\nabla}\varphi$$

 ← small element of liquid

total force acting on element:

$$\vec{F} = \underbrace{-\oint p\, d\vec{S}}_{\text{pressure}} - \underbrace{\int dV\cdot\rho\cdot\vec{\nabla}\varphi}_{\text{gravity force}}$$

$$-\oint p\, d\vec{S} = -\int \vec{\nabla}p\cdot dV \Rightarrow$$

$$\vec{F} = -\int(\vec{\nabla}p + \rho\vec{\nabla}\varphi)\, dV$$

2nd equation of Newton:

$$\rho \frac{d\vec{v}}{dt} = -\vec{\nabla} p - \rho \vec{\nabla} \varphi$$

$$\vec{v} = \vec{v}(x, y, z, t) \Rightarrow$$

$$\frac{d\vec{v}}{dt} = \frac{\partial \vec{v}}{\partial t} + \frac{\partial \vec{v}}{\partial x} \cdot \frac{\partial x}{\partial t} + \frac{\partial \vec{v}}{\partial y}\frac{\partial y}{\partial t} + \frac{\partial \vec{v}}{\partial z}\frac{\partial z}{\partial t} =$$

$$= \frac{\partial \vec{v}}{\partial t} + (\vec{v} \cdot \vec{\nabla})\vec{v}$$

Final equation:

$$\boxed{\frac{\partial \vec{v}}{\partial t} + (\vec{v} \cdot \vec{\nabla}) \cdot \vec{v} + \frac{1}{\rho}\vec{\nabla} p + \vec{\nabla}\varphi = 0}$$

Euler, 1755

One of the main equations of hydrodynamics.

Let us consider small perturbations:

$$\delta p, \ \delta \varphi, \ \delta \vec{v}, \ \delta \rho$$

$$\begin{cases} \nabla^2 \delta\varphi = 4\pi G \delta\rho \\ \frac{\partial \delta\rho}{\partial t} + \vec{\nabla}(\rho_0 \delta\vec{v}) = 0 \\ \frac{\partial \delta\vec{v}}{\partial t} + \frac{1}{\rho_0}\vec{\nabla}\delta p + \vec{\nabla}\delta\varphi = 0 \end{cases}$$

From general physics (Lectures of Meitter)

acustical waves:

$$v_s^2 = \frac{\delta p}{\delta \rho} \quad \text{(entropy is constant)}$$

$\uparrow$ speed of sound $\Rightarrow$ $\qquad \delta p = v_s^2 \delta\rho$

$$\frac{\partial \delta \vec{v}}{\partial t} + v_s^2 \frac{1}{\rho_0} \vec{\nabla} \delta\rho + \vec{\nabla} \delta\varphi = 0 \quad \checkmark$$

$$\frac{\partial \delta\rho}{\partial t} + \rho_0 \vec{\nabla} \delta \vec{v} = 0 \quad \checkmark \quad \text{(ok)}$$

$$\Downarrow$$

$$\frac{\partial^2 \delta\rho}{\partial t^2} + \rho_0 \vec{\nabla} \left(\frac{\partial \delta v}{\partial t} \right) = 0 =$$

$$= \frac{\partial^2 \delta\rho}{\partial t^2} + \rho_0 \vec{\nabla} \left(- v_s^2 \frac{1}{\rho_0} \vec{\nabla} \delta\rho - \vec{\nabla} \delta\varphi \right) =$$

$$= \frac{\partial^2 \delta\rho}{\partial t^2} - v_s^2 \nabla^2 \delta\rho - \rho_0 \cdot 4\pi G \delta\rho = 0$$

Take $\rho \sim e^{ikx - i\omega t} \Rightarrow$

$$-\omega^2 + v_s^2 K^2 - 4\pi G \rho_0 = 0 \Rightarrow$$

$$\omega^2 = v_s^2 K^2 - 4\pi G \rho_0$$

for $K \gtrsim \frac{\sqrt{4\pi G\rho_0}}{v_s} \equiv K_J \quad \omega^2 > 0$ – stability

$K \lesssim \frac{\sqrt{4\pi G\rho_0}}{v_s}$ – instability:

perturbations increase in amplitude

The reason of structure formation!

What happens in expanding universe when relativistic effects are taken into account? (Qualitative analysis only).

– we can neglect expansion if

$$\omega \gg H \sim \sqrt{G\rho}$$

└ Hubble „constant"

– So, instability cannot appear if

$$\underline{\underline{K \gg K_J}} \text{ even in expanding Universe.}$$

$K_J \Rightarrow$ Jeans lenght,

$$\lambda_J = \frac{2\pi}{K_J}$$

Jeans mass: mass inside a sphere with radius λ_J:

$$M_J = \frac{4}{3}\pi \cdot \left(\frac{2\pi v_s}{\sqrt{4\pi G\rho_0}}\right)^3 \cdot \rho_0$$

$$= \frac{4}{3}\pi\rho_0 \left[\frac{\pi v_s^2}{G\rho_0}\right]^{3/2}$$

rad. dom.: $\rho \sim T^4$

$M_J \sim \frac{M_{pl}^3}{T^2}$

$t \sim \frac{M_{pl}}{T^2} \Rightarrow$

$M_J \sim M_{pl} \cdot t$

Radiation-dominated universe:

$$v_s^2 = \frac{1}{3} \qquad \left(\text{from } p = \frac{\rho}{3}\right) \Rightarrow$$

$$M_J \sim \frac{4}{3}\pi \frac{M_{pl}^3}{\sqrt{\rho_0}} \simeq \frac{M_{pl}^3}{T^2} \sim 10^{17} M_\odot$$

at recombination.

typical galaxy: $10^{11} M_\odot$

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develop of instability: $\not\propto t$

$$K \cdot H \sim \sqrt{\rho_0 G} * H^{-1} \sim \frac{T^2}{M_{pl}} \frac{M_{pl}}{T^2}$$

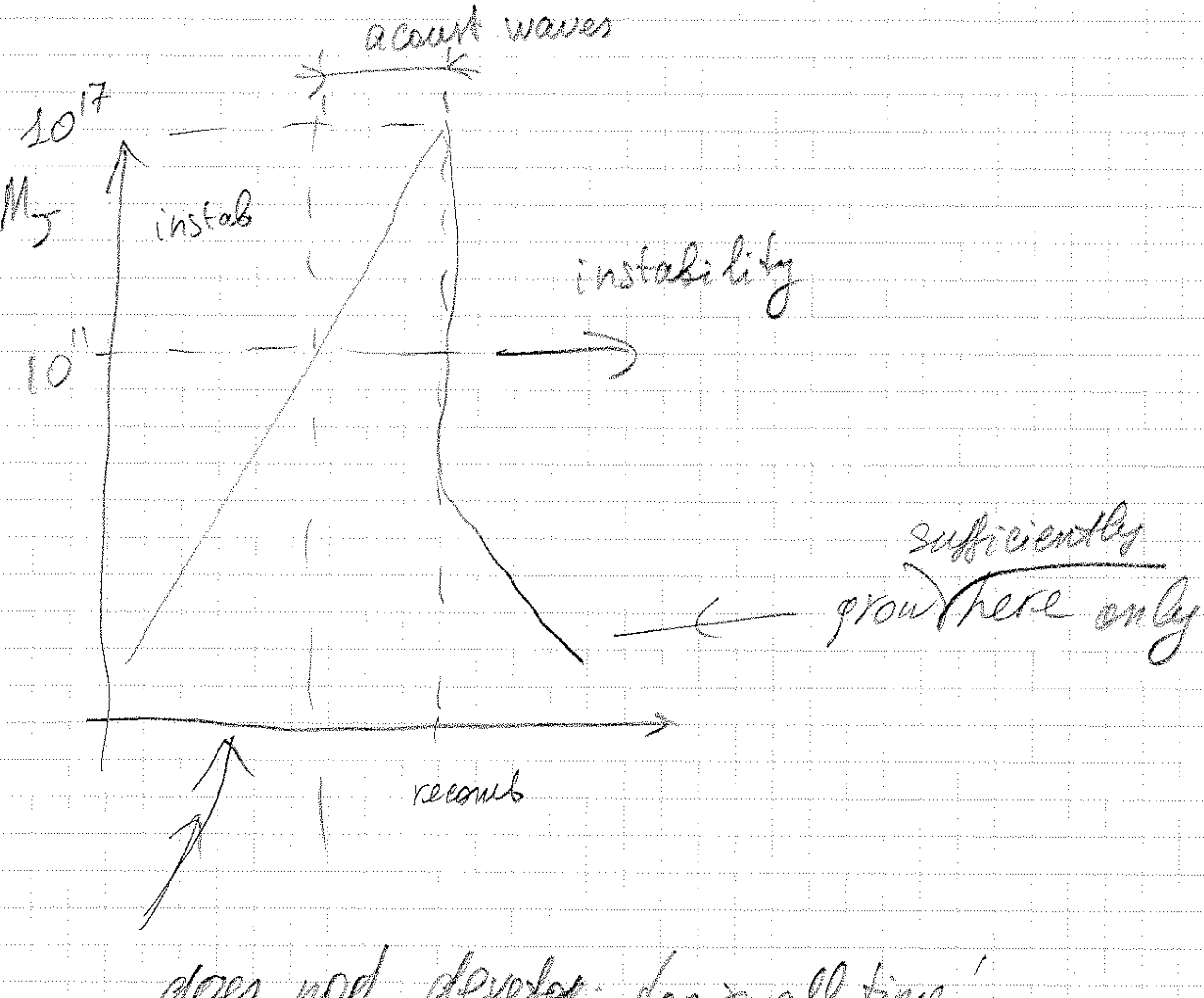
no amplification

Recombination:

$$v_s^2 \to \frac{5}{3}\frac{T}{m_p} \simeq \frac{5}{3}\,\frac{0{,}25\,eV}{10^9\,eV} = 4\cdot 10^{-10} \Rightarrow$$

$$v_s \simeq 2\cdot 10^{-5}\ ;\ v_s/v_{rel} = 2\sqrt{3}\cdot 10^{-5} \approx 3.4\cdot 10^{-5}$$

$$M_J \to \underline{10^{17}} * 3\cdot 10^{-14} \approx 3\cdot 10^{3}\, M_\odot$$



does not develop: too small time!

Problem: from where one can get initial fluctuations — inflation!

So: The early universe history:

beginning: quantum gravity – nobody knows.

– inflation

– radiation dominated universe,

– baryogenesis:

$T \simeq 100 - 10^{15}\,\mathrm{GeV}$

– nucleosynthesis

$T \simeq 1\,\mathrm{MeV}$

– matter dominated universe

$z \simeq 12000$

– recombination and decoupling

$z \simeq 1100, \quad 0.25\,\mathrm{eV}$

– accelerated universe expansion

$\Omega_M \simeq 0.3, \; \Omega_\Lambda \simeq 0.7$