Defraction can produce multiple images:

\[ \text{source} \xrightarrow{\text{defraction}} \text{observer} \]

This method can study the distribution of matter in the universe (G. Meylan @ EPFL)
Closed orbits: Mercury perihelion

Newton gravity:

\[ \frac{d\theta}{dt} = \frac{d\phi}{dy} = 0 \] (x)

In GR, the orbit is more complicated and (x) holds, of course.

Integrals of motion:

\[ \frac{1}{r^2} = -\frac{m^2}{y^2} + \frac{E^2}{B(r^2)y^2} = 0 \]

We can find 2 integrals of motion:

\[ \frac{m^2}{E^2} = \frac{\frac{1}{r^2} - \frac{1}{R^2}}{B(r^2) - B(R^2)} \]

\[ \frac{J^2}{E^2} = \frac{\frac{1}{y^2} - \frac{1}{R^2}}{B(\infty) - B(0)} \]
Change of angle, \( \psi \) if we go from \( r \) to \( r' \):

\[
2(\psi(r') - \psi(r))
\]

If this is a closed curve, the it should be equal to \((2\pi)\).

In general, the shift of perihelion is given by

\[
\Delta \psi = 2(\psi(r') - \psi(r)) - 2\pi,
\]

where

\[
\psi(r') - \psi(r) = \int_r^{r'} \frac{d\theta}{\sqrt{\frac{R^2(B^{-1}(r') - B^{-1}(r))}{r^2} - \frac{\frac{1}{2}}{r^2} - \frac{1}{4} \left(\frac{1}{r'} \right)^{-1/2}}}
\]

Computation: again, expand with respect to small parameter(s) \( \frac{GM\, \theta}{r} \).

\[
A = 1 + \frac{2\, GM}{r} + \ldots
\]

\[
B = 1 - \frac{2GM}{r}
\]
Computation is left for exercise:

\[ \delta \gamma = 6\pi \frac{MG}{L} \]

\[ \frac{L}{\Delta L} = \frac{1}{2} \left( \frac{1}{r_+} + \frac{1}{r_-} \right) \]

\( \delta \gamma > 0 \): perihelion moves in the same direction as the particle.

Mercury: (why Mercury—\( L \) is smaller)

\( L = 55.3 \times 10^6 \text{ km} \)

\( MG = 1.475 \times 10^8 \text{ km} \)

\( \delta \gamma = 0.1038'' \) for one turn

100 years: 415 turns \( \Rightarrow \)

\( \delta \gamma \Rightarrow N \delta \gamma = 43.03'' \) for hundred years

Observations exist starting from 1765.

Comparison with observations is non-trivial since there are other reasons for change of the orbit.
Newton theory gives

\[ \Delta \varphi = 532'' \] (for 100 years), due to perturbation
related to Venus, Earth, Jupiter,
so that their contribution is just \( \approx 10\% \)
other possible effects: sun is not
spherically symmetric.

Yet another test (we will not
analyze it) - delay of radar echo

\[ \oplus \quad \oplus \quad \oplus \quad \oplus \]
Earth  Planet  Sun  Planet

Travel time Earth \( \rightarrow \) planet \( \rightarrow \) earth