Extras : statistical analysis of CMB
for lecture 12

CMB anisotropies

Main idea of the analysis:

Consider

$$\frac{\Delta T(\theta,\varphi)}{T} = \sum_{lm} Y_{lm}(\theta,\varphi)\, a_{lm}$$

Y_{lm}: spherical harmonic,

$$L^2\, Y_{lm} = l(l+1)\, Y_{lm}$$

$$L_3\, Y_{lm} = m\, Y_{lm}$$

$\theta \in 0,\pi$

$\varphi \in 0, 2\pi$

$$\int_0^{2\pi}\int_0^{\pi} Y^*_{l'm'}\, Y_{lm}\, d\cos\theta\, d\varphi = \delta_{ll'}\,\delta_{mm'}$$

$$\left\langle \frac{\Delta T(\theta,\varphi)}{T}\,\frac{\Delta T(\theta',\varphi')}{T}\right\rangle =$$

$$\sum_{ll'}\sum_{mm'} Y_{lm}(\theta,\varphi)\, Y_{l'm'}(\theta'\varphi')\langle a_{lm}\, a^*_{l'm'}\rangle =$$

spherical symmetry:

$$\langle a_{lm}\, a^*_{l'm'}\rangle = C_l\,\delta_{ll'}\,\delta_{mm'} \Rightarrow$$

$$\overset{*}{=} \sum_{ll'}\sum_{mm'} Y_{lm}(\theta,\varphi)\, Y_{l'm'}(\theta'\varphi')$$

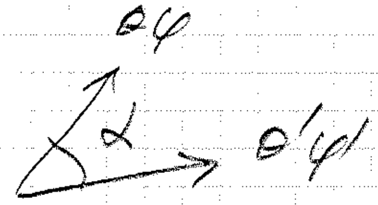
$$\cdot\, C_l\,\delta_{ll'}\,\delta_{mm'}$$

$$= \sum_\ell \sum_m Y_{\ell m}(\theta\varphi)\, Y^*_{\ell m}(\theta'\varphi')\, C_\ell$$

Q. from mathematics:

$$\sum_m Y_{\ell m}(\theta,\varphi)\, Y^*_{\ell m}(\theta',\varphi') = \frac{2\ell+1}{4\pi} \underbrace{P_\ell(\overset{\cos}{\alpha})}_{\text{Legendre polinomials}} \Rightarrow$$

α:



$a=-1,\ b=1,\ \omega=1$

$P_n(x) = \frac{1}{2^n}\frac{1}{n!} D^n[(x^2-1)^n]$

$\underline{0 \le \alpha \le \pi}$

$$\left\langle \frac{\Delta T}{T}\frac{\Delta T}{T} \right\rangle = \sum_\ell \frac{2\ell+1}{4\pi} P_\ell(\overset{\cos\alpha}{\alpha})\, C_\ell$$

$P_0 = 1$

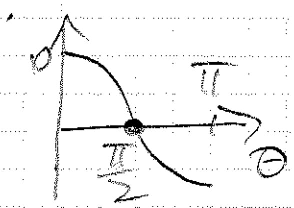
$P_1 = x$

$P_2 = \frac{3}{2}x^2 - \frac{1}{2}$

So: Find avarage of $\left\langle \frac{\Delta T}{T}\frac{\Delta T}{T} \right\rangle$ over all the sky as a function of α.

orthogonality condition:

$$\int_{-1}^{1} P_\ell(x)\, P_{\ell'}(x)\, dx = \delta_{\ell\ell'} \frac{2}{2\ell+1},$$ i.e.



$$C_\ell = \int d\cos\alpha \left\langle \frac{\Delta T}{T}\frac{\Delta T}{T} \right\rangle P_\ell(\alpha) \cdot 2\pi$$

estimate of relation between angle and ℓ:

$$P_\ell \sim (\cos\alpha)^\ell \sim \cos \ell\alpha \Rightarrow \underline{\ell\alpha \simeq \frac{\pi}{2}} \Rightarrow$$

angle - ℓ relation:

$$\theta \sim \frac{\pi}{2\ell}, \quad \text{in degrees: } \theta^\circ \sim \frac{100}{\ell}$$

large ℓ: small resolution, small distance scale

small ℓ: large angle, large distance scale.

$\ell \sim 100$: $\theta \sim 1^\circ$

MAP: launched in June 2001, $\ell \sim 1000$

Planck (European) will be launched in 2007, will go up to $\ell \sim 2000$

Power spectrum:

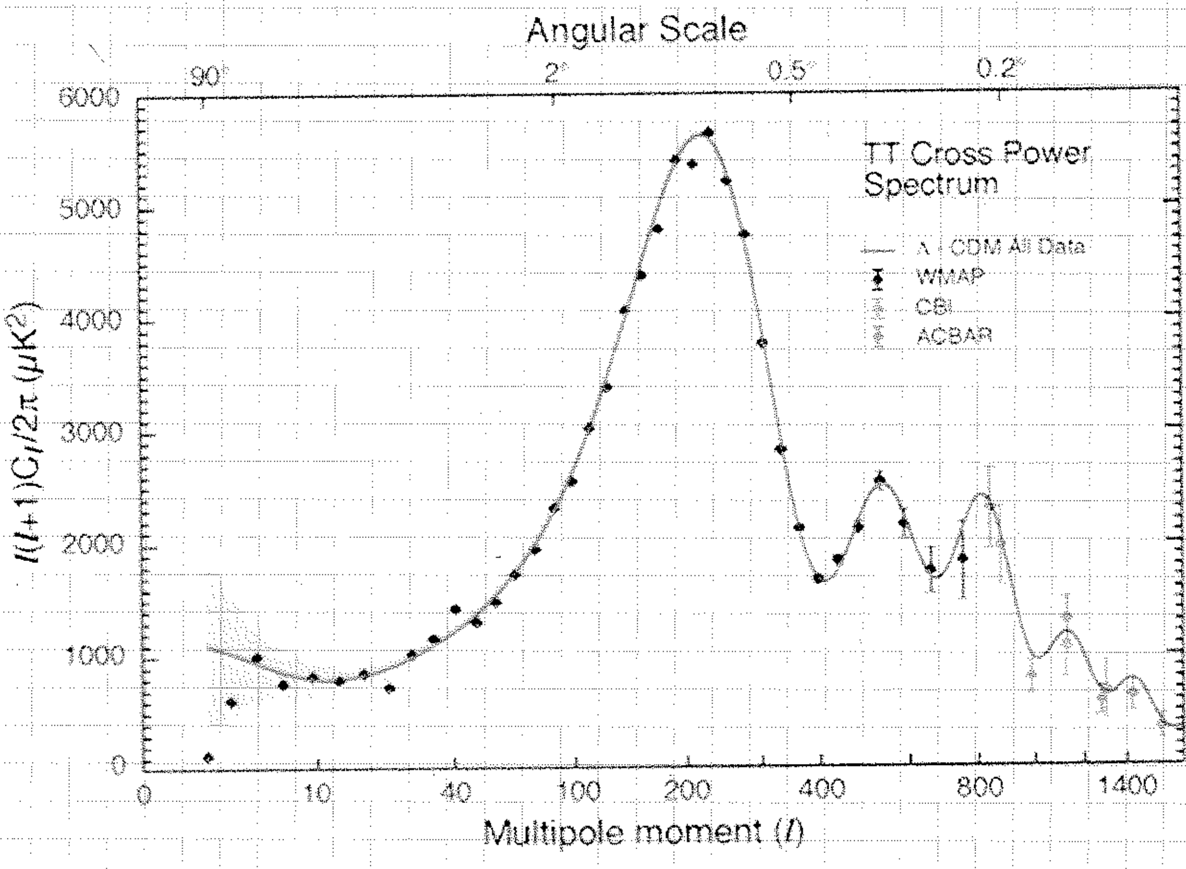
$$\Delta_T^2 = \frac{\ell(\ell+1)}{2\pi} C_\ell T^2,$$

see figures.

WMAP Power Spectrum

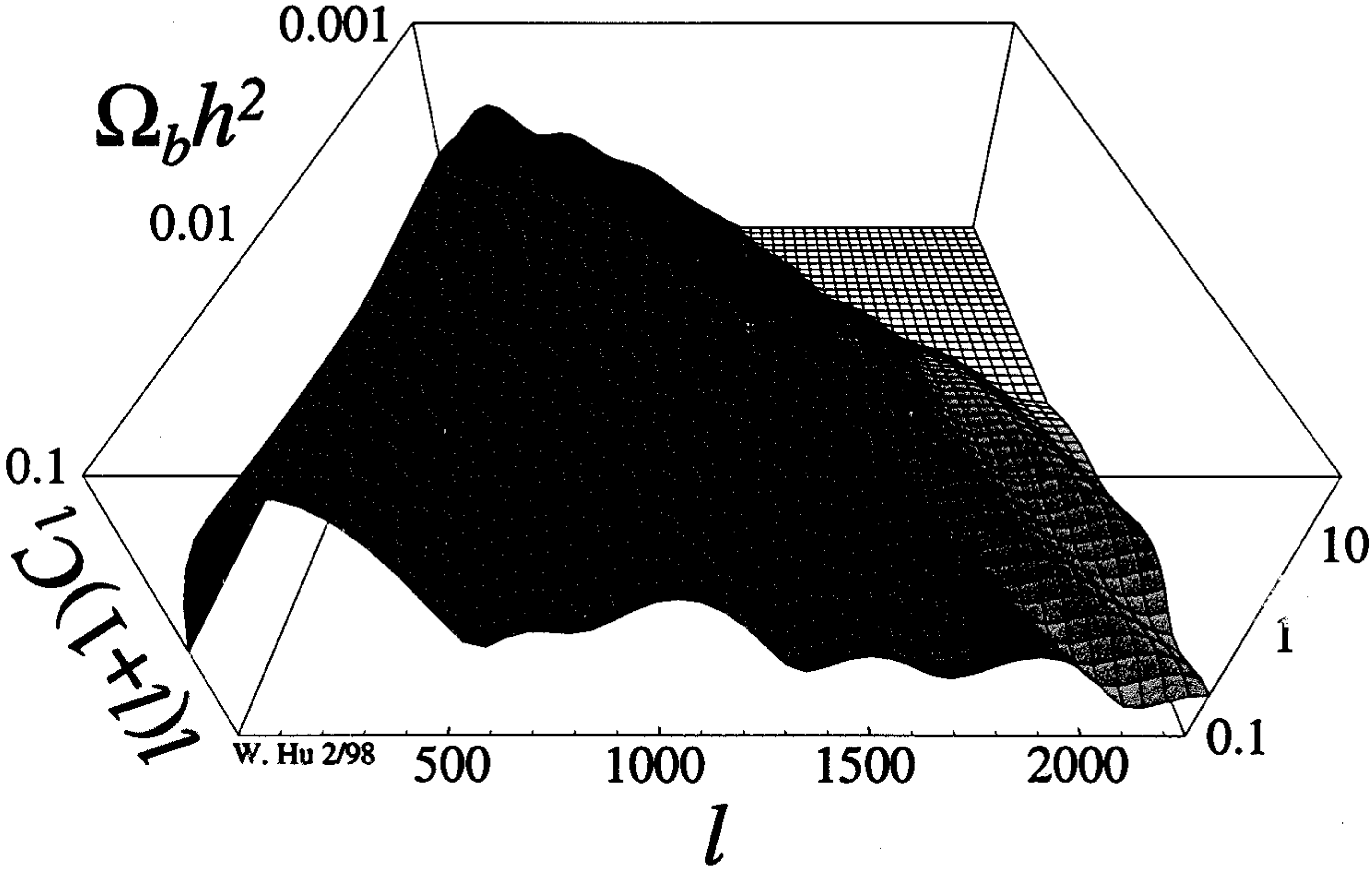
Basic flat WMAP parameters: $\Omega_\Lambda = 0.71, \Omega_{\mathrm{m}} = 0.29\ (\Omega_{\mathrm{c}} = 0.24, \Omega_{\mathrm{b}} = 0.047)$, $n = 0.93, h = 0.71$.

WMAP + other: $\Omega_\Lambda = 0.71, \Omega_{\mathrm{m}} = 0.27\ (\Omega_{\mathrm{c}} = 0.23, \Omega_{\mathrm{b}} = 0.044)$, $n = 0.93, h = 0.71$.

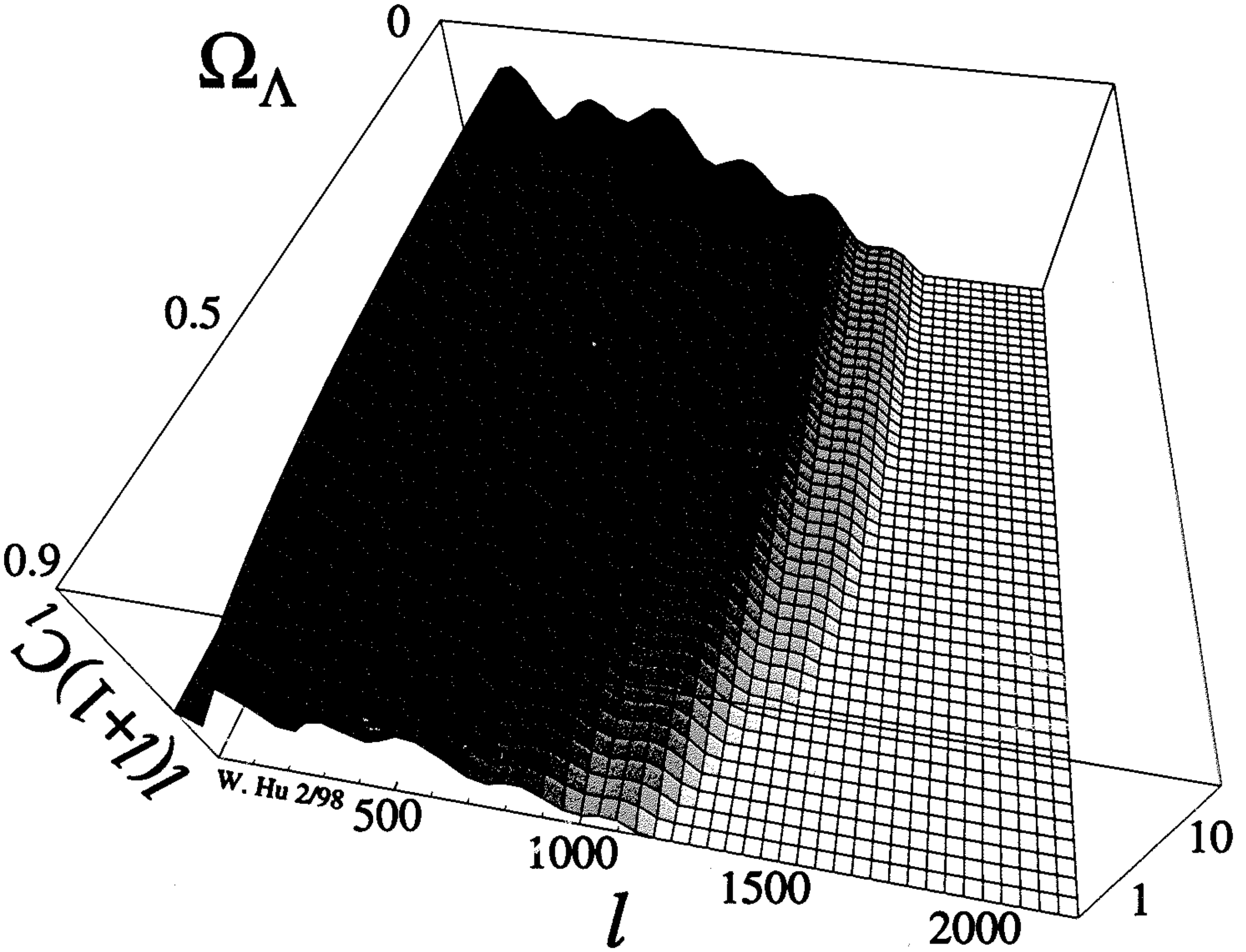


IRAS PSCz Survey

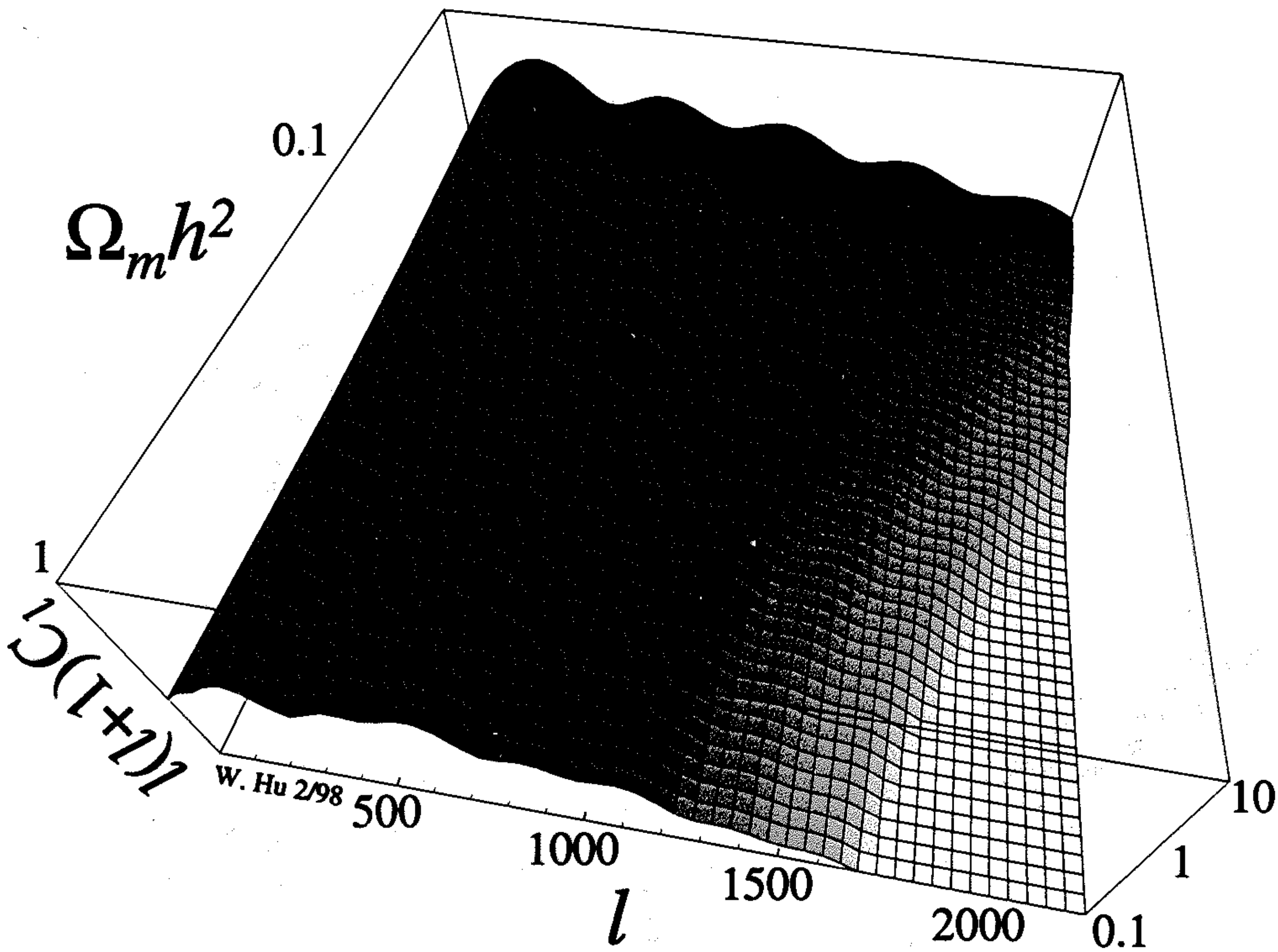
Baryon–Photon Ratio in the CMB



Cosmological Constant in the CMB



Matter-Radiation Ratio in the CMB



Curvature
in the CMB