# Formulary

**1)** *Fundamental constants*

$$\begin{aligned}
\hbar &= 1.05 \cdot 10^{-34}\,[\mathrm{Js}] \\
k_b &= 1.38 \cdot 10^{-23}\,[\mathrm{J/K}] \\
c &= 3 \cdot 10^{8}\,[\mathrm{m/s}] \\
G &= 6.67 \cdot 10^{-11}\,[\mathrm{Nm^2/kg^2}] \\
e &= 1.6 \cdot 10^{-19}\,[\mathrm{C}] \\
M_{Pl} &= 2.18 \cdot 10^{-8}\,[\mathrm{kg}] = 1.2 \cdot 10^{19}\,[\mathrm{GeV}]
\end{aligned}$$

**2)** *Relations between units*

$$\begin{aligned}
1\,[\mathrm{GeV}] &= 1.6 \cdot 10^{-10}\,[\mathrm{J}] = 0.51 \cdot 10^{14}\,[\mathrm{cm^{-1}}] \\
&= 1.5 \cdot 10^{24}\,[\mathrm{s^{-1}}] = 1.2 \cdot 10^{13}\,[\mathrm{K}] \\
&= 1.8 \cdot 10^{-24}\,[\mathrm{g}] \\
1\,[\mathrm{g\ cm^{-3}}] &= 4.3 \cdot 10^{18}\,[\mathrm{eV^4}] \\
1\,[\mathrm{pc}] &= 3 \cdot 10^{16}\,[\mathrm{m}]
\end{aligned}$$

**3)** *Cosmological parameters*

$$\begin{aligned}
H_0 &= 70\,[\mathrm{km\ s^{-1}\ Mpc^{-1}}] \\
\rho_c &= 0.9 \cdot 10^{-26}\,[\mathrm{kg/m^3}] = 4 \cdot 10^{-47}\,[\mathrm{GeV^4}] \\
M_\odot &= 2 \cdot 10^{30}\,[\mathrm{kg}] = 1.1 \cdot 10^{57}\,[\mathrm{GeV}]
\end{aligned}$$

Age and current temperature of the Universe :

$$t_0 = 14 \cdot 10^9\,[\mathrm{a}],\ T = 2.73\,[\mathrm{K}].$$

Age and temperature of the Universe at the time of equality between matter and radiation densities :

$$t_{eg} = 1400(\Omega_m h^2)^{-2}\,[\mathrm{a}],\ T_{eg} = 5.5\,\Omega_m h^2\,[\mathrm{eV}],$$
$$\Omega_m = 0.3,\ h = 0.7.$$

**4)** *Thermodynamic*

Non relativistic :

$$\begin{aligned}
n &= g\left(\frac{mT}{2\pi}\right)^{3/2} \exp\left(-\frac{m-\mu}{T}\right) \\
s &= n\left[\frac{5}{2} + \frac{m-\mu}{T}\right]
\end{aligned}$$

Ultra relativistic :

$$\begin{aligned}
\rho &= \frac{\pi^2}{30} g_* T^4 \\
n_b &= \frac{\zeta(3)}{\pi^2} g T^3 (\mathrm{bosons}) \\
n_f &= \frac{3\zeta(3)}{4\pi^2} g T^3 (\mathrm{fermions}) \\
s &= \frac{2\pi^2}{45} g_* T^3
\end{aligned}$$

**5)** *Christoffel's symbols*

$$\Gamma^\gamma_{\alpha\beta} = \frac{1}{2} g^{\gamma\delta}(\partial_\alpha g_{\beta\delta} + \partial_\beta g_{\alpha\delta} - \partial_\delta g_{\alpha\beta})$$
$$R^\alpha{}_{\beta\gamma\delta} = \partial_\gamma \Gamma^\alpha_{\beta\delta} - \partial_\delta \Gamma^\alpha_{\beta\gamma} + \Gamma^\alpha_{\mu\gamma}\Gamma^\mu_{\beta\delta} - \Gamma^\alpha_{\mu\delta}\Gamma^\mu_{\beta\gamma}$$
$$R_{\alpha\beta} = R^\mu{}_{\alpha\mu\beta} \qquad R = g^{\alpha\beta} R_{\alpha\beta}$$

**6)** *Friedmann's equations*

$$\begin{aligned}
\frac{\dot{R}^2}{R^2} + \frac{k}{R^2} - \frac{\lambda}{3} &= \frac{8\pi G}{3}\rho \\
2\frac{\ddot{R}}{R} + \frac{\dot{R}^2}{R^2} + \frac{k}{R^2} - \lambda &= -8\pi G p
\end{aligned}$$

**7)** *Structure formation*

Jeans mass and length :

$$M_J = \frac{4}{3}\pi\rho_0 \left[\frac{\pi v_s^2}{G\rho_0}\right]^{3/2}$$
$$k_J = \frac{\sqrt{4\pi G\rho_0}}{v_s}$$
$$\lambda_J = \frac{2\pi}{k_J}$$