\( \frac{\Delta T(\theta, \varphi)}{T} = \sum_{l,m} Y_{lm}(\theta, \varphi) a_{lm} \)

\( Y_{lm} \) : spherical harmonics,

\[
\begin{align*}
L^2 Y_{lm} &= l(l+1) Y_{lm} \\
L_3 Y_{lm} &= m Y_{lm} \quad & \theta \in 0, \pi \\
& \varphi \in 0, 2\pi
\end{align*}
\]

\[
\int_{0}^{2\pi} \int_{0}^{\pi} Y_{lm}^* Y_{lm'} \sin \theta \, d\theta \, d\varphi = \delta_{ll'} \delta_{mm'}
\]

\[
\left\langle \frac{\Delta T(\theta, \varphi)}{T} \frac{\Delta T(\theta', \varphi')}{T} \right\rangle = \sum_{l,m} \sum_{l',m'} Y_{lm}(\theta, \varphi) Y_{lm'}^*(\theta', \varphi') \langle a_{lm} a_{lm'} \rangle
\]

spherical symmetry:

\[
\langle a_{lm} a_{lm'} \rangle = c_l \delta_{ll'} \delta_{mm'} \Rightarrow
\]

\[
= \sum_{l,m} \sum_{l',m'} Y_{lm}(\theta, \varphi) Y_{lm'}^*(\theta', \varphi') \cdot c_l \delta_{ll'} \delta_{mm'}
\]
\[ \sum_{l} \sum_{m} Y_{lm}(\theta, \phi) Y_{l'm'}(\theta', \phi') C_l \]

from mathematics:

\[ \sum_{m} Y_{lm}(\theta, \phi) Y_{l'm'}(\theta', \phi') = \frac{2l+1}{4\pi} P_{l}\left(\cos\theta'\right) \]

Legendre polynomials:

\[ a=1, \ b=1, \ c=1 \]

\[ P_n(x) = \frac{1}{2^n n!} \frac{d^n}{dx^n} \left[(x^2-1)^n\right] \]

\[ \theta' \rightarrow \theta' + \pi \]

\[ 0 < \theta < \pi \]

\[ 0 \leq \theta \leq \pi \quad \text{and} \quad \frac{2l+1}{4\pi} P_{l}(\cos(\theta)) C_l \]

\[ l_0 = 1 \]

\[ l_2 = \frac{3}{2} \times \frac{3}{2} \times \frac{1}{2} \]

So: Find average of \( \frac{\Delta T}{T} \) over all the sky as a function of \( \lambda \).

or orthogonality condition:

\[ \int_{-1}^{1} P_{l}(x) P_{l'}(x) dx = \delta_{ll'} \frac{2}{2l+1} \quad \text{i.e.} \]

\[ C_l = \int \cos(\theta) \frac{\Delta T}{T} P_{l}(x) \cdot 2\pi \]

estimate of relation between angle and \( \theta \):

\[ P_l \sim (\text{Calm})^l \sim \text{Calm} \theta \Rightarrow \theta \approx \frac{\text{Calm}}{2} \]
angle-\(\ell\) relation:

\[ \Theta \sim \frac{\pi}{\ell}, \text{ in degrees; } \Theta \sim \frac{100}{\ell} \]

large \(\ell\): small resolution, small distance scale

small \(\ell\): large angle, large distance scale

\[ \ell \sim 100; \quad \Theta \sim 1^\circ \]

MAP: launched in June 2001, \(\ell \sim 1000\)

Planck (European) will be

launched in 2007, will go up to

\(\ell \sim 2000\)

Power spectrum:

\[ \frac{\Delta^2}{\ell} = \frac{\ell(l+1)}{2\pi} C_\ell \ell^2, \]

see figures.
WMAP Power Spectrum

Basic flat WMAP parameters:
\[ \Omega_\Lambda = 0.71, \Omega_m = 0.29 \ (\Omega_c = 0.24, \Omega_b = 0.047), \]
\[ n = 0.93, h = 0.71. \]

WMAP + other:
\[ \Omega_\Lambda = 0.71, \Omega_m = 0.27 \ (\Omega_c = 0.23, \Omega_b = 0.044), \]
\[ n = 0.93, h = 0.71. \]

**Angular Scale**

**TT Cross Power Spectrum**

- \( \Lambda \) CDM All Data
- WMAP
- CBI
- ACBAR

**IRAS PSCz Survey**
Baryon–Photon Ratio in the CMB
Cosmological Constant in the CMB

$\Omega_\Lambda$

$\ell(l+1) C_\ell$

W. Hu 2/98

500 1000 1500 2000

1 10
Matter-Radiation Ratio in the CMB

$\Omega_m h^2$
Curvature in the CMB

\[ \Omega_K \]