Lecture # 8. 

Main points of #7

Considered photon decoupling.

Physics: (i) $\gamma$ interact mainly with free e.

(ii) the rate of interaction is mainly determined by concentration of free e

(iii) this concentration is determined by process $p + e \leftrightarrow H + \gamma$. Important parameter: I - ionisation energy

(iv) equation for decoupling:

$$\sigma_{e\gamma} \cdot n_e = H(t)$$

$\nearrow$ determined by Saha equation and baryon asymmetry,

$$\frac{n_B}{n_\gamma} = \eta$$

Solution: $\frac{I}{2T^*} = 26.6 \Rightarrow T^* \approx 0.25\,eV = 3000\,K$

# of free electrons: $\frac{n_e}{n_H} \sim 10^{-12}$

Important numbers:

$z_{equality} = 3000 \leftarrow$ change from radiation domination to matter domination

$z_{decoupling} = 1100$

Today

- neutrinos in the early universe

# Neutrino temperature



neutrino:

weakly interacting particle

most probably has some mass (very small, in the eV range or smaller)

we will derive cosmological limit later.

there are 3 types of neutrinos

$\nu_e$ $\nu_\mu$ $\nu_\tau$

lepton number $= +1$

electron number: $\nu_e = 1$ $\nu_\mu = \nu_\tau = 0$

muon —"— $\nu_e = 0$, $\nu_\mu = 1$, $\nu_\tau = 0$

tauon —"— $\nu_e = \nu_\mu = 0$, $\nu_\tau = 1$.

Spin $= 1/2$

# of degrees of freedom: 1 !

electron: spin ↑, spin ↓ = 2

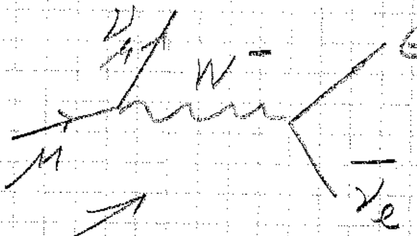
massless neutrino is "left", spin is antiparallel to momentum

for antineutrino spin is parallel to momentum

**Question** what is the temperature of relic neutrino gas now (assuming that neutrinos are massless) (4)

Decoupling of neutrinos: "charged currents"

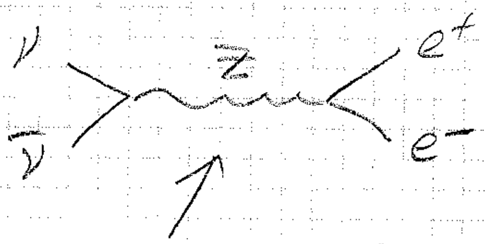
$\mu \to \nu_\mu e^- \bar{\nu}_e$




$m_W = 80.4$ GeV

intermediate vector boson

neutral currents:




neutral vector boson "Z", $m_Z = 91.2$ GeV

cross - section

$$\sigma \sim G_F^2 \cdot E^2$$

$E$: 1 MeV $\Rightarrow$

$$\sigma_W = 10^{-10} \cdot 10^{-6} = 10^{-16}\ \text{GeV}^{-2}$$

$$G_F \approx \frac{g^2}{M_W^2} \approx 10^{-5}\ \text{GeV}^{-2}$$

$$\sigma_{EM} \approx \frac{\alpha^2}{E^2} \approx 10^{-4} \cdot 10^{6} = 10^{2}\ \text{GeV}^{-2}$$

decoupling of neutrinos:

$$\frac{\sigma_W}{\sigma_{EM}} \approx 10^{-18}$$

$$\sigma n v \approx \underbrace{G_F^2 T^2 \cdot T^3}_{\text{rate of } \nu \text{ interactions}} \approx \underbrace{\frac{T^2}{M_0}}_{\text{rate of Universe expansion}}$$

(5)

$$T^* = \frac{1}{(M_0 G_F^2)^{1/3}} = \frac{1}{(10^{18}\cdot 10^{-10})^{1/3}} =$$

$= 2\cdot 10^{-3}$ GeV $\approx 2$ MeV

This temperature is $T^* \gtrsim m_e$, but $T^* \ll$ masses of all particles $\Rightarrow$

at $T \approx T^*$ we have equilibrium mixture of
$e^+, e^-, \gamma, \nu_e, \nu_\mu, \nu_\tau$

at $T \ll m_e$ $e^+e^-$ annihilate into photons, but not neutrinos, as neutrinos do not interact.

at $T \ll m_e$ we have : $\gamma, \nu_e, \nu_\mu, \nu_\tau$

So : $e^+e^-\gamma \rightarrow \gamma$ at $T \simeq m_e$

entropy is conserved $\Rightarrow$ (thermal equilibrium!)
adiabatic process, $H \ll \Gamma$

$$\Big[\underbrace{2}_{\gamma} + \underbrace{\frac{7}{8}(2+2)}_{e^+e^-}\Big] T_{in}^3 a_{in}^3 = 2\cdot T_\gamma^3\, a_{out}^3$$

$$\underbrace{T_{in}^3 a_{in}^3 = T_\nu^3 a_{out}^3}_{\text{free expantion of } \nu}$$

$$\Downarrow$$

$$T_\nu^3 = T_\gamma^3 \left[\frac{2 + \frac{7}{8}(2+2)}{2}\right]^{-1};$$

$$T_\nu = \left(1 + \frac{7}{4}\right)^{-1/3} T_\gamma = \left(\frac{4}{11}\right)^{1/3} T_\gamma \approx 2^\circ K.$$

$$n_\gamma = 400\ ^1/_{cm^3};\quad n_{\nu_e} = \frac{3}{4} \cdot \frac{4}{11} \cdot \frac{1}{2}\, n_\gamma$$

Consider now cosmological constraints on neutrino.
We will see that a bound on their mass can be derived from cosmological observations

Require: energy density in $\nu$
$\leq$ critical (observed) energy density

(7)

$$\rho_c = \frac{3H^2}{8\pi G} \simeq 1.88 \times 10^{-29} h^2 \frac{g}{cm^3} =$$

$$= 10^{-5} h^2 \frac{GeV}{cm^3} = \frac{10^4 h^2 \ eV}{cm^3} \Rightarrow$$

$$\sum m_\nu \cdot 112 \lesssim \frac{10^4 h^2 \ eV}{cm^3} \cdot \Omega_{DM}$$

$$\sum m_\nu \lesssim 100 \ h^2 \ eV \cdot \Omega_{DM}$$

if $h = 0.7$, then

$$\sum m_\nu \lesssim 50 \ eV \ \Omega_{DM} \simeq 10 \ eV$$

Direct Experimental particle physics limits:

$m_{\nu_\mu} \lesssim 0.17$ MeV
$m_{\nu_\tau} \lesssim 18.2$ MeV

Cosmology gives much better limits (if $\tau_\nu \gtrsim$ universe age)

$m_{\nu_e} \lesssim 3$ eV

Lecture #9 

Main points of #8

1 - found neutrino decoupling temperature,

$T_\nu \approx 2 MeV$

2 - since $T_\nu \gtrsim m_e$ – $\nu$ are colder than photons now –

found $T_\nu = \left(\frac{4}{11}\right)^{1/3} T_\gamma \approx 2^\circ$

3 - obtained a constraint on $\nu$-masses:

$\sum m_\nu \leq 10 eV$ from requirement

$\rho_\nu < \Omega_{DM} \cdot \rho_c$

Today - BBN

## Big Bang Nucleosynthesis

If $T \gg O(10 MeV) \Rightarrow$

protons and neutrons are not binded together

Binding energies:

$$B_A \equiv Z m_p + (A - Z) m_n - m_A$$

$Z$ - charge of nucleus A (# of protons)

$A$ - atomic number (# of neutrons)

$m_A$ - mass of nucleus

| $^AZ$ | $B_A$ [MeV] | $g_A$ |
|---|---|---|
| $^2H$ | 2.22 | 3 |
| $^3H$ | 6.92 | 2 |
| $^3He$ | 7.72 | 2 |
| $^4He$ | 28.3 | 1 |
| $^{12}C$ | 92.2 | 1 |

⇐ example, see also transparency on next page

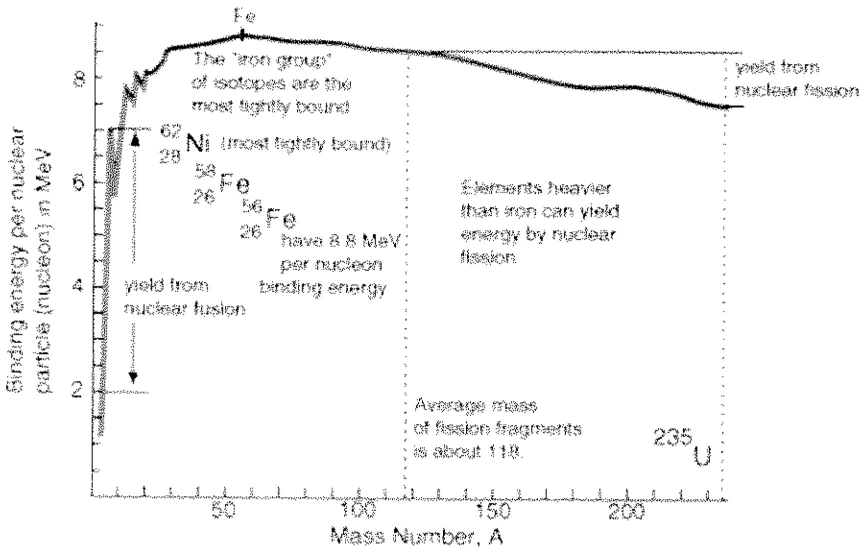
Energetically more favorable to hide protons & neutrons in nuclei!

**Best: in Fe!**

Questions: Can we explain __all__ chemical content of the Universe by processes occurring at $T \sim O(1 MeV)$?

If not, what elements can be created.

3



Fe
The "iron group" of isotopes are the most tightly bound
yield from nuclear fission
62 28 Ni (most tightly bound)
58 26 Fe
56 26 Fe
have 8.8 MeV per nucleon binding energy
Elements heavier than iron can yield energy by nuclear fission
yield from nuclear fusion
Average mass of fission fragments is about 118.
235 U
Binding energy per nuclear particle (nucleon) in MeV
Mass Number, A
2
4
6
8
50
100
150
200

Everything in Fe ??

No, as dynamics of decoupling is important.

important reactions to consider

$\nu + n \leftrightarrow p + e^-$ (for neutron concentration)

$n + p \rightarrow D + \gamma$

$D + D \rightarrow {}^3H + p$

$D + D \rightarrow {}^3He + n$

${}^3H + D \rightarrow {}^4He + n$

${}^3He + n \rightarrow {}^3H + p$

etc

Initial conditions:

$1\,GeV \gg T \gg 1\,MeV$ $\quad (t \ll 1\,sec)$

$p\,;\; n\,;\; e^+;\; e^-;\; \gamma\,;\; \nu;\; \bar{\nu}.$

$\eta = \dfrac{n_B}{n_\gamma} \sim 10^{-10} \Leftarrow$ very important parameter of BBN.

reactions:

$$n \leftrightarrow p + e^- + \bar{\nu}_e$$

$$\nu + n \leftrightarrow p + e^-$$

$$e^+ + n \leftrightarrow p + \bar{\nu}_e$$

Suppose these reactions are in thermal equilibrium:

$$\mu_n + \mu_\nu = \mu_p + \mu_e$$

$$\Downarrow$$

$$\frac{n_n}{n_p} = \exp\left(-\frac{Q}{T} + \frac{(\mu_e - \mu_\nu)}{T}\right) \Leftarrow$$ similar to Saha f-la we derived for $n_e, n_\gamma, n_p$

$$Q = m_n - m_p = 1.293\,MeV$$

neutrality of the universe: $n_e = n_p \Rightarrow$

$\dfrac{\mu_e}{T} \sim \eta \sim 10^{-10}$

<u>Assume</u> that $\dfrac{\mu_\nu}{T} \sim \eta \Rightarrow \dfrac{n_n}{n_p} \approx \exp\left(-\dfrac{Q}{T}\right)$

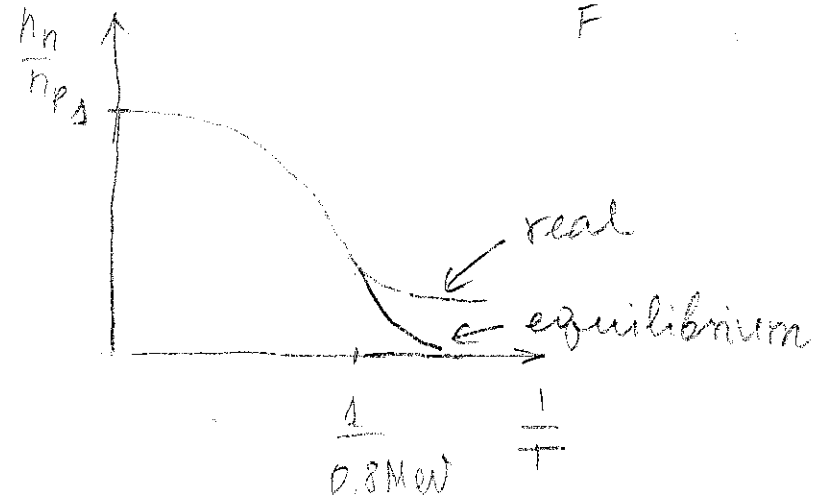
decoupling of these reactions:

(parametrically, as in $\nu$ decoupling)

$$\Gamma \sim G_F^2 T^5 \approx \frac{T^2}{M_0} \Rightarrow T \approx \left(\frac{M_0}{G_F^2}\right)^{1/3}$$

$$\Downarrow$$

$$T_F \approx 0.8\,\text{MeV} \qquad t \approx 1.2\,s$$




Definition of abundance:

$$X_A = \frac{A \cdot n_A}{n_B}$$

$A$: # of nucleons in nuclei $A$, $n_A$ - concentr.

Step # 1

$T \approx 10\,\text{MeV}$, $t \approx 10^{-2}\,s$

$e^{\pm}$, $\gamma$, neutrinos

weak reactions are much faster than the rate of univ. expansion

equilibrium:

$X_n \approx X_p \approx 0.5$

$X_2 \approx 6 \cdot 10^{-12}$ (deuterium)

$X_3 \sim 2 \cdot 10^{-23}$ ($^3He$)

$X_4 \approx 2 \cdot 10^{-34}$ ($^4He$)

$X_{12} \approx 2 \cdot 10^{-108}$ ($^{12}C$)

Step # 2 (7)

at $T \approx 1$ MeV neutrino decouple, later $e^{\pm}$ annihilate

$T \approx 1$ MeV, $t \approx 1$ s

freezout of neutrino reactions

$$\left(\frac{n_n}{n_p}\right)_{\text{freez-out}} \approx \exp\left(-\frac{Q}{T_F}\right) \simeq \frac{1}{5}$$

$X_n = 1/6$

$X_p \simeq 5/6$

$X_2 \simeq 10^{-12}$

$X_3 \simeq 10^{-23}$

$X_4 \approx 10^{-28}$

$X_{12} \approx 10^{-108}$

Step # 3: $t \simeq 5$ mins, nuclear reactions:

$p + n \leftrightarrow D + \gamma$

$D + D \to {}^3He + n$ ; ${}^3He + D \to {}^4He + p$

$D + D \to {}^3H + p$ ; ${}^3H + D \to {}^4He + n$

$D + D \to {}^4He + \gamma$ , ...

The first reaction in the chain:

$p + n \leftrightarrow D + \gamma$ determines the nucleosynthesis temperature:

temperature of Nucleosynthesis:

$p + n \Leftrightarrow D + \gamma$

analogue: $e + p \Leftrightarrow H\gamma$ — already considered.

Saha formulae:

$$m_D - m_p - m_n = -\Delta_D = -2.23\ \text{MeV}$$

$$\frac{n_p \cdot n_n}{n_D} \approx \left(\frac{m_p T}{2\pi}\right)^{3/2} e^{-\Delta_D/T}$$

$n_D \sim n_p$ at $\quad n_n \approx \left(\frac{m_p T}{2\pi}\right)^{3/2} e^{-\Delta_D/T} \approx$

$$\eta T^3 \Rightarrow$$

T is determined from

$$\left(\frac{m_p}{2\pi T}\right)^{3/2} e^{-\Delta_T/T} \approx \eta \sim 10^{-10} \cdot 6.1$$

$$T_{NS} \simeq 70\ \text{keV}$$

$$t_{NS} \simeq \frac{1}{2H(T_{NS})} \approx 265\,s \approx 4.5\ \text{min}$$

Concentration of neutrons at nucleosynthesis temperature:

$$\frac{1}{5}\exp\left(-\frac{t_{ns}}{\tau_n}\right) \approx \frac{1}{7}$$

$\tau_n$ – neutron life-time: $\tau_n \approx 886\,s \approx 15\,min$

The most „attractive" nuclei: $^4He$:

binding energy per nucleon:

$$\frac{28.3}{4} \approx 7\,MeV$$

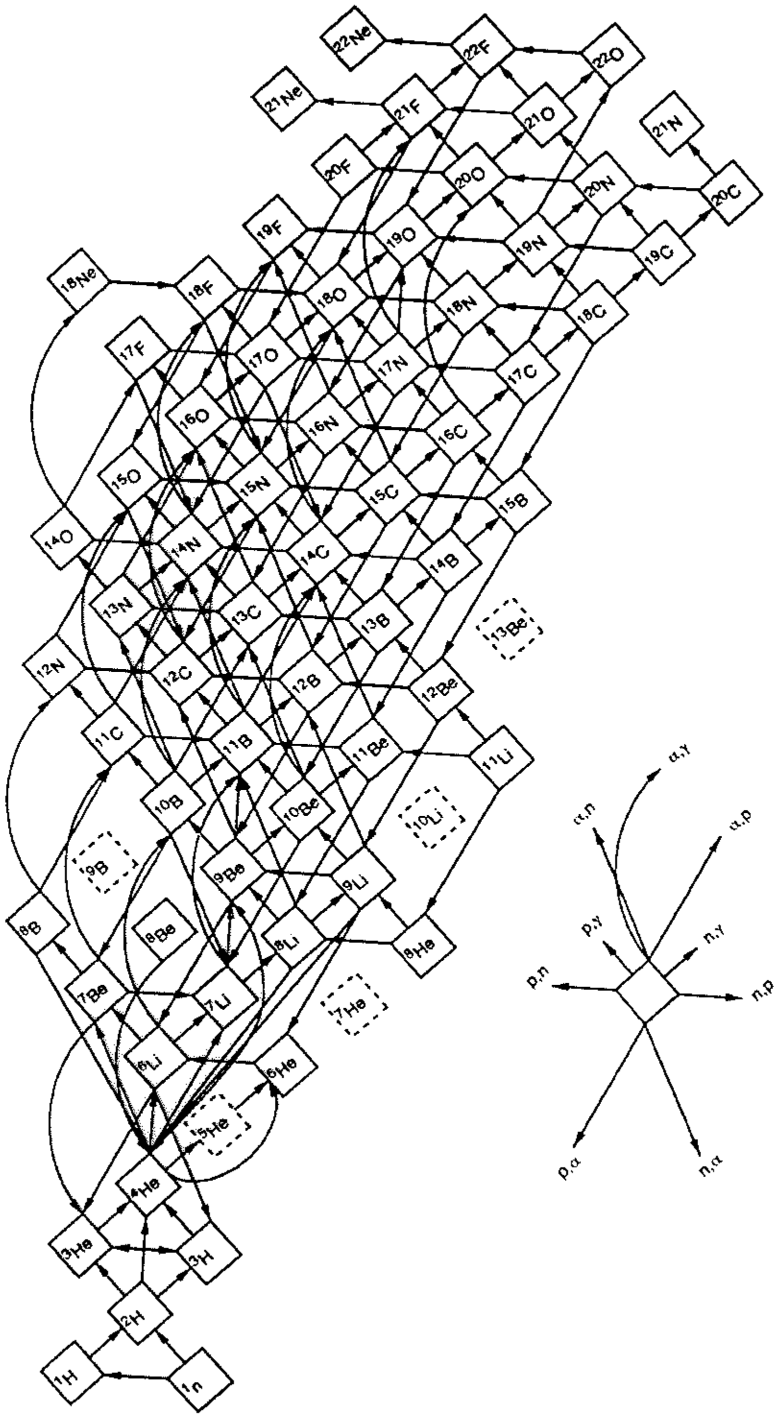
All $p$ & $n$ „want" to be in $^4He$, but in $^4He$ # of $p$ = # of $n$ $\Rightarrow$

$$n_{He_4} \approx \frac{1}{2} n_n$$

$$X_4 \approx \frac{4 n_{He}}{n_p + n_n} = \frac{2 n_n}{n_p + n_n} = \frac{2\, n_n/n_p}{1 + \frac{n_n}{n_p}} \approx$$

$$\approx \frac{2/7}{1 + 1/7} = \frac{2}{7}\,\frac{7}{8} = \frac{1}{4} = 0.25$$

Some $^7Li$ is synthesizes also




The nuclear network used in BBN calculations.

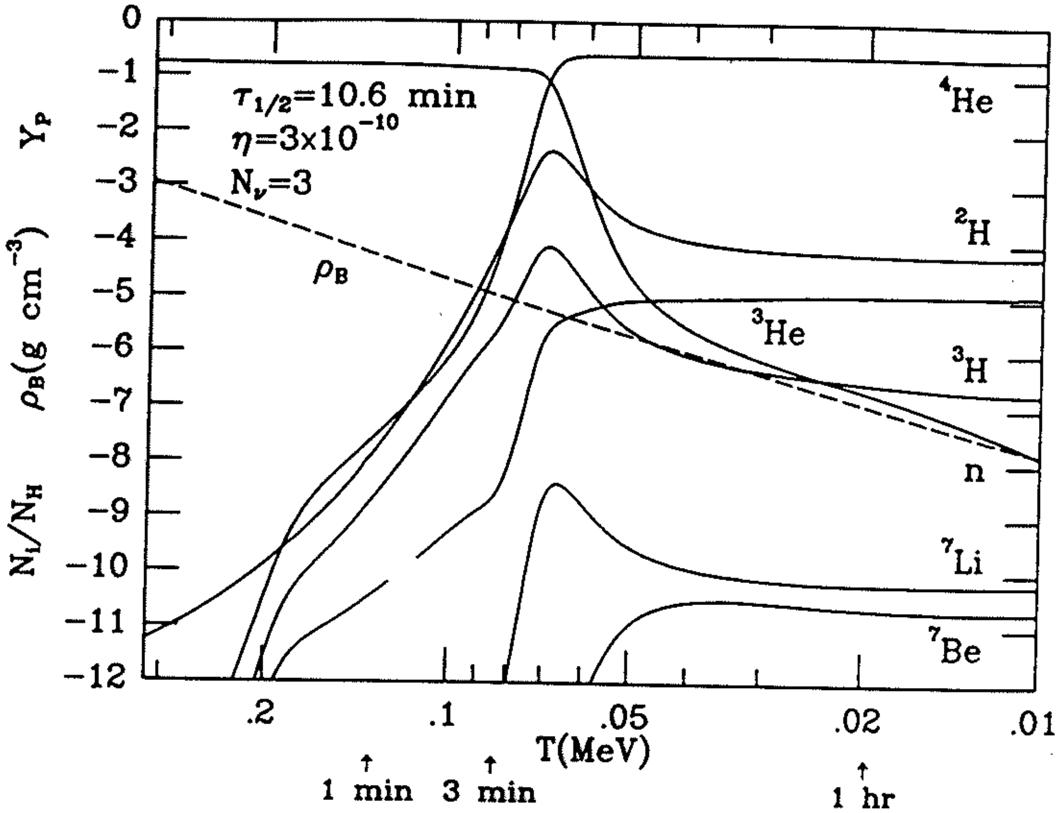


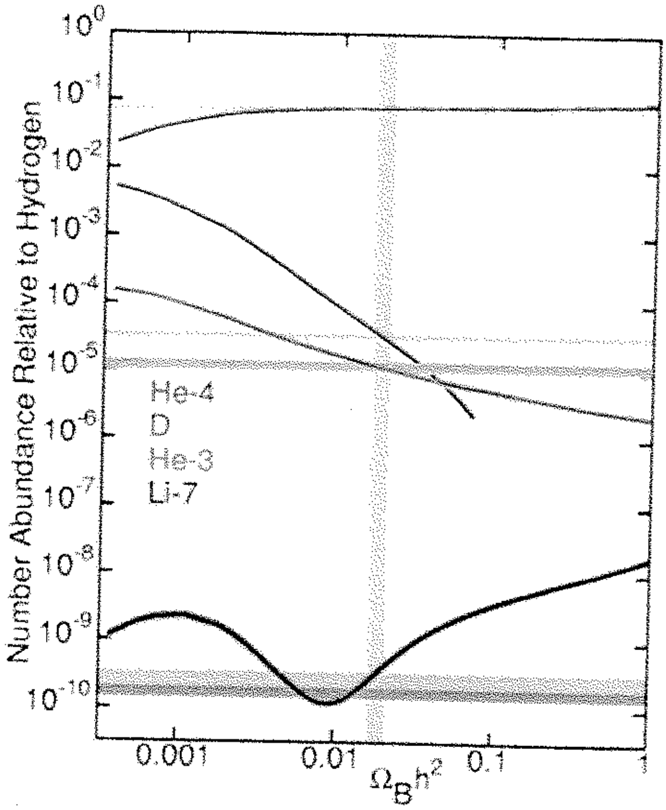

Fig. 4.3: The development of primordial nucleosynthesis. The dashed line is the baryon density, and the solid lines are the mass fraction of $^4$He, and the number abundance (relative to H) for the other light elements.

Kolb & Turner book.

Other elements:

heavy, $A \geq 12$ - massive stars

light, B, $Be^6$, Li - cosmic ray spallation processes.

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