Lecture # 11

Main points of # 10

- for $T \gtrsim 1\,\text{GeV}$, $t \lesssim 10^{-6}\,s$ we had small baryon asymmetry,

$$\left.\frac{n_B - \bar{n}_B}{n_B + \bar{n}_B}\right|_{T \approx 1\,\text{GeV}} \approx \left.\frac{n_B}{n_\gamma}\right|_{\text{now}} \simeq 6 \cdot 10^{-10}$$

- origin of asymmetry is related (most probably) to
  - baryon # non-conservation
  - C and CP - violation
  - deviations from thermal equilibrium

- Considered specific mechanism of BAU generation: leptoquark decays

$$X \to q\ell,\ \bar{q}\bar{q}$$
$$\bar{X} \to \bar{q}\bar{\ell},\ qq$$

Today:
- $\sqrt{}$ Evidence for Dark matter
- particle physics candidates for DM
- particle horizons and problems of Big-bang cosmology

Dark matter

② Rotational curves of Spiral galaxies

Orbital velocity of the disk as a function of radius

Luminosity $I(r) = I_0 \exp(-r/r_D)$

at $r \gg r_D$: (assume that only luminous matter gravitate)

$$m v^2 \sim \frac{Mm}{r} G_N \Rightarrow$$

$$v \sim \frac{1}{\sqrt{r}} (M G_N)^{1/2}$$

Kepler's law

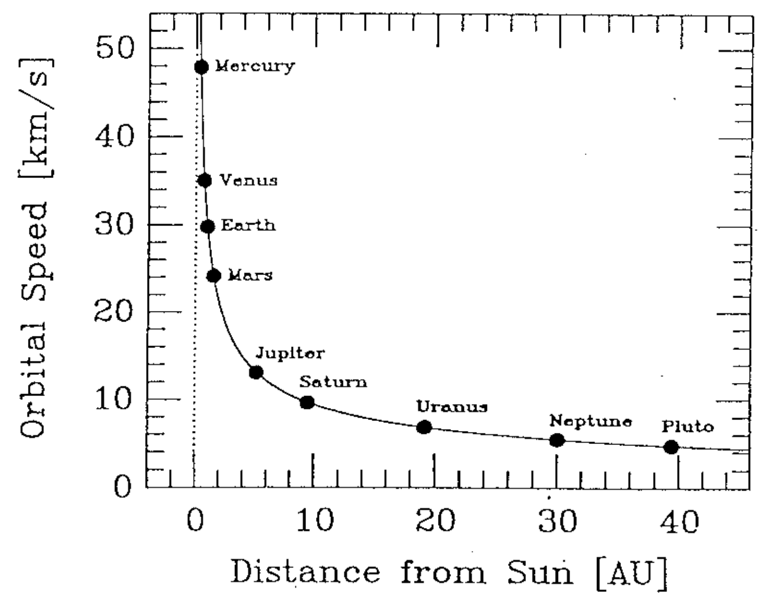


Figure 1: Rotation curve of the solar system which falls off as $1/\sqrt{r}$.

(M)

In reality: flat curves $\Rightarrow$

$$M_{DARK} \sim r, \quad \rho_{DARK} \sim \frac{1}{r^2}$$

$$\rho_{DARK}(r) \simeq \frac{v_\infty^2}{4\pi G_N r_c^2} \frac{r_c^2}{r^2 + r_c^2}$$

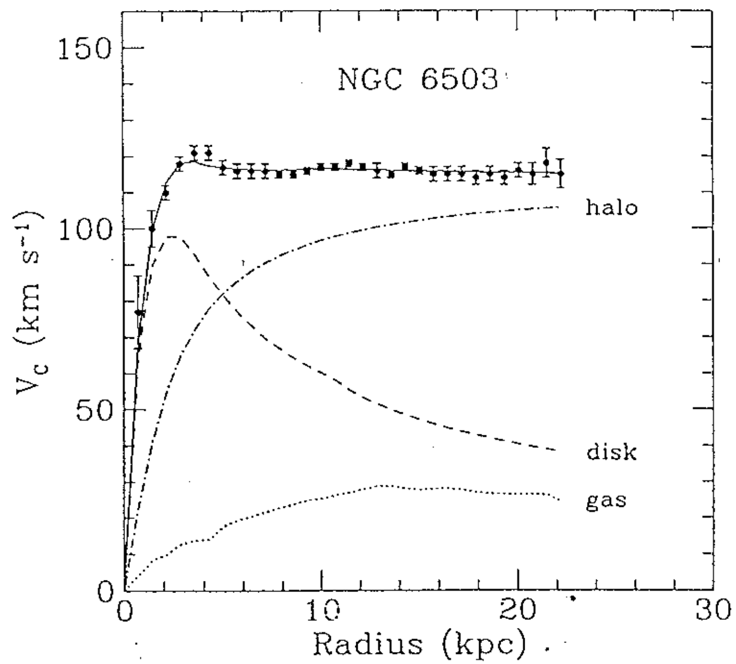


Figure 2: Rotation curve of the spiral galaxy NGC 6503 as established from radio observations of hydrogen gas in the disk. The last measured point is at 12.8 disk scale-lengths. The dashed line shows the rotation curve expected from the disk material alone, the dot-dashed line from the dark matter halo alone.

CONCLUSION FOR Ω_M:

UNIVERSE CONTAINS NON-BARYONIC DARK MATTER WHICH CLUSTERS BUT DOES NOT SHINE,

$$\Omega_M \simeq 0.3 \; ;$$

$$\frac{\Omega_B}{\Omega_M} \simeq 1/7$$

2. Candidates for dark matter particles

1. active neutrinos?

Suppose we have an object with size r (galaxy halo) and with typical velocities v.

Number of fermions in this object (Pauli principle):

$$N \sim \frac{1}{(2\pi)^3}\int d^3p\, d^3x\, n \simeq O(p^3 r^3)$$

Mass of the object: $M < m_\nu p^3 r^3 \simeq$

$$\simeq m_\nu^4 v^3 r^3$$

at the same time $\frac{MG}{r} \sim v^2$ (Kepler) $\Rightarrow$

$$v^2 < \frac{G}{r} \cdot m_\nu^4 v^3 r^3 \Rightarrow$$

It must be

$$m_\nu \gtrsim \left[\frac{1}{G v r^2}\right]^{1/4} \simeq$$

$$120 \left(\frac{100\ \mathrm{km/s}}{v}\right)^{1/4} \cdot \left(\frac{1\ \mathrm{kpc}}{r}\right)^{1/2}$$

"our galaxy": $r \simeq 10\,\mathrm{kpc}$, $\sigma \simeq 220\,\mathrm{km/s}$ $\Rightarrow$

$m_\nu \gtrsim 30\,\mathrm{eV}$

But for dwarf spheroidal galaxies

$M \sim 10^6 M_\odot$: $m_\nu \gtrsim 300-500\,\mathrm{eV}$ $\Rightarrow$

ν cannot be a d.m. candidate.

(2) protons? Does not work, since from BBN one finds that

$$\frac{\Omega_B}{\Omega_{DM}} \simeq \frac{1}{7}$$

If one takes another number, the abundancies of light elements cannot be explained [show slide with Ω-dependence of BBN]

Then : improvisation

about – sterile ν

– SUSY DM

INFLATION

particle horizons :

Static Universe : if we have two events separated by distance Δl and time Δt, they are independent provided $\Delta l > ct$ $\quad (ds^2 = c^2dt^2 - \Delta l^2 < 0$

Universe expands – how to get similar statement ?

$$\frac{dl}{dt} = c + \frac{\dot{a}}{a} l \Rightarrow$$

$$l(t) = c \int_{t_0}^{t} \frac{a(t)}{a(t')} dt'$$

Radiation dominated Universe :

$$a(t) \sim t^{1/2}$$

matter dominated Universe :

$$a(t) \sim t^{2/3}$$

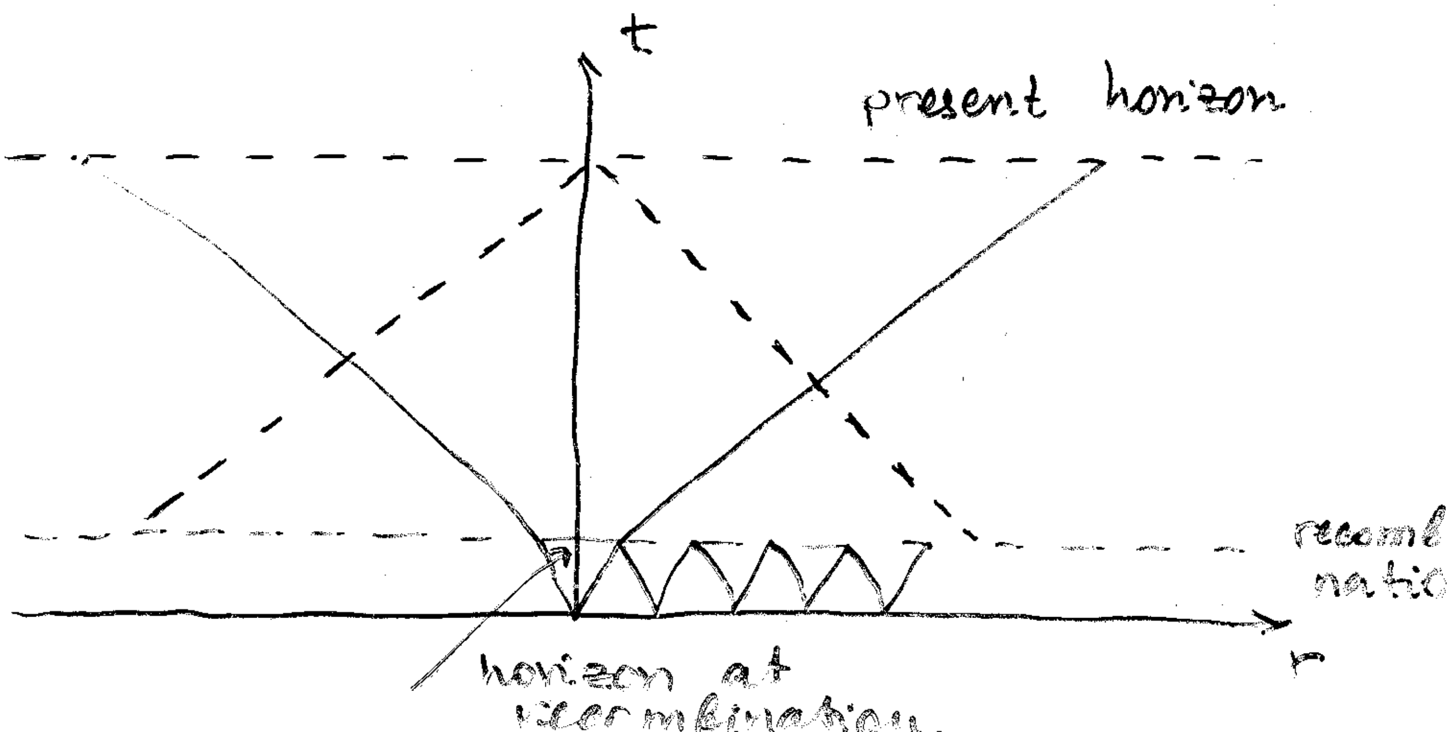
If we put $t_0 = 0$, $l(t)$ converges:

$$l_H(t) = \begin{cases} 2t, & rd \\ 3t, & md \end{cases} \quad \text{particle horizon}$$

Only points at distances $\lesssim l_H(t)$ were in causal contact and thus at $d \gtrsim l_H(t)$ we should not expect to have homogeneity, isotropy, etc.

<u>Horizon and homogeneity problem</u>

rd + md epoch



If we look at different points of the sky separated by $\theta \gtrsim \theta_H$ we see CMB emitted from regions that were never in causal contact (?!)

Estimate of θ_H :

Horizon size at t_d : $3t_d$; $t_d \simeq 5 \cdot 10^5 y$

Size of this region today :

$$3t_d \cdot \left(\frac{t_{now}}{t_d}\right)^{2/3} \Rightarrow$$

$$\theta_H \simeq \frac{3t_d}{3t_{now}} \left(\frac{t_{now}}{t_d}\right)^{2/3} \simeq \left(\frac{t_d}{t_{now}}\right)^{1/3} \approx$$

$\approx \left(\frac{5 \cdot 10^5}{15 \cdot 10^9}\right)^{1/3} = \left(\frac{1}{3 \cdot 10^4}\right)^{1/3} = \approx \frac{1}{30}$

$\theta_H \simeq 1^\circ$, $t_{now} \sim 15 \cdot 10^9$ years

$\theta \approx \frac{180}{30\pi} \approx 2^\circ$

Number of different regions within our horizon today ~

$\pi : 180$

$\frac{1}{30} : x$

$$\left(\frac{t_{now}}{t_d}\right) \sim 3 \cdot 10^4$$

BUT: CMB is isotropic with accuracy $O(10^{-5})$ at $\theta \gtrsim 1^\circ$!!

<u>Flatness problem</u>

$$\Omega - 1 = \frac{k}{a^2 H^2}$$

for rd or md:

$$H \sim \frac{1}{t}\,; \quad a \sim t^\alpha \,, \quad \alpha = \frac{1}{2} \text{ or } \frac{2}{3}$$

$$\Downarrow$$

$$\Omega - 1 \sim t^{2(1-\alpha)}$$

increases with t

Therefore, to have $\Omega \approx 1$ now, Ω must be very close to 1 in the past

e.g. at nucleosynthesis time

$$\Omega - 1 \approx 10^{-15} \quad !!$$

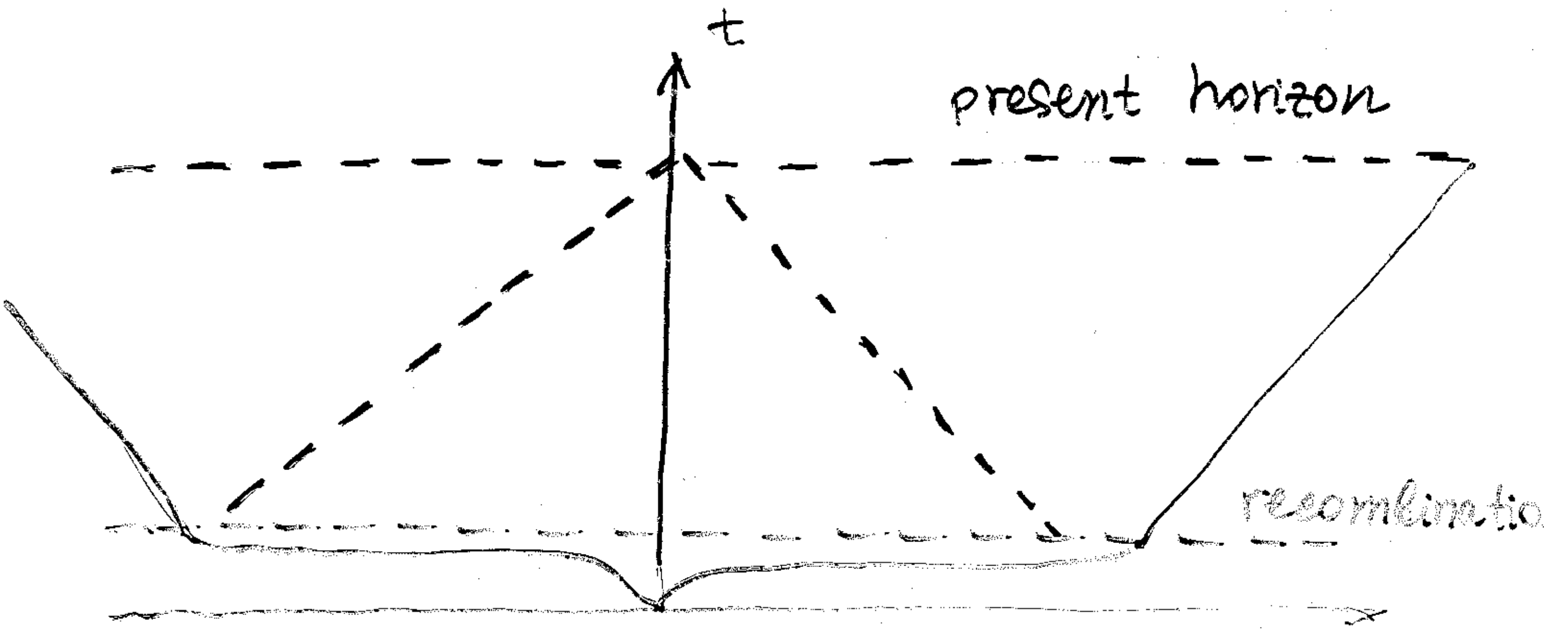
Two problems are related:

if $\alpha \geq 1 \Rightarrow$

particle horizon at recombination is ~~in~~finite $\left[d \sim \int_0^{t_r} \frac{dt}{a(t)} \right]$

and $\Omega - 1$ is decreasing function of time.

Solution: modification of expansion before recombination (before nucleosynthesis)



Lectures # 12

Main points of # 11

- There is an observational evidence for non-baryonic dark matter,

  $\Omega_{DM} \simeq 0.22$, $\Omega_b \simeq 0.05$ $\Omega_\Lambda \simeq 0.73$

  coming from : - rotational curves of galaxies
  - structure formation and fits of CMB
  - BBN

- Dark matter cannot be neutrinos
  - some rotational curves require $m_\nu \gtrsim 500\, eV$ (coming from Fermi-statistics of ν)
  - analysis of CMB & structure formation tells that

    $m_\nu \leq 0.5\, eV$,

    direct experiments tell $m_\nu \lesssim 1 eV$

- Dark matter should be some unknown particle (- sterile ν, neutralino, axion, gravitino, Q-ball

or Einstein Dynamics must be changed

Found particle horizons:

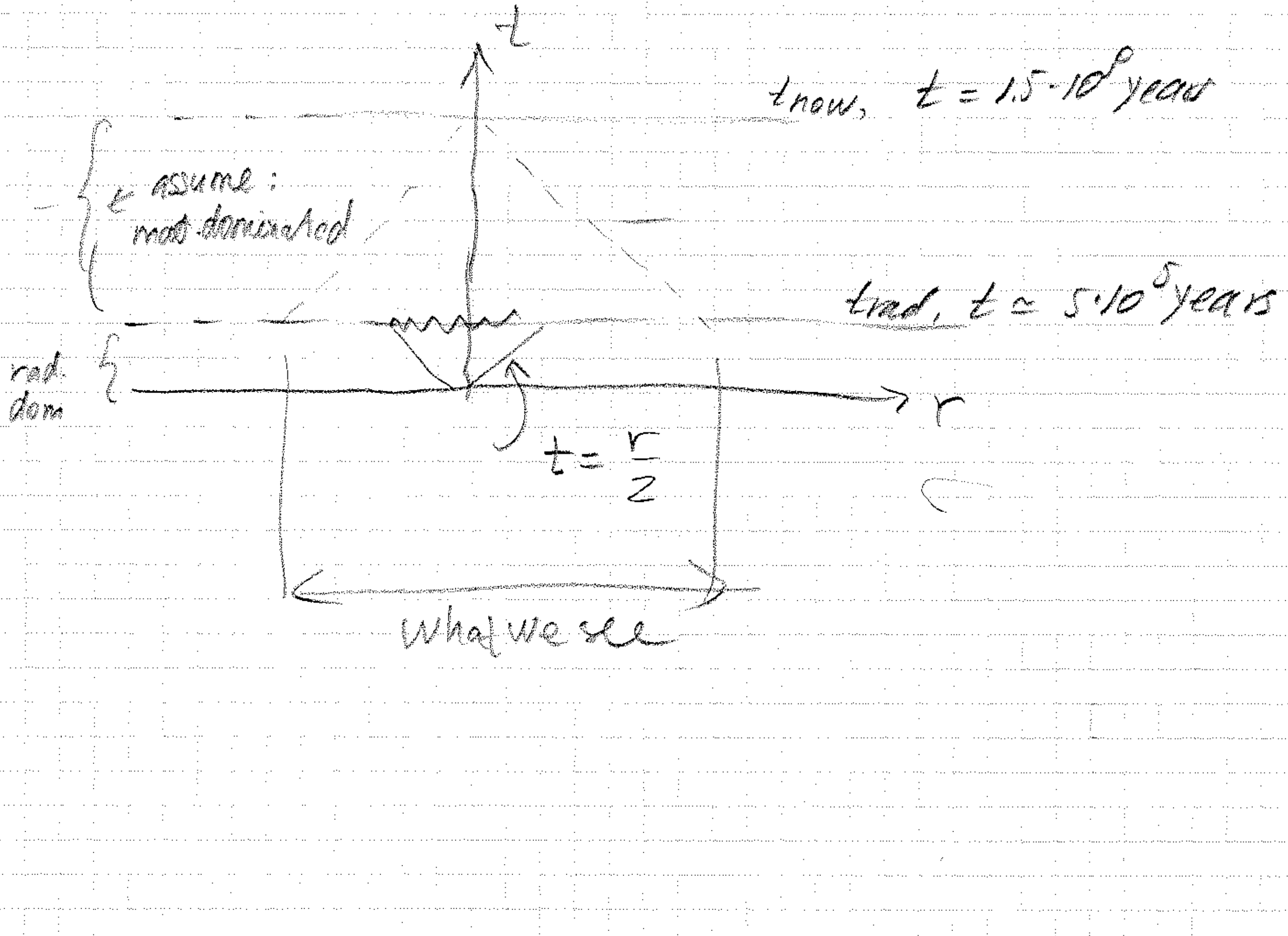
radiation - dominated Univers

$$l = 2t$$

matter - dominated Univ:

$$l = 3t$$

Causal structure of cosmic microwave background:



Today: continuation:

Problems of homogeneous univers

- idea of inflation
- inflation & scalar field dynamics
- parameters of the universe from CMB

Idea of inflation: modification of expansion law at $T > T_{\text{Baryogenesis}}$:

new expansion law, l goes much faster than $2t$ or $3t$!!

Suppose : vacuum energy density dominates at $t \leq t_{nuc}$:

$$a \sim a_0 \exp\left[H(t-t_0)\right]; \; H = \text{Const} = \sqrt{\frac{8\pi G \varepsilon}{3}}$$

for $t_0 \leq t \leq t_1$

and then for $t > t_1$ - rd epoch

$$a(t_r) = a_0 \exp\left[H(t_1 - t_0)\right] \left(\frac{t_r}{t_1}\right)^{1/2} \Rightarrow$$

Horizon problem :

horizon at recombination $\sim$

$$\boxed{l_H \simeq \frac{1}{H} \exp\left(+H(t_1 - t_0)\right)}$$

and is $\gg t_r$ if $H(t_1 - t_0) \gg 1$

if, say, $\varepsilon \sim (10^{15}\,GeV)^4$, for

$H(t_1 - t_0) \gtrsim 65$ the horizon problem is solved!

Flatness problem:

$$\Omega - 1 \sim \frac{k}{a^2 H^2} \sim k e^{-2H(t_1 - t_0)}$$

very small, if $H(t_1 - t_0) \gg 1$.

Simplest realization of inflation "Chaotic inflation"

Scalar field:

$$S = \int \sqrt{-g} \left[\frac{1}{2} g^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi - V(\varphi) \right] d^4x$$

$$V(\varphi) = \frac{1}{2} m^2 \varphi^2 + \frac{\lambda \varphi^4}{4}.$$

Initial condition at $t \sim \frac{1}{M_{pl}} \sim 10^{-43} s$:

$$\varepsilon \sim M_{pl}^4 \quad ; \quad \varepsilon \sim \frac{1}{2} \dot{\varphi}^2 + \frac{1}{2} (\nabla \varphi)^2 + V(\varphi)$$

assume: $m^2 \ll M_{pl}^2$; $\lambda \ll 1$

Take a region where fluctuation of potential energy dominates:

$$V(\varphi) \gg (\nabla\varphi)^2,\ (\dot{\varphi})^2$$

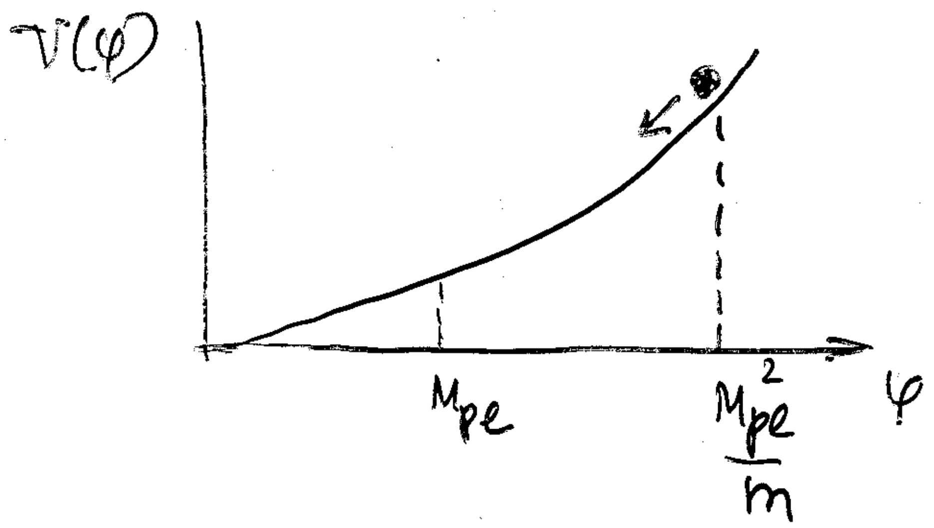
$$m^2\varphi^2 \approx M_{pl}^4 \Rightarrow$$

$$\varphi \approx \frac{M_{pl}^2}{m} \gg M_{pl}$$

φ - field dynamics:

$$\ddot{\varphi} + 3H\dot{\varphi} + V'(\varphi) = 0 \qquad (*)$$

$$H^2 = \frac{8\pi G}{3}\left(\frac{1}{2}\dot{\varphi}^2 + V(\varphi)\right)$$



let us solve eq. (*)

assuming that $H = \mathrm{const}$,

$H \gg m^2$ (we also take

$x = 0$ for simplicity)

Then,

$\varphi \sim C e^{i\omega t}$, eq. for ω:

$-\omega^2 + 3Hi\omega + m^2 = 0 \Rightarrow$

$\omega \approx 3iH, \quad \omega \approx + \frac{m^2}{3H} \cdot i$

and the solutions are

$\varphi \sim C_1 e^{-3Ht} + C_2 e^{-\frac{m^2 t}{3H}}$.

The first term quickly disappears, and

we have $\varphi \sim C_2 e^{-\frac{m^2 t}{3H}}$.

in other words, in eq (*) one

can neglect $\ddot{\varphi}$

This is valid provided $H \gg m$.

in this case we have also

$\dot{\varphi} = \frac{m^2}{3H} \cdot \varphi \Rightarrow \dot{\varphi}^2 \ll m^2 \varphi^2$

Overdamped regime:

$$H \gg \frac{\ddot{\varphi}}{\dot{\varphi}}$$

$$H \sim \frac{m\varphi}{M_{pl}}$$

$$H^2 \approx \frac{4\pi m^2 \varphi^2}{3M_{pl}^2} \; ; \; V' = m^2 \varphi$$

$$H = \frac{\sqrt{4\pi}\, m\varphi}{\sqrt{3} M_{pl}}$$

$$\frac{\sqrt{12\pi}\; m \varphi \dot{\varphi}}{M_{pl}} + m^2 \varphi = 0 \Rightarrow$$

$$\boxed{\varphi = \varphi_0 - \frac{m M_{pl}}{\sqrt{12\pi}} \cdot t \approx \varphi_0 \left(1 - O\left(\frac{m^2 t}{M_{pl}}\right)\right)}$$

Approximation breaks down at

$\dot{\varphi}^2 \sim V(\varphi) = m^2 \varphi^2$, i.e. at

$$\boxed{\varphi \simeq M_{pl} \, , \; t^* \approx \frac{M_{pl}}{m^2}}$$

$$\dot{\varphi} = \frac{m M_{pl}}{\sqrt{12\pi}} ; \quad \dot{\varphi} = m\varphi \Rightarrow \quad \varphi = \frac{M_{pl}}{\sqrt{12\pi}} \Rightarrow$$

$$m^4 \varphi_0^4 = M_{pl}^4$$

$$m\varphi_0 = M_{pl}^2 ; \; \varphi_0 = M_{pl}^2 / m \qquad t \approx \frac{\varphi_0 \sqrt{12\pi}}{m M_{pl}} = \frac{\sqrt{12\pi}\, M_{pl}}{m^2}$$

Universe inflates by a factor

$$e^{+Ht} \approx \exp\left[+\frac{M_{pl}^2}{m^2}\right] \gg 1.$$

What happens at $\varphi \lesssim M_{pl}$?

Energy of φ is transferred to radiation

[reheating, preheating ....]

Many models for inflation :

hybrid, power, new, old, ...

Another gain: quantum scalar field fluctuations produce primordial energy perturbation $\Rightarrow$

Structure formation !

Most important predictions of inflation:

- $\Omega_{tot} = 1$

- scale-invariant spectrum of primordial density perturbations

Density perturbations, inflation

$$\frac{\delta\rho(\vec{x})}{\rho} = \frac{1}{(2\pi)^3}\int \delta_k \exp(-i\vec{k}\vec{x})\, d^3k$$

k: comoving momentum

Power spectrum, homogeneous case

$$\left\langle \frac{\delta\rho(x)}{\rho}\,\frac{\delta\rho(y)}{\rho}\right\rangle = \frac{1}{(2\pi)^3}\frac{1}{(2\pi)^3}\int \delta_k \delta_p \exp(-ikx - ipy)\, d^3k\, d^3p$$

$$\Rightarrow \langle \delta_k \delta_p \rangle = (2\pi)^3\, \delta(\vec{k}+\vec{p}) \cdot |\sigma_k|^2$$

Ordinary assumption: $|\sigma_k|^2 \sim k^n$

Inflation, $|x-y| \lesssim \frac{1}{H}\exp(+Ht)$:

$$\langle \varphi(x)\varphi(y)\rangle \sim \mathrm{const} \Rightarrow$$

$$\mathrm{Const} \sim \frac{\delta\rho(x)}{\rho}\frac{\delta\rho(y)}{\rho} = \frac{1}{(2\pi)^3}\int |\sigma_k|^2 d^3k\, e^{ik(x-y)} \Rightarrow$$

$$|\sigma_k|^2 \sim \frac{1}{k^3}\underbrace{k^{n_s-1}}$$

$n_s = 1$:

Zeldovich–Harrison spectrum

Definition of n_s

CMB and cosmological parameters:

- give spectrum of initial perturbations from deep theory (inflation) at initial time well before recombination

- study its evolution as a function of cosmological parameters
  - $\Omega_K = \Omega_M + \Omega_\Lambda + \Omega_\gamma$ (gives $\frac{\rho}{\rho_c}$)
  - Ω_Λ
  - $\eta = \frac{n_b}{n_\gamma}$
  - Ω_M

compare with observations.